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SPECIAL REPORT

Some aspects of fluid flow—Conference at Leamington Spa, 1950

The Institute of Physics held a conference on "Some aspects of fluid flow" at Ashorne Hill, Leamington Spa, from 25–28 October, 1950. The conference opened with a survey of industrial problems and this was followed by groups of papers on fundamental problems, techniques, and applications of present knowledge and techniques. Five discussion groups dealing with the following subjects met in the afternoon: (1) combined flow of fluids and solids; (2) fundamentals of mixing and flow patterns; (3) flow problems in industries employing high temperature furnaces (steel, glass, etc.); (4) flow problems in the process industries (gas, oil, chemical, etc.); (5) flow problems in the industries based on steam generation (power industries). The Chairmen of these groups presented reports to a final session. After the presentation of the reports from the discussion groups the conference concluded with a summing up of its findings by a member of the Organizing Committee. A number of films was shown during the conference. The papers and the discussions are summarized in this report and the full proceedings will be published shortly.

The aim of the conference was outlined by SIR CHARLES GOODEVE, Director of the British Iron and Steel Research Association, in his opening remarks as being the need to bring together those who have been developing the fundamental aspects of the science of aerodynamics and those who are studying the many industrial systems the performance of which is largely governed by fluid flow. An apt example of a practical fluid flow problem was, he said, the mixing of a custard powder, where the shearing forces necessary to disperse the particles cannot be obtained after the mixture is diluted more than a definite amount.

The first session of the conference was largely devoted to a series of reports on the results of a questionnaire which had been circulated to a wide variety of industries to ascertain the problems of fluid flow with which they were concerned. The replies were grouped according to the type of problem involved under the headings of (i) turbulent mixing, (ii) combined flow of fluids and solids, (iii) pattern of fluid flow, and (iv) hydromechanics; authorities on these subjects prepared reviews of the problems and the available knowledge under these headings. For the purposes of this summary it may, however, be preferable to depart from the strict arrangements of the conference and attempt to classify the whole subject matter under different main headings, namely: (i) turbulent flow in complex chambers, (ii) combined flow of fluids and solids, and (iii) miscellaneous problems.

TURBULENT FLOW IN COMPLEX CHAMBERS

The heading "Turbulent flow in complex chambers" is introduced to denote those flow problems where the direction of average flow is not substantially constant across the vessel as it is in pipes, nor can the effects of the walls be neglected as they are in wind tunnel studies of the flow round aerofoils. It also includes systems where the average flow direction is constant, but where two or more streams are mixing. In such appliances as industrial furnaces and the combustion chambers of boilers there may be a fairly well-defined three-dimensional flow pattern with fluctuations, or there may be instability of a considerable part of the flow system. One is, therefore, concerned with both mean flow conditions and deviations from them. A very attractive technique for presenting the flow direction pattern was demonstrated by DR. J. H. CHESTERS, MR. R. S. HOWES and MR. I M. D. HALLIDAY, who used a three-dimensional model with sets of curved wires of three different colours to represent unburnt fuel, air and combustion gases respectively. If this type of model can be developed to include the representation of flow velocity as well as of flow direction, for example by using wire of different diameters or adjusting the density of wires at right-angles to the flow direction, then it will be of very great value for demonstrating the results of investigations into the flow in complex chambers.

Techniques described at the conference as having been used to evaluate the flow pattern in such systems, included water models (by DR. CHESTERS, MR. HOWES and MR. HALLIDAY, and also by MR. M. P. NEWBY), and air models (also by MR. NEWBY); cinematography and the static photography of rounded aluminium particles and air bubbles have also been used by DR. CHESTERS and his colleagues to give a record of the direction of flow in water models. Slit illumination is used to enable the flow on any plane cross-section of the model to be observed. Velocity components can be determined from the lengths of the streaks photographed with a known exposure time. The same authors also describe the use of wood blocks inserted into actual hot furnaces to make the flow direction visible by the resulting flame. During the group 2 discussion (Chairman, MR. D. O. OLIVER), DR. CHESTERS described his work on an improved furnace design resulting from these model experiments.

MR. W. A. SIMMONDS described the Schlieren technique as applied to the photography of the flow patterns of a

stream of heated air and also the use in the Gas Research Board's laboratories of very fine solid particles to make visible the flow of air into a Bunsen flame.

There was some discussion at the conference on the appropriate similarity laws for such models, although not perhaps as much discussion as their extended use would justify. The Reynolds criterion could ensure that the ratio of momentum to viscosity was correct, but provided this exceeded a certain minimum value it could probably be ignored where some other criterion was inconsistent. One particularly important problem was the similarity criterion for a model of a furnace with a fluid atomized oil burner where the density ratio of fuel stream and combustion air was not the same as that of the air or water used to represent both streams. Dr. Chesters and his colleagues proposed the use of equal ratios of momentum of air entrainment to fuel port Reynolds numbers, as between the oil flame and a corresponding gas flame of equal calorific input, while Mr. Newby, Mr. R. D. Collins and Mr. J. A. Leys proposed the use of the assumption that the flow pattern is independent of the density ratio but depends on the momentum flux ratio of the two streams. In this connexion Mr. Newby mentioned the study of the flow pattern in a cold open hearth furnace with the oil flame replaced by a compressed air jet.

In addition to the problem of determining the flow pattern in complex chambers the study of the rapidity of mixing of two streams or, more usually, of the entrainment of one fluid by a jet of another is of considerable practical importance. Mr. M. W. Thring and Mr. Newby quoted Hinzes' conclusions that on the theoretical side the earliest assumption, that of Boussinesq, that there is an eddy viscosity and an eddy diffusion each of which is very much greater than the corresponding molecular property, was more fruitful in explaining the experimental results than any more recent and more elaborate hypothesis. A jet in free space could be accurately described as a cone of constant angle inside which there was uniform turbulence and which had a similar velocity profile along its length, but as soon as walls were introduced the theory was inadequate and model studies were necessary. Mr. Newby described a technique for measuring mixing contours on an air model using carbon dioxide or methane as tracers in one air stream and passing the samples through an infra-red gas analyser. In discussion groups 2 and 3 (Chairman, Dr. E. Seddon) there was considerable discussion on the effect of various steel and glass furnace port designs on the mixing rate of fuel and air; by port design is meant the shape of the part of the furnace chamber in which the fuel and air streams flow just before and just after they begin to mix. So-called premixing chambers or parts of the furnace in which some mixing takes place but which are separated from the main melting chamber by a constriction or throat, are in use in glass tanks where the glass is melted by several flames across the width of the furnace, but are avoided in steel furnaces where there is a single flame running the full length of the furnace. It was agreed that the ideal mixing rate is that giving the shortest flame which just does not overheat the roof at the ingoing end and that in practice the flame is usually considerably longer than this. The use of models for studying the gas flow in water tube boilers was decided at discussion group 5 (Chairman, Dr. R. S. Silver) to be complicated by the presence of large temperature differences.

The technique of using radon discussed by Mr. R. Mayorcas and Mr. K. P. Perry may also be appropriately mentioned under this heading, as although it can be used for metering flow quantities it can also be used for measuring mixing contours by injecting it into one stream before it begins to mix with another or the self-mixing of a fluid stream by injecting it at a point and measuring the distribution on a cross-section downstream.

The distribution of air flow across the natural draught cooling towers of a power station is a matter of considerable importance and was discussed by group 5. At first sight one might expect the greatest flow to be near the walls since the air enters under the base of the wall, but in actual fact the reverse appears to be the case; the best form of model technique for studying this flow was discussed, and it was possible to make the definite recommendation that a cold model with fan induced flow could be used provided a packing could be introduced which would have approximately the same relative resistance distribution as the full-scale packing.

To summarize the gaps in knowledge thrown up in this field, considerably more is required to be known about the best approximations for obtaining similarity in cold models to combustion systems, especially those with fluid atomized oil burners and those with appreciable temperature changes, and the development of a theory to deal with jet entrainment and mixing in the presence of surrounding walls is greatly needed. Methods of controlling the course of mixing along a stream to any given history would be of great value in connexion with the optimum utilization of luminous radiation.

THE COMBINED FLOW OF FLUIDS AND SOLIDS

The title of this section is taken from Mr. R. L. Brown's review of industrial problems covering gas cleaners, fluidized reactors, pneumatic and hydraulic conveyors, coal and ore washers, and combustion and gasification apparatus, which was one of the opening papers of the conference. The title can appropriately also cover the papers by Mr. P. G. W. Hawksley, Dr. H. E. Rose and Dr. R. A. Mott, discussion group 1 (Chairman, Mr. Brown), and a cinema film of a blast furnace model shown by Mr. Newby. The subject was appropriately opened with a slide of a sixteenth-century ore-washing plant indicating that industrial practice

was fully fledged at that date! MR. BROWN emphasized that this type of flow can be regarded as a "spectrum" of systems ranging from fluids carrying solids in very dilute suspension to tightly packed beds of solids through which a fluid is forced. The most important example of the first extreme is the cleansing of dusty gases in Cyclones and other air classifiers (usually centrifugal); in all these, the laws governing the relation between the patterns of gas flow and the relative motion of the particles require further study.

Fluidized beds, which are so much in fashion at the moment in the industrial world, represent another "colour" along the "spectrum." These beds give very rapid mixing between all particles and all the gas in a chamber and hence very rapid reaction rates, but lose the thermodynamic advantage of the counterflow system. The flow of gases and particles in fluidized beds can be described by a "phase diagram" with gas velocity and pressure drop as co-ordinates and different types of flow as phases. Theoretical treatments of these systems are not at all adequate yet.

Next come gravity concentration systems in which a fluidized system is used to act as a liquid with a density which can be adjusted to lie between that of the constituents to be separated. The operation of these systems was nicely illustrated by a cinema film introduced by MR. BROWN. The application of experimental studies of corresponding systems in this field should provide important developments.

The flow of fluids through packed beds can be divided into shallow beds (e.g. in boiler and furnace fuel beds), where accidental blowholes can be of great importance, and deep beds (blast furnaces and gas producers), where the method of feeding at the top determines the top configuration and may result in channelling in areas where large particles move downwards. MR. NEWBY'S blast furnace model films, using water and soda particles, demonstrated clearly how the flow pattern in packed beds is entirely different when the particles are loosened by consumption, to that when they remain constant in size.

MR. HAWKSLEY developed a theoretical formula for the rate of settling of suspensions. Particles tend to assist each other's fall if there is adequate space elsewhere for the return flow of the fluid dragged down with them, but if the cloud fills the whole chamber they fall more slowly than would a single particle. The equation relates this reduction in settling rate to the concentration and gives reasonable agreement with the experimental values. MR. HAWKSLEY also applied the same equation to the flow of fluids through a packed bed, replacing the driving force of the weight of the particles by that of the externally applied pressure drop, and thus obtained a theoretical evaluation of the constant in the normal pressure drop formula.

DR. ROSE applied dimensional analysis to the relation between the pressure drop in a bed of particles and the independent variables such as flow velocity and particle size, and deduced that head of fluid lost over bed divided by particle size d is equal to the product of five functions of, respectively, Reynolds number, depth of bed/d, container diameter D/d, voidage, and Froude number. He then discussed the form of these five functions in the light of available experimental data; as far as D/d is concerned the wall effect vanishes when D/d exceeds 50; the effect of voidage is complex and no reliable formula is available (this may be where MR. HAWKSLEY'S more basically theoretical approach can assist). The function of Reynolds number is the main effect and it is shown by logarithmic plots.

DR. MOTT'S treatment of the same subject is like that of MR. HAWKSLEY in so far as he relates the resistance to flow through a bed to the free falling speed of a single particle. Like Dr. Rose he uses a logarithmic plot of the resistance coefficient against the Reynolds number, but he is more interested in the coarser beds, considering even cases where the bed diameter is only three times that of the particles. Under these conditions the effect of the parameter D/d is clearly large and DR. MOTT gives a number of experimental results for this effect.

In discussion group 1, MR. W. H. DENTON described some further work on packed beds of spheres at Reynolds numbers up to 100 000 and gave a qualitative explanation of the hump in the curve of resistance coefficients against Reynolds numbers based on the curve for flow past a single cylinder or sphere. The form of the variation with voidage was discussed and a theoretical treatment was proposed to give this variation. It was suggested that the Stokes' diameter should be used universally as the definition of size of irregular particles.

In summarizing the discussion of group 1, MR. BROWN pointed out that information is still lacking on the trajectory of irregular particles in non-rectilinear fluid flow, on the changeover criterion between separation by density and by size in a dense suspension, and on the effect of accidental channels on fluid flow in shallow packed beds.

EJECTORS, PUMPS, SHIPS, ORIFICES, ATOMIZATION AND BOUNDARY LAYERS

It seems to be impossible to find a single grouping for the remainder of the subjects discussed at the conference and they are accordingly dealt with here piecemeal.

There was considerable discussion on the theory of ejectors based mainly on the paper presented by MR. R. A. SMITH and on an earlier one by DR. SILVER. These papers derive the behaviour of ejectors under various conditions from the application of Bernoulli's theorem and MR. SMITH gave some new experimental data for air ejectors and steam-driven water ejectors which confirms his formula well, except at high entrainment rates. In discussion group 2, DR. SILVER pointed out that this type of treatment is satisfactory for ejectors

with a low pressure-recovery ratio but exaggerated their efficiency when a high recovery is aimed at. MR. N. H. JOHANNESEN pointed out that this is due to the presence of shock waves, so this became one of the points of contact between the industrial problems of this conference and the problems of the aircraft physicist. In the discussion on ejectors (group 4; Chairman, MR. SIMMONDS) MR. G. H. BYGRAVE described some experiments on jet pumps giving efficiencies varying between 2 and 25% and showing that with these liquid-liquid ejectors the ratio of quantity of entrained to entraining fluid fell to zero where the latter quantity was reduced to zero.

MR. H. L. GREEN discussed the theories of swirl chamber, air blast and centrifugal types of so-called atomizers, i.e. apparatus for breaking up liquids into very fine sprays. The first is used in agricultural spraying, oil firing of some furnaces and of gas turbines and internal combustion engines; the fluid is forced tangentially into a chamber and leaves through an orifice of small diameter on the axis. Taylor has developed a simple boundary layer theory for such systems which shows that there is a main body of fluid with a constant circulation and a boundary layer retarded by viscosity. This retardation deprives this layer of the necessary centrifugal force so that it flows along the boundary inwards to the orifice. The fluid leaving comes from this boundary layer and that of the hollow core. This theory shows that a swirl chamber to produce a $10\,\mu$ thick boundary layer and hence $10\,\mu$ droplets would require an impracticably small hole. The break-up of the issuing sheet is not yet sufficiently understood. With air (or steam) blast atomizers which are used in many furnaces much more energy is available and finer droplets can be obtained, although they are very inefficient in using this energy. A study of high-speed photographs has shown that ligaments are torn from the stream of liquid and subsequently break into drops, but the wide size range found in the droplets is not understood. Centrifugal atomizers can give a very uniform drop size, the lowest obtainable size being $10\,\mu$ for an air-driven top. The value of the attempts to develop basic theories for these systems was clearly brought out by MR. GREEN.

MR. H. E. DALL gave a very fascinating qualitative physical explanation of the characteristic curves of various orifices and nozzles. In particular he showed how the hump in the characteristic curve for an orifice or nozzle would mean that a more viscous fluid such as a light oil would empty out of a can through a hole of the right size more rapidly than a less viscous fluid such as petrol. Also, with the light oil the can could empty more quickly with a hole of the right size punched inwards than one of the same size punched outwards, because the greater surface for viscous drag in the latter case more than offset the smaller area of the *vena contracta* in the former case. One would like to see these qualitative explanations made quantitative by suitable calculation, although this would probably have to use numerical methods such as that of relaxation. In discussion group 4 MR. DALL made further reference to the advantages of orifices of small area ratio.

MR. L. E. PROSSER and MR. R. C. WORSTER dealt with two illustrations of the general problem of designing hydraulic equipment—the centrifugal pump and the pipe line. In these fields the application of mathematical theory has in the past been more extensive than that of experimental physics, but it was suggested that the latter should increase in order to provide a firmer basis for the assumptions of the former. The problem in pumps is essentially that of flow at high speeds within complex boundaries and although high efficiencies can already be obtained in any given pump, the reasons for the optimum design are not very fully understood and cavitation and gland seals present further problems. The chief lack of knowledge about friction in commercial pipes relates to the wall roughness effects, and no simple method of assessing the roughness of a pipe is available to avoid carrying out a pressure drop measurement. Cavitation on propeller blades was also discussed in group 5 and it was concluded that measurements of the physical conditions causing the bubbles could be of great value.

MR. J. F. O'CONNOR described the measures which have been found necessary to avoid laminar flow on the wax models used in ship tanks, a type of flow which does not correspond to the full-scale system. A trip wire on the model which ensured turbulence is found to increase the resistance up to 10%, and such a wire should be used. Methods of indicating the extent of turbulence in the flow are desirable—ink stream released into the boundary layer has been found useful for this purpose. This paper provoked considerable discussion, many new suggestions for ship model experiments being put forward, although DR. J. C. CONN pointed out that a proposal to use the heat transfer friction analogy to study friction was hardly practicable with wax models!

The last paper to be mentioned in this summary was particularly valuable at the conference because it brought a breath of the upper atmosphere of aerodynamics into the more earth-bound atmosphere of many of the other papers. This was a contribution by PROF. A. D. YOUNG on the present state of our knowledge, particularly theoretical knowledge, of the behaviour of boundary layers in compressible flow. In this type of flow there is a temperature boundary layer as well as a velocity boundary layer; in these regions and inside shock waves viscous and heat conduction effects are important, elsewhere they can be neglected. Also in such flow, small changes in the boundary affect the whole flow and not only the boundary layer. A full boundary layer theory will show effects of the Mach number, Reynolds number, Prandlt number, the exponent in the viscosity-temperature relation, and the ratio of wall temperature to free stream temperature, so that numerical computation with the most complex modern machines seems the most helpful way of solving the differential equations.

On the experimental side data on the velocity distributions in the boundary layers are highly inadequate. PROF. YOUNG also discussed boundary layer shock-wave interaction.

CONCLUSION

The conference concluded with a summary by the author of this report, in which it was suggested that the conference showed that industrial design of fluid flow systems of the types discussed was steadily being supplemented by the development of a "fluid flow physics." There was still, however, considerable need for a master hypothesis on which a general theory could be developed analogous to the dislocation hypothesis on which so much of metal physics is based. The argument was illustrated by a diagram purporting to show the relation between engineering and physics in such a field.

It is, of course, impossible in such a short summary to do justice either to the subject matter or to the conclusions of the conference, and it is hoped that readers will obtain the full *Proceedings*.

M. W. THRING

The proceedings of the conference, containing the fifteen contributed papers, the summaries of the discussions held by the five groups and the summing up by Mr. Thring, are being published for The Institute of Physics by Messrs. Edward Arnold and Co., of 41 Maddox Street, London, W.1, to whom orders should be sent.

ORIGINAL CONTRIBUTIONS

An apparatus for studying the laws of the magnetic field due to an electric current in a long straight conductor

By DAVID OWEN, B.A., D.Sc., F.Inst.P., London

[*Paper first received* 29 *August*, 1950, *and in final form on* 26 *September*, 1950]

It is impracticable to measure directly the magnetic field due to an electric current in a single long straight conductor, since a complete circuit is necessary. By using a narrow rectangular coil the field is, within reasonable limits of accuracy, the same as that due to the two long sides regarded as infinite in length. The formulae for the field are simple for points along two axes relative to the coil, provided the points are not too far removed from its centre. The linearity of suitable graphs of a series of observations constitutes a verification of the law derived for a single straight conductor, and from the graphs values of the horizontal component of the earth's magnetic field can be determined. Two experimental methods, involving the location of neutral points, are described.

This paper describes a simple form of circuit for verifying the laws of the magnetic field due to a current in a long straight conductor, and methods of making the magnetic measurements. Usually the attempt is made to test directly the expression $2I/10r$ for the field strength H at a point r cm, from the axis of a long wire carrying a current I in amperes, by using a roughly square vertical circuit about a metre in side. Observations being confined to points near the middle of one of the vertical sides, it is assumed that the fields due to the far vertical side and the two horizontal sides can be neglected. This is, however, far from justifiable. Let us suppose, for the size of circuit specified, that the working point is in the plane of the coil and 5 cm from the centre of one vertical side. The error of neglecting the field due to the opposite vertical side is then about 2·5%; but that due to neglecting the effect of the horizontal sides is much more serious, amounting in this instance to over 10%, so that the total error incurred is over 12%.

The attempt to determine the magnetic field of the current in a single long straight conductor is in fact impracticable. The errors above indicated can be obviated by employing a narrow rectangular coil with the long sides vertical. Consider a coil of n close turns of breadth $2a$ and height $2b$, carrying a current of I in amperes. The magnetic field at the centre of the rectangle is exactly expressed by the equation $H = \frac{4nI}{10a}\sqrt{1 + \frac{a^2}{b^2}}$. If $a^2/b^2 = 1/50$ the actual field at the centre differs from that due to two infinite parallel wires at the same distance apart by only 1%, which may be taken as within the limits of ordinary experimental error. This fact forms the basis of the apparatus here proposed. In practice a coil of width 10 cm and height 70·7 cm ($50\sqrt{2}$) has been found suitable.

It will be shown that the field due to such a coil may be investigated simply and accurately along two lines in the horizontal plane midway up the coil: the first line (the x-axis) being in the plane of the coil, the second

line (the z-axis) at right angles to the plane of the coil and through its centre.

Two experimental modes of procedure may be adopted. In the first the coil is set perpendicular to the magnetic meridian, and neutral points in the combined field of the coil and the earth's horizontal field are located. In the second method the plane of the coil is turned about a vertical axis through a small angle, of 5° or a little less, from the position occupied in the first method. In this case points are located which virtually coincide with neutral points, so that the formulae to be employed in both methods of procedure are identical.

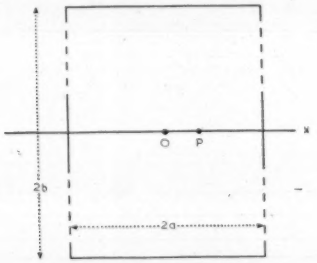

Fig. 1. The coil, and the x-axis

Consider the first method, applied to find the field at a point on the x-axis, at x cm from the centre O of the coil (Fig. 1). For current in one direction the field due to the coil at a point on that axis and between the wires

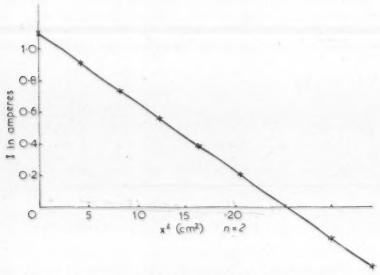

Fig. 2. Graph of observations on the x-axis

is opposite to the earth's field. Since this field is to be regarded as due to the current through two infinitely long wires at the distance $2a$ apart, we have

$$H = \frac{2nI}{10(a-x)} + \frac{2nI}{10(a+x)} = \frac{4naI}{10(a^2-x^2)} \qquad (1)$$

The same formula applies to points on the x-axis but outside the coil, bearing in mind that the sign of I is reversed.

Neutral points occur at places where H is equal and opposite to the earth's horizontal field H_0. At such points, therefore,

$$H_0 = \frac{4naI}{10(a^2-x^2)} \qquad (2)$$

which may be written

$$I = \frac{10H_0a}{4n} - \frac{10H_0}{4na} \cdot x^2 \qquad (2')$$

On plotting observed values of the current I as ordinates against x^2 a straight line will be found (Fig. 2), and by measuring the intercepts on each axis a value of H_0 can be determined. Alternatively if a is measured directly H_0 can be deduced from (2′) using the intercept on the I-axis.

The neutral points can be accurately located by the method of directional loci* in a simplified form. The first locus, that of points at which a compass needle sets magnetic north or south, is obviously the x-axis. The second locus, that of points at which the needle sets magnetic east or west, intersects the first locus at a neutral point. Thus all that is required to locate that point is to place the compass near the x-axis, say with its centre half a diameter away, and to slide it parallel to the axis until the needle sets perpendicular to that axis. The needle then points to a neutral point.

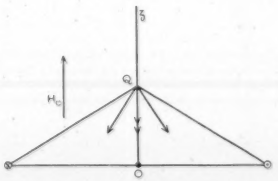

Fig. 3. Field on the z-axis

It will be noted that the neutral points occur in pairs for any value of the current, being at equal distances on either side of O. They coincide at O when the current rises to a certain critical value, thus giving a "double neutral point." If the current is increased beyond this value the neutral points of a pair separate out along the z-axis, at distances z from O (see Fig. 3). In this case the field at Q due to the coil will be seen to be

$$H = \frac{4nI}{10\sqrt{a^2+z^2}} \cdot \frac{a}{\sqrt{a^2+z^2}} = \frac{4naI}{10(a^2+z^2)} \qquad (3)$$

* OWEN. *Proc. Phys. Soc.*, **57**, p. 294 (1945).

and at a neutral point

$$H = H_0 = \frac{4naI}{10(a^2 + z^2)} \qquad (4)$$

or

$$I = \frac{10H_0 a}{4n} + \frac{10H_0}{4na} \cdot z^2 \qquad (4')$$

The type of graph obtained in observations of neutral points on the z-axis is shown in Fig. 4. The points lie

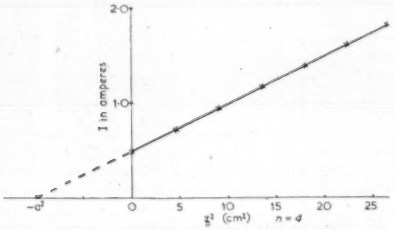

Fig. 4. Graph of observations on the z-axis

on a straight line passing (when produced) through the point $I = O$, $z^2 = -a^2$, and again from the graph a value of H_0 can be determined.

Consider next the second mode of experimentation, when the plane of the coil is turned through a small angle α. Suppose the field is being investigated at points along the x-axis (Fig. 5). If a compass needle is placed

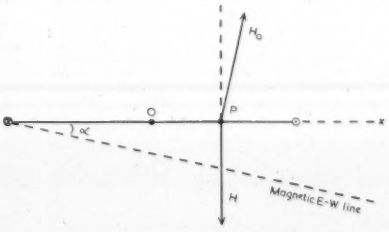

Fig. 5. Use of formula $H = H_0 \cos \alpha$

with its centre at P on that axis the needle will set along that line provided $H = H_0 \cos \alpha$, which, since α is small, is practically equivalent to $H = H_0$. The points P thus located are practically indistinguishable from neutral points, and therefore equations (2) and (2′) again apply.

Similarly equations (4) and (4′) apply when the points at which the compass is placed lie on the z-axis, and the compass sets perpendicular to that axis.

It is important to remark that this second method is of quite general application, and that the deflecting field $H_0 \sin \alpha$ remains the same, whatever the value of x or z, provided α is the same. In experiments such as those here described this method has the advantage over the first method, since at and very near double neutral points the method of directional loci becomes insensitive. In general, by either method, a neutral point can be located within half a millimetre.

The field at any point on the x- or z-axis of the coil may also, of course, be investigated, if desired, with the oscillation magnetometer or the deflexion magnetometer; the plane of the coil being set perpendicular to the magnetic meridian in oscillation experiments, and parallel to it in deflexion experiments.

Some details of the experimental procedure adopted, and examples of measurements, may be of service. A sheet of squared paper, preferably ruled in cm and mm, was placed on the platform. By cutting two holes in the paper, about 1 cm in diameter and 10 cm apart and round each vertical side, the paper can be accurately adjusted so that the centre O of the coil coincides with the intersection of two of the thicker rulings, one parallel to the coil and the other perpendicular to it. The measurements of x and z can then be made directly on the paper. The electric circuit usually included a 4 V battery, a wire rheostat (with a carbon plate rheostat for fine adjustment), an ammeter, a reversing key, and the rectangular coil separated from the rest of the circuit by using twin flex leads about a yard long.

The coil was constructed of 4 close turns, and tappings made so that the central 2 turns may be used or all 4 turns. Using the 2-turn coil a series of values of I satisfying the condition $H = H_0$ was obtained at values $x = 0,\ 0{\cdot}5, \ldots$ up to $3{\cdot}5$ cm. A good straight-line graph of I against x^2 was plotted, from which the value $1{\cdot}07$ A was read as the intercept on the I-axis. The observed value of $2a$ was $9{\cdot}97$ cm, hence

$$H_0 = \frac{4 \times 2 \times 1{\cdot}07}{10 \times 4{\cdot}98} = 0{\cdot}172 \text{ oersted } (\pm 0{\cdot}002)$$

A second experiment was made at the same spot, for points on the z-axis, using the 2-turn coil for the lower values and the 4-turn coil for the higher values of z. The plotted points lay on a straight line, passing, when produced, through the point $I = 0$, $z^2 = -a^2$, and applying formula (4′) the value $H_0 = 0{\cdot}173$ oersted was determined. At the higher values of z the points are observed to deviate from the line slightly, though quite distinctly, the graph falling towards the axis of z^2.

Replica techniques in electron microscopy

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[*Paper first received* 1 *September*, 1950, *and in final form* 18 *October*, 1950]

Precautions for obtaining clean and reliable replicas for the study of large areas of metal surfaces are outlined. Modifications to the backed replica technique, and the application of preshadowing in metallographic studies are described.

An investigation was recently carried out on the various structures obtained by heat treatment of a low alloy steel. Plain, shadowcast, and preshadowed replicas were used, and the structures observed were compared with those seen under the optical microscope. In order to facilitate the comparison, the replicas were photographed at a magnification comparable with that obtainable on the optical microscope (× 2 000). In the electron microscope type E.M.3 by Metropolitan-Vickers Electrical Co. Ltd., a micrograph at this magnification corresponds to an area of the object 40 μ square. This area will normally include a number of different metal grains permitting examination of the general features of the metallographic structure. This helps to avoid ambiguous or misleading interpretations which may result from the examination of a portion of a single grain at higher magnification. These relatively large areas must be clean and free from artefacts if reliance is to be placed on the interpretation. In order to realize these conditions, it was found necessary to take stringent precautions throughout the replica processes and these are outlined below.

PREPARATION OF THE METAL SURFACE

Washing.—In experiments where a grease-removing wash was not used, the replicas frequently reproduced pseudo-structures which were later identified as due to grease on the metal surface. Fig. 1 shows a Formvar replica of a pearlite specimen which was grease-contaminated. A particularly bad patch was selected to illustrate the resultant artefacts, which include particulate contamination and a number of holes in the replica film. The grease itself is reproduced as structure which might be interpreted as carbide precipitates in the ferrite grains; their presence amongst the lamellae of the pearlite shows them to be artefacts. This micrograph may be compared with Fig. 3, which is a replica from the same metal specimen after it had been properly cleaned. After polishing, the specimens were washed successively in running water, distilled water and ether in absolute alcohol, and again in distilled water and ether in alcohol after etching. All these washes were found to be essential if clean specimens were to be obtained.

Mounting.—In order to avoid possible tempering, some martensite specimens were mounted in a low temperature thermo-setting plastic at 80° C or below. Replicas taken from these specimens, however, were always marred by small white dots (on the print) which might have been confused with tempering precipitates if these had not been known to be absent (Fig. 2). The phenomenon was never noted on unmounted specimens or specimens mounted in Bakelite, and it was concluded that the contamination was secreted in the small gap that was usually present between the metal specimen and the thermo-setting plastic. Subsequent martensite specimens were not mounted and other specimens were examined to ensure that the Bakelite was in good contact around the periphery of the specimen.

Cleansing replicas.—Despite the washing processes described above, quantities of small particles (possibly polishing dust) appeared on the replica surface. The surface was therefore cleaned further by successive stripping of thick Formvar films. Two such cleansing replicas were usually found adequate. It was, however, found to be extremely difficult to remove all contaminating particles from certain types of structure.

REPLICA PROCESSES

Negative Formvar replica.—The dry stripping technique has been used extensively for obtaining replicas of metallographic structures (Nutting and Cosslett).[1] When the specimens had been cleaned as described above, however, it was found to be exceedingly difficult to detach the Formvar films by this method. Nearly all the films that could be detached exhibited strain lines. It was noted that the presence of a very small amount of grease on the metal surface enabled the replica to be stripped with ease. It was therefore concluded that the method was inapplicable when proper precautions to ensure cleanliness had been taken. Formvar replicas backed with a thick collodion film were then employed (Cuckow and Trotter),[2] this system enabling thin replicas of high contrast to be removed without any evidence of strain in the Formvar (Figs. 3 and 4). After the composite film has been floated off the metal on to a water surface, the normal method of mounting on to specimen grids is as illustrated in Fig. 5(*a*). This method leaves the contour surface of the Formvar film in contact with the specimen grid. Shadowcasting, when required, must, therefore, be carried out through the grid bars, which are approximately 25 μ thick. Thus, even with a shadowing angle of 45°, only about 45 μ width of each

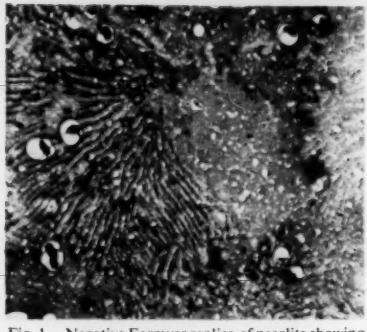


Fig. 1. Negative Formvar replica of pearlite showing artefacts due to grease contamination
Magnification × 2 600

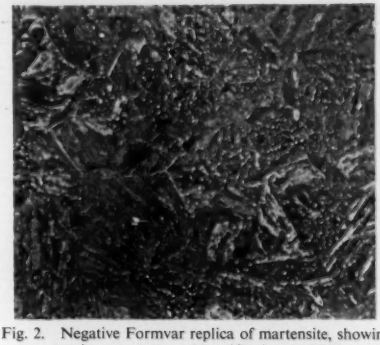


Fig. 2. Negative Formvar replica of martensite, showing spurious structure. Magnification × 2 000

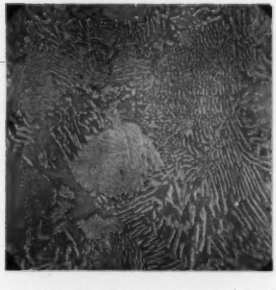


Fig. 3. Negative Formvar replica of pearlite.
Magnification × 2 000

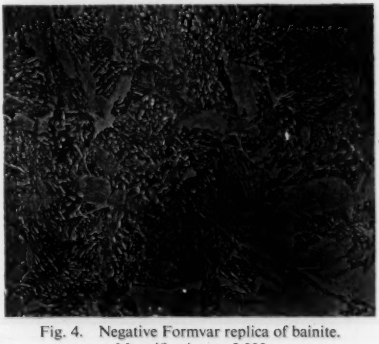
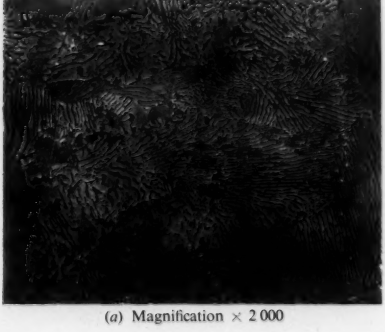
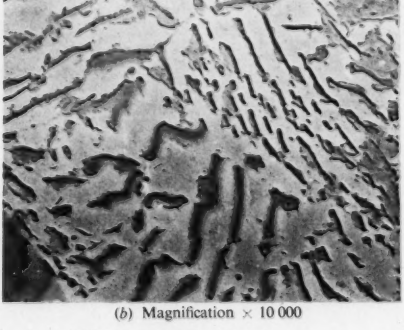


Fig. 4. Negative Formvar replica of bainite.
Magnification × 2 000



(*a*) Magnification × 2 000



(*b*) Magnification × 10 000

Fig. 6. Shadowcast Formvar replica of pearlite. 60% gold-platinum at cot−12.

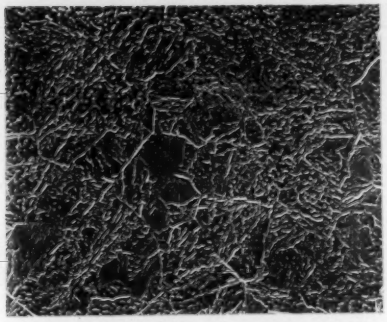


Fig. 7. Shadowcast Formvar replica of bainite.
60% gold-platinum at cot^{-1}2. Magnification × 2 000

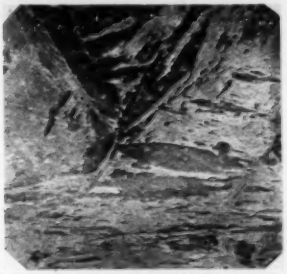
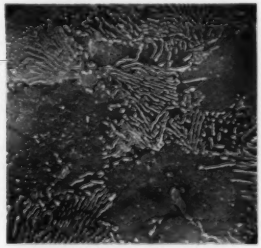
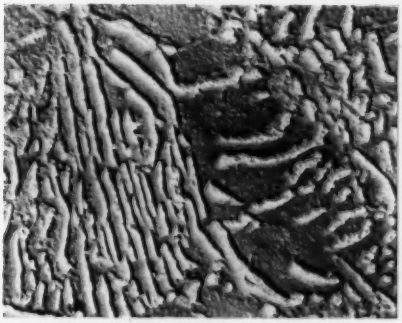


Fig. 8. Shadowcast Formvar replica of martensite.
60% gold-platinum at cot^{-1}2.
Magnification × 2 000



(*a*) Magnification × 2 000



(*b*) Magnification × 10 000

Fig. 9. Preshadowed replica of pearlite. 60% gold-platinum at cot^{-1}2

Fig. 10. Preshadowed replica of martensite.
80% gold-platinum at cot^{-1}2. Magnification × 4 000

grid aperture will be shadowcast. Since, for some very shallow structures a shadowing angle of $\cot^{-1}4$ is required, the mounting procedure was modified as illustrated [Fig. 5(*b*)]. The composite film is picked up from the water surface on a clean microscope slide; the collodion is dissolved away, and the clean Formvar film is floated off again on to the water surface. It can then be picked up on specimen grids in the normal manner. This leaves the contour surface outermost from the grid and the whole of the structure can now be shadowcast at any desired angle. This process has the added advantage that the film adheres securely to the grid, which was not always the case previously.

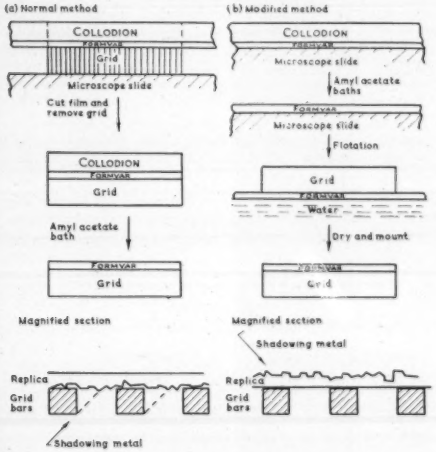


Fig. 5. Backed replica process subsequent to flotation on water surface

Shadowcast replicas.—Shadowcasting the replicas was found to improve the contrast considerably and to render visible a great deal more fine structure without confusing the interpretation of the main structures (Figs. 6, 7, 8). The metal used was a 40% palladium gold alloy evaporated at an angle of $\cot^{-1}2$. Since the resolution of the Formvar replica was reputed to be limited to structures coarser than 200 A, the preshadowed replica technique offered obvious advantages in the study of surfaces where fine structure may be present. This technique has been little used, presumably because of the difficulty of dry stripping the replica, when any metal other than gold is used. A palladium gold alloy was used as the shadowing metal in these experiments, and it was found that wet stripping by the process described above was quite successful (Figs. 9, 10). In the study of pearlitic steel specimens, a 40% palladium gold alloy was used as the shadowing metal. For metallographic structures such as bainite and martensite, it was not in general possible to strip 40% palladium gold in this way. When the gold content was increased to 80%, however, the shadowing metal could be completely removed from all such structures encountered. Normal beam intensities in the electron microscope cause no observable aggregation of the shadowing metal, despite the high gold content. Preshadowed replicas of a number of different structures prepared in this way show a striking increase in the amount of fine surface structure resolved, while reproducing the main structural details in excellent agreement with the Formvar replica. Measurements on some of the detail resolved by the preshadowed replicas, but not revealed by the Formvar replicas even when shadowcast [Figs. 6(*b*) and 9(*b*); Figs. 8 and 10] suggests that the limit of resolution of the Formvar replica may be slightly better than 200 A—probably of the order of 150 A.

CONCLUSIONS

(*a*) It is possible to obtain electron micrographs of surface replicas giving consistent and reliable interpretations. Even at the low magnifications required for a proper study of the structures, the particle contamination and artefacts can be kept negligible over the large areas required if proper precautions in the replica preparation are observed.

(*b*) Shadowcasting of the replicas yields additional information about the structures being studied without in any way confusing the interpretation. The modified replica technique described greatly increases the application of shadowcasting to replicas of this type.

(*c*) Preshadowing renders visible fine details of structure not resolved by Formvar replicas without obscuring the main structural features. No difficulty was experienced in the preparation when a wet stripping method was adopted.

(*d*) For a complete study of surface structure, it is recommended that all three replica processes are used.

ACKNOWLEDGMENTS

The authors wish to thank Sir Arthur P. M. Fleming, Director of Research and Education, and Mr. B. G. Churcher, Manager of the Research Department, Metropolitan-Vickers Electrical Co. Ltd., for permission to publish this paper.

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An air-flow analogy for the solution of transient heat conduction problems

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[*Paper first received* 6 *October*, 1950]

This method has been developed primarily for the solution of transient heat conduction problems in which the specific heat and thermal conductivity of the medium are functions of temperature. In these cases it constitutes an alternative to the more familiar resistance-capacity analogy since variations of specific heat and conductivity may be simulated automatically by a preliminary setting up procedure. The paper summarizes the principles of the method and gives a brief description of the small prototype unit so far constructed.

The use of analogy methods for the solution of complex heat conduction problems has received a good deal of attention in recent years. For transient problems a hydraulic analogy has been used by Moore,[1] and the electrical resistance-capacity analogy has been developed by a number of workers, the best-known installation being probably the very elaborate Heat and Mass Flow Analyser due to Paschkis.[2] The present writer's particular interest is in heat conduction problems in metals of which the specific heat and thermal conductivity may vary appreciably with temperature. These conditions complicate the working of the resistance-capacity network considerably because they necessitate progressive alteration of the circuit values during the course of a calculating run. This has to be done either by hand, involving a number of operators, or by means of a multiplicity of electronic control circuits. These complications are not beyond the scope of a large installation like that mentioned above, but they form a serious obstacle to the construction and operation of a simpler laboratory unit. For such cases the air-flow analogy to be described is put forward as a possible alternative. A small prototype was shown in London at the 34th Physical Society Exhibition in March, 1950, and this paper has been prepared in response to the inquiries received.

GENERAL DESCRIPTION

In common with the other methods mentioned above, the air-flow analogy involves a "lumping" procedure in which the second-order differential equation of heat conduction is replaced by a set of first-order equations. For the benefit of those unfamiliar with the process a physical interpretation is given below which illustrates very clearly the principle of the analogy. Fig. 1(*a*) shows the simple case of heat flow in a slab of constant thickness and infinite extent, over the surfaces of which the temperature is everywhere uniform though varying with time. Since the isothermals are parallel to the surfaces we may consider any arbitrary region of the slab, such as that bounded by the lines AB and $A'B'$. In the lumping procedure we imagine that this region is subdivided into a number of cells 1, 2, 3 . . . as shown, that the heat capacity of the material in each cell is represented by an equivalent heat reservoir at its centre, and that the heat-flow paths between the centres of adjacent cells are represented by equivalent conducting rods, as shown in Fig. 1(*b*). It is this latter system of heat reservoirs interconnected by conducting rods which is simulated by the air-flow system. How closely it represents the conditions in the original slab depends, of course, on the fineness of

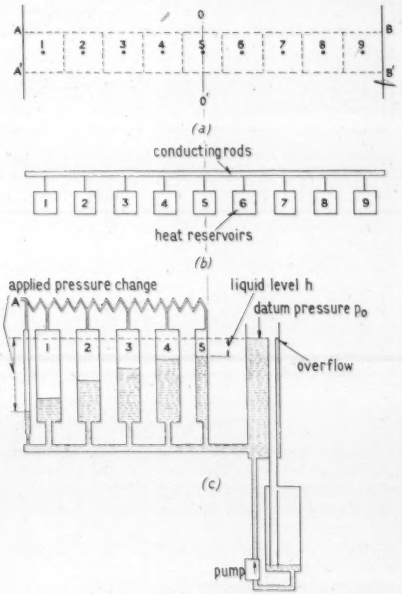

Fig. 1. (*a*) heat-flow problem; (*b*) effect of lumping; (*c*) air-flow analogue

the lumping, a subject which is discussed in a later section. The lumped system of Fig. 1(*b*), then, consists of heat reservoirs, the heat content of which is a function of the temperature, interconnected by conducting rods along which the heat flux is proportional to the temperature difference between the two ends. In the

air-flow analogy this system is simulated by the system shown in Fig. 1(*c*), which consists of air reservoirs, each containing a mass of air which is a function of the air pressure within it, interconnected by restricting ducts through which the mass air flow is proportional to the pressure difference between the ends. In this particular case the temperature distribution in the slab is symmetrical about the centre line OO' so that it is only necessary to simulate one half of the heat-flow system. In the model so far constructed the ducts are capillary tubes while each reservoir is, in effect, one limb of a U-tube with a containing liquid. The U-tubes may be independent or they may have a common second limb incorporating a constant level device as shown. Detailed examination of the conditions for equivalence of the heat- and air-flow systems is deferred until the next section, but if we regard air pressure as corresponding to temperature and mass of air to quantity of heat it will be seen at once that the equations describing the behaviour of the two systems are of the same form. The pressures in the apparatus are measured relative to the pressure (usually atmospheric) above the liquid surface in the common second limb of the U-tubes. The level of the liquid in a reservoir is thus itself an indication of the pressure of the air contained above it and may be read against a graduated scale. The scale reading corresponds directly to the temperature at the equivalent point in the heat-flow system, to which it is related by a

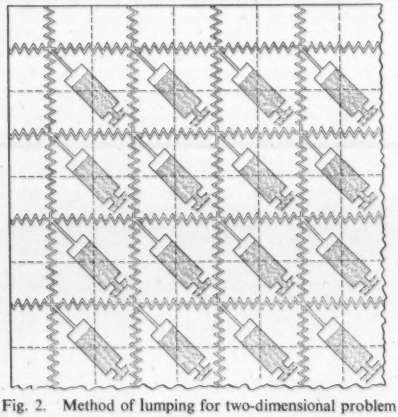

Fig. 2. Method of lumping for two-dimensional problem

conversion factor. Thus to study the temperature changes within the slab resulting from a sudden change of surface temperature (to take a simple example) the reservoir levels are first adjusted to correspond to the initial temperature distribution. The appropriate pressure change is then applied to the point A in the air-flow system which corresponds to the surface of the slab. The subsequent changes of the liquid levels in the reservoirs correspond exactly with the temperature changes in the heat reservoirs of Fig. 1(*b*), though the time scales for the two systems are in general different. It will be shown in the next section that, as with other analogy methods, by suitable choice of component dimensions in the air-flow system the time scale for the heat-flow system may be expanded or contracted at will to give a speed of operation which is convenient for taking readings. In this next section it will also be shown that, using a special form of relation between pressure and temperature the case of variable thermal conductivity and specific heat can be simulated by making each reservoir in such a way that its cross-sectional area is a function of the liquid level. The small model so far constructed has only six reservoir units and its use is therefore restricted to one-dimensional problems. Given more units the method can, of course, be extended to bodies of more complicated shape. Fig. 2, for example, illustrates the method of lumping and the connexions between the reservoirs in a two-dimensional problem.

FUNDAMENTAL PRINCIPLES

Preliminary.

The following symbols will be used in connexion with the air-flow system:—

h = The liquid level in a reservoir measured from a datum as shown in Fig. 1(*c*).

m = The mass of air contained in a reservoir when the liquid level is h (including the air contained in the dead space above the datum level and in the connexions to the capillaries).

p_0 = The absolute pressure of the air in the reservoir when $h = 0$.

d_0 = The density of the air at pressure p_0.

q = A factor converting units of h into units of p_0.

S = The cross-sectional area of the reservoir at level h.

v = The volume of the dead space, etc., above the datum level.

m_0, S_0 = The values of m and S when $h = 0$.

ϕ = A characteristic of the capillary defined below.

T = Time.

The following symbols are used for heat flow:—

θ = Temperature, measured for convenience from a datum such that $\theta = 0$ when $h = 0$.

c, k = Specific heat and thermal conductivity of the material, both functions of θ.

u = Surface heat transfer coefficient for heat exchange between the solid and its surroundings. A function of θ.

c_0, k_0 = Values of c and k when $\theta = 0$.

H_0^θ = The total heat of the material $\int_0^\theta c\,d\theta$.

ρ_0 = The density of the material, assumed independent of θ.

A = The cross-sectional area of the heat-flow path between the centres of adjacent cells.
x = The length of such a path.
V = The volume of a cell.
Z = The surface area to which u is applicable.
t = Time.

The following scale factors are used:—

α = In the relation between θ and h. See equations (5) and (16) below.
β = Relating the volume of a cell of the conductor to the size of the corresponding reservoir. See equation (14).
γ = Relating the value of A/x for a heat-flow path to the value of ϕ for the corresponding capillary. See equation (14).
ϵ = Relating the time scales. See equations (9) and (15).

The necessary conditions for a valid analogy between the air-flow and lumped heat-conduction systems can be established by comparing the mass balance equation for an air reservoir with the heat balance equation for the corresponding cell of the conductor. Before writing down these equations, however, we must consider briefly the properties of the two basic components of the air-flow system—the U-tube reservoir and the capillary. The reservoir [see Fig. 1(*c*)] requires little explanation. When the liquid level is h, the absolute pressure of the air above it is $p_0 + qh$ and its density is therefore $d_0[1 + (qh/p_0)]$. Its volume is $v + \int_0^h S dh$, so that

$$m = d_0[1 + (qh/p_0)]\left(v + \int_0^h S dh\right)$$

$$m_0 = d_0 v$$

and $$m - m_0 = d_0[1 + (qh/p_0)]\int_0^h S dh + d_0 v(qh/p_0) \quad (1)$$

For the mean velocity of flow through a capillary to be proportional to the pressure difference between the ends the following conditions must be satisfied:—

(1) The air flow within the capillary must be laminar.

(2) The pressure losses at entry and exit, which are proportional to the square of the velocity, must be negligibly small. The practical criterion is that, under the maximum pressure difference used in the apparatus, the combined entry and exit loss must not exceed 1% of the total drop. If the above conditions are complied with, the mass flow of air through a capillary with entry and exit pressures $p_0 + qh_1$ and $p_0 + qh_2$ is given by

$$\frac{dm}{dt} = \phi d_0(h_1 - h_2)\left(1 + q\frac{h_1 + h_2}{2p_0}\right) \quad (2)$$

where ϕ is a constant depending upon the dimensions of the capillary and the viscosity of the air. For air at a given temperature, therefore, ϕ is a constant for a particular capillary and, being easily determined from flow measurements, it is a convenient quantity by which the flow characteristics of the capillary may be specified.

Conditions for similarity.—In setting up the air-flow analogue of a heat conduction problem two obvious conditions for geometrical similarity must be satisfied. Firstly, the dimensions of the reservoirs must be made proportional to the volumes of the cells which they represent. Secondly, the ϕ-values of the capillaries representing the various heat-flow paths must be made proportional to the values of A/x for those paths. Provided that these two rules are followed it can be shown that there is no loss of generality in deriving the conditions for exact simulation from a consideration of the heat balance for a cell at temperature θ_i associated with a single adjacent cell at temperature θ_j, the cross-sectional area and length of the heat-flow path between the centres of these cells being A and x respectively. The heat balance equation for cell i at time t is therefore

$$\frac{A}{x}\int_{\theta_i}^{\theta_j} k d\theta = \rho_0 V c \frac{d\theta}{dt} \quad (3)$$

The air-flow analogue of this system consists merely of two reservoirs, in which the liquid levels are h_i and h_j, connected by a capillary ϕ. The mass balance for reservoir i at time T is

$$\phi d_0(h_j - h_i)\left(1 + q\frac{h_j + h_i}{2p_0}\right) = \frac{dm}{dT} \quad (4)$$

That k is a function of θ and that the mass flow through the capillary depends to some extent upon $h_j + h_i$ as well as upon $h_j - h_i$ can be allowed for if we relate θ and h by the expression

$$\alpha(k/k_0)d\theta = [1 + (qh/p_0)]dh \quad (5)$$

where α is an arbitrary scale factor and k_0 is introduced so that α has the dimensions of h/θ. Integrating we have

$$\frac{\alpha}{k_0}\int_0^\theta k d\theta = h + \frac{qh^2}{2p_0} \quad (6)$$

For the behaviour of the air reservoir i on the time scale of T to simulate exactly the behaviour of the heat cell i on the time scale of t the equations (3) and (4) must be identical. Using equation (5) integrated between appropriate limits the condition for this is found to be

$$dm = \lambda c d\theta \quad (7)$$

where $$\lambda = \alpha\epsilon\frac{d_0 V\phi}{(A/x)}\left(\frac{\rho_0}{k_0}\right) \text{ a constant} \quad (8)$$

and $$\epsilon = T/t \text{ the constant time scale factor} \quad (9)$$

Integrating (7) we have

$$m - m_0 = \lambda\int_0^\theta c d\theta = \lambda H_0^\theta \quad (10)$$

This means that for the analogy to be valid and for the effects of variable c and k to be simulated we merely have to shape the reservoirs in such a way that at any liquid level h the value of $m - m_0$ is proportional to the total heat of the material at the temperature θ which corresponds to h in equation (6).

The shape of the reservoirs.—The manner in which S, the cross-sectional area of a reservoir, must vary with h for equation (10) to be satisfied may be found from equations (1), (5), and (6). It is convenient to use S_0 as a characteristic dimension in terms of which the variation of S may be specified. It is found that

$$\frac{S+\Delta}{S_0+\Delta}=\frac{k_0}{c_0}\left\{\frac{c}{k}-\frac{\alpha q H_0^n}{k_0 p_0[1+(qh/p_0)]^2}\right\} \tag{11}$$

and

$$S_0+\Delta=\lambda c_0/\alpha d_0 \tag{12}$$

where

$$\Delta=\frac{qv}{p_0[1+(qh/p_0)]^2} \tag{13}$$

In practice Δ is usually very small and may be taken as qv/p_0, a constant, with negligible error. The second term in the curly brackets of equation (11), representing the effect of the compressibility of the air in the reservoir, is also usually small, though in accurate work it cannot be neglected altogether.

Time scales.—The conditions for geometrical similarity mentioned earlier in this section mean that we can use scale factors, β and γ, say, by which the value of V for a cell and A/x for a heat-flow path may be multiplied to give the appropriate values of $S_0+\Delta$ and ϕ for the reservoir and capillary representing them. That is, we may write

$$S_0+\Delta=\beta V \text{ and } \phi=\gamma(A/x) \tag{14}$$

where β and γ are constant throughout the system. The factor ϵ relating the time scales can now be obtained in terms of β and γ and of the physical constants of the material by eliminating λ between equations (8) and (12). We find

$$\epsilon=\frac{T}{t}=\frac{\beta}{\gamma}\left(\frac{k_0}{\rho_0 c_0}\right) \tag{15}$$

Relation between θ and h.—In equation (6) as it stands we still have an undetermined scale factor α. If h_m denotes the maximum value of h attainable on the model it is usually convenient to choose α so that h_m corresponds to θ_m, the upper temperature limit in the heat-flow system. Then from (6)

$$\alpha=k_0\frac{h_m+(qh_m^2/2p_0)}{\int_0^{\theta_m}k\,d\theta} \tag{16}$$

so that h and θ are related by the equation

$$\frac{h+(qh^2/2p_0)}{h_m+(qh_m^2/2p_0)}=\frac{\int_0^{\theta}k\,d\theta}{\int_0^{\theta_m}k\,d\theta} \tag{17}$$

With the pressures used in practice, however, qh_m is small compared with p_0, and the maximum error introduced by using the simpler expression

$$\frac{h}{h_m}=\frac{\int_0^{\theta}k\,d\theta}{\int_0^{\theta_m}k\,d\theta} \tag{18}$$

is only about 0·3%.

Heat exchange at a surface.—If a particular cell of the heat conduction system at temperature θ is exchanging heat with surroundings at θ_A over a surface area Z and with heat transfer coefficient u the heat exchange may be simulated on the air-flow system by a capillary ϕ_A connecting the appropriate reservoir with a pressure level h_A, where h_A is the value of h corresponding to θ_A. From a comparison of heat and mass balance equations the expression for ϕ_A is found to be

$$\phi_A=\frac{\gamma Zu(\theta-\theta_A)}{\int_{\theta_A}^{\theta}k\,d\theta} \tag{19}$$

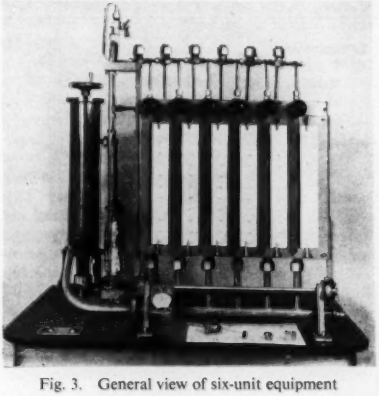

Fig. 3. General view of six-unit equipment

PRACTICAL DETAILS

The actual design of a model based on the foregoing principles may be varied very considerably to suit the needs of particular problems. For this reason only a brief description is given of the model so far constructed. Fig. 3 is a general view of the equipment. The six reservoirs are formed by machining six apertures 2 in wide × 11 in high in a brass plate $\frac{1}{2}$ in thick. Each aperture is closed by Perspex windows front and back [see sectional view of Fig. 4(*a*)], thus forming a rectangular duct with cross-section $\frac{1}{2}$ in × 2 in. The 11 in height gives a working scale length of 10 in or 250 mm. Renewable paper scales are clipped to the front windows as shown in the photograph, and illumination from the back makes the levels easy to read. The containing liquid used in the reservoirs must have reasonably low viscosity and vapour pressure. In practice a grade of transformer oil is used which happens to be readily available. The passages through which the oil flows should be of generous proportions to minimize pressure losses. Referring again to Fig. 3, the oil enters each reservoir at the bottom left-hand corner from a manifold

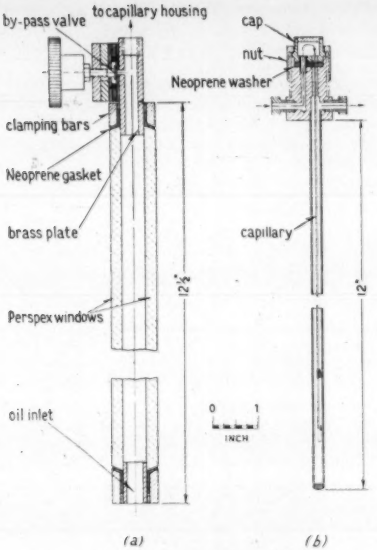

Fig. 4. Sectional views
(*a*) Reservoir; (*b*) Capillary housing

connected to the constant-level container seen at the left of the photograph. It is sometimes convenient to have the constant level at the lower end of the working range instead of at the top. For this purpose the overflow may be lowered by the handwheel. The housings containing the capillaries will be seen above the reservoir block. They are connected to each other and to the reservoirs by simple tee-pieces. The black knobs seen in the photograph operate valves with which to bypass the capillaries when setting the reservoir levels. Throughout the system demountable pipe joints are made from standard oil-pipe unions slightly modified to accommodate Neoprene washers. The capillaries are made from cupro-nickel tubing supplied by Johnson, Matthey and Co. Ltd., 0·017 in bore being the size most used. The construction of the capillary housing is shown in section in Fig. 4(*b*). It is largely made from standard pipe-fitting components. The length of capillary tubing is merely pressed through a pin-hole in a Neoprene washer which itself fits in a recess in the housing where it is clamped by a cap and nut. With this design the capillary in its washer can be easily removed without breaking the main air connections. In spite of the number of bends in the path of the air through the housing, the assembly as a whole satisfies the criterion for turbulence losses mentioned in the previous section. One of these housings, with cap and nut removed and the capillary in its washer partly withdrawn, may be seen in the foreground of Fig. 3. To construct reservoirs with variable cross-sectional areas as required by equation (11), the plain rectangular ducts are partially filled by cores extending the full height and shaped so as to give the desired variation. These cores are inserted by removing the back Perspex windows from the reservoirs, and to facilitate this the whole reservoir block can be lowered into the horizontal position. Cores of simple shape can be made up from available bar and sheet material. More complex ones are cast in hard plaster in a mould with an adjustable side, as shown in the photograph, Fig. 5, which is self-explanatory. Examination of equation (19) shows that if k is variable the value of ϕ for the capillary representing a surface heat transfer coefficient varies with θ (and h) even when u is constant. This variation has to be performed by hand and the construction of a convenient "variable air resistance" is still under development. A simple though rather cumbersome arrangement is a capillary the bore of which can be partially blocked for a varying proportion of its length by a wire sliding within it. As an alternative, when only a few cells involve surface heat exchange, instead of keeping h_A fixed at the value corresponding to θ_A we may keep ϕ_A fixed and vary h_A for each cell separately so that the mass air flow through ϕ_A always corresponds to the heat flux $u(\theta - \theta_A)$.

ACCURACY

Solutions obtained on the model are subject to errors of two different kinds—those due to inaccuracies of construction and calibration and those introduced by the lumping procedure. The latter are, of course, common to all methods employing such a procedure and depend, broadly speaking, only on the coarseness of the lumping.

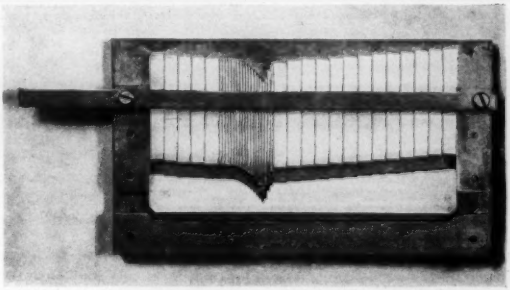

Fig. 5. Adjustable mould for making cores

Since these errors are not peculiar to the air-flow method and since they have been examined in some detail in connexion with the electrical analogy[3] they will not be discussed here except to say that surprisingly accurate results can be obtained using quite a small number of lumps, say five or six for a one-dimensional problem. To illustrate this we may quote the results of comparing the solution obtained on the model for the slab of Fig. 1, lumped as shown under the most unfavourable conditions possible ($\theta = 0$ everywhere for $t < 0$, $\theta = 1$ at surfaces for $t > 0$) with the mathematical solution to the same problem. To make a mathematical solution possible the case of constant c and k was taken, and to separate the lumping errors from the constructional errors the mathematical solution of the set of first-order equations representing the lumped heat-flow system was also worked out. As might be expected, the maximum lumping errors occur in the cell nearest the surface soon after the start. The error at point 1 reached a maximum of about 13% of the total temperature change, but soon died away to less than 1%. The maximum error at point 2 was 2·7% and at the remaining points 1% or less. Constructional errors nowhere exceeded 0·7%, though it is only fair to say that they might be considerably greater than this in cases where large cores have to be used in the reservoirs. The cross-sectional area then depends on the difference between two relatively large quantities so that the effect of dimensional errors in the cores is magnified. Unfortunately, no quantitative information on this point is as yet available.

CONCLUSION

The air-flow analogy has been developed to provide a method by which heat conduction problems involving variable c and k can be handled by an investigator who has not the resources of a large electrical computer at his disposal. By means of the simple arrangement of reservoirs with shaped cores the adjustment of component values during a run is obviated so far as internal points are concerned and the operator is left free to attend to the boundary conditions. Compared with an *RC* network with fixed component values, however, the setting up of the air-flow model is admittedly tedious. For this reason it is not suggested as an alternative to the electrical analogy in cases where c and k are constant, though it is possible that a greatly simplified version, using glass tubes for reservoirs and with no provision for the insertion of cores, might even in these cases have certain advantages, such as freedom from leakage effects. The six-unit model so far constructed is, of course, only a prototype, and a larger model is proposed for solving certain two-dimensional problems. At least twenty-five reservoirs will be needed and in such a case the capillary housings will be mounted in a separate bank and not immediately above the reservoirs as at present. Permanent piping from capillaries and reservoirs will be brought out to a central exchange panel where the necessary cross-connexions for a particular problem can be quickly made with rubber tubing. The desirability of some such arrangement in an installation with many units was one of the main reasons for the choice of a gas rather than a liquid as the working fluid. With a liquid-flow system freedom of layout is greatly restricted by the need to guard against air-locks and the effects of gravity. Throughout the description it has been assumed that the working fluid is air. Other gases could, of course, be used and from the point of view of reducing end losses in the capillaries there is some advantage in using a light gas. It is thought, however, that this would, in general, be outweighed by the disadvantage of working with a closed system.

In this paper emphasis has been laid on fundamental principles rather than on applications. Extension of the method to two- and three-dimensional heat conduction systems has been mentioned only briefly and applications to other processes, such as diffusion with variable diffusion constant, which involve the same fundamental equations, have not been dealt with at all. This is partly because practical details vary so much with the nature of the problem that their adequate treatment would occupy too much space, and partly because many of these matters will be better discussed after a larger installation has been constructed and operated. It is hoped, however, that the basic principles have been set down fully enough to serve as a useful guide to those interested in applying the method to their own particular problems.

ACKNOWLEDGMENTS

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A new method of preparing paste and powder specimens for the electron microscope

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A new method of preparing specimens of pastes and powders for examination by the electron microscope which avoids aggregation of the solid particles is described. A sample of the paste or powder is dispersed in a liquid medium which gels on drying. This dispersion is sprayed on to a heated pre-coated specimen grid and the gelling agent is finally washed off.

For the examination of paste samples in the electron microscope it is important to avoid aggregation of the particles during the preparation of the specimen. Paste samples, generally, have been examined by techniques involving either the drying down of the sample and subsequent dispersion in a film-forming medium or by the direct drying of a drop of the diluted sample on a pre-coated specimen grid. Both methods result in aggregation, frequently severe, during the drying process. By dispersing the samples in a solution which gels at an early stage of drying, it has been found possible to prevent aggregation.

DESCRIPTION

Aqueous pastes.—A spray method, which requires mainly dilution of the sample, is less likely to modify the condition of a suspension than, for example, drying the suspension and treating it as a powder. Aqueous pastes diluted with water and sprayed, using a compressed air supply of about 1 atm pressure, did not give satisfactory specimens due to the poor spraying qualities of water and aggregation during the drying of the drops. Preliminary optical investigation indicated that a spray driven by an 80 lb/in^2 air supply would be suitable for spraying water without requiring any alcohol. To overcome these disadvantages isopropyl alcohol was added to improve the spraying quality and gelatin solution to prevent the aggregation. Spraying compositions containing about 50% of alcohol and 0·02% of gelatin were found to be satisfactory, but the quantities were not critical. Dilutions of the sample with this medium of the order of 1000 : 1 were found to give suitable concentrations of particles on the final micrograph. In practice it was found preferable to add the gelatin before the alcohol as the latter tended to produce flocculation with some samples. In general, when flocculation was encountered it was overcome by increasing the amount of gelatin, reducing the quantity of alcohol and by adding the alcohol as a water-alcohol mixture. The diluted sample was sprayed on to a specimen grid, pre-coated with a supporting film, which was mounted in front of an electric hot-plate in order to ensure rapid drying of each drop before another drop reached the same area. During the drying period the presence of gelatin caused a high viscosity and the formation of a gel, and prevented the aggregation of the sample particles. Thin disks of gelatin were formed in which the sample particles were distributed. The specimen grid was immersed twice in distilled water (2 min at 50° C) when the gelatin was removed by solution, apparently without affecting the specimen except by removing any water soluble dispersing agent. The latter effect was advantageous and avoided the difficulty of distinguishing between sample particles and dispersing agent. After washing, the specimens were ready for examination in the electron microscope; they could be metal shadow cast, if desired.

Optical examination of the technique.—Attempts were made optically, using a $\frac{1}{8}$ in objective, to detect changes in the sample as a result of the preparation procedure. Flocculation during addition of alcohol was detected occasionally in this manner but was overcome by increasing the amount of gelatin used. Optical examination at the washing stage did not yield evidence of any change in the sample. Before washing, disks of gelatin were visible in which the particles were well distributed. All particles were in focus at once except where disks overlapped. After washing no change could be detected apart from the absence of gelatin and of the out-of-focus particles. A faint coloration of the water was detected, in some cases, when using highly coloured pigment samples. This was attributed to the removal of the second layer in overlapping drops.

Extension to powder samples.—Examination of powder samples by a spray method required the preparation of a dispersion of the powder. This was achieved by rubbing-out in gelatin solution plus a dispersing agent, Dispersol L. The resulting paste was treated by the method already described. The choice of gelatin as a rubbing-out medium was made because gelatin was used at a later stage and because, together with the dispersing agent, it could be removed at the washing stage. Rubbing of gelatin solution to dryness tends to produce an insoluble gelatin. This was avoided by rubbing nearly to dryness and, after dilution, heating the bulk liquid to ensure complete solution of the gelatin. More recently glycerol has been used as a rubbing-out medium by Mr. A. C. Cooper of Imperial Chemical Industries Ltd. (Dyestuffs Division).

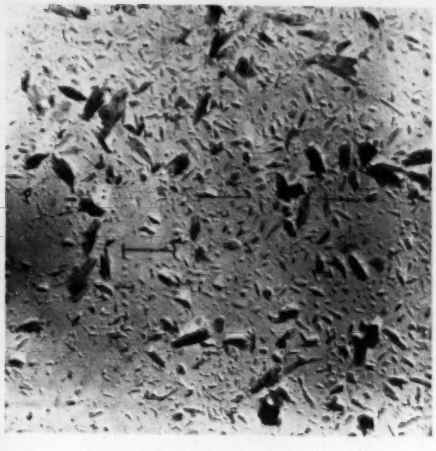

Fig. 1. An organic pigment paste.
Magnification × 10 000

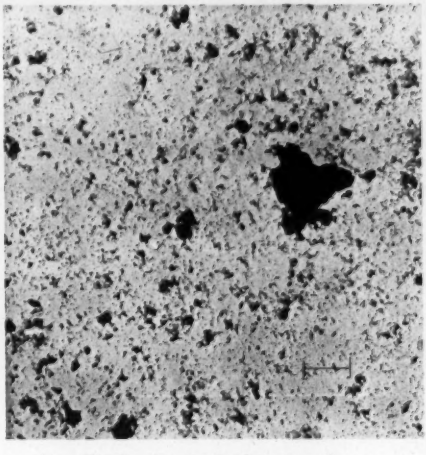

Fig. 2. An organic pigment powder.
Magnification × 10 000

Non-aqueous media.—As an alternative to a gelatin solution, a medium viscosity grade of ethyl cellulose in isopropyl alcohol was used. A paste heavily diluted with ethyl cellulose solution (about 1 : 1000) was sprayed on to a clean microscope slide supported in front of a hot-plate. After removing the ethyl cellulose by solution in acetone, the slide was gold shadowcast and coated with a thin film of nitrocellulose which, finally, was floated off on a water surface. The particles and the metal shadowing adhered to the film. Direct spraying on to a nitrocellulose-coated grid was not possible as the nitrocellulose would have dissolved in the acetone at the washing stage. Alternative film materials, e.g. aluminium, could have been used.

DISCUSSION

Figs. 1 and 2, of an organic pigment paste and an organic pigment powder, show that the method described is capable of presenting large numbers of particles with minimum overlapping of particles. The good dispersions which have been achieved by this method together with the variation in dispersion which the author has observed in a graded series of paste samples suggests that the method of preparation has not altered the state of dispersion of paste samples. Optical examination at various preparation stages has not revealed any discrepancies apart from avoidable effects of alcohol. From these considerations the method, for pastes in particular, is considered to be reliable and superior to the earlier methods which can cause extensive aggregation. The method is believed to be a basic one and not restricted to the substances mentioned. Its relative simplicity should make the method widely applicable provided care is taken to avoid solvents liable to modify the sample (e.g. by crystal growth).

ACKNOWLEDGMENT

The author wishes to thank Mr. W. E. Durrant for able assistance during the investigation.

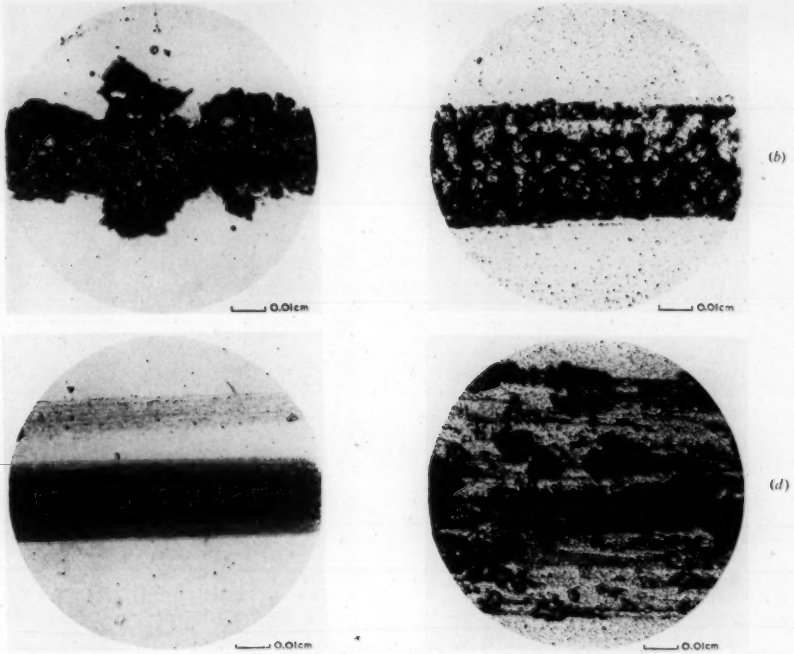

Fig. 2. Tracks formed on a photographic plate after rubbing with various hemispherical sliders and developing. Load, 2 kg; speed, 2 cm/sec. (*a*) Tungsten carbide, (*b*) Perspex, (*c*) copper, (*d*) cadmium. (*a*), (*c*) and (*d*) photographed by reflected light. (*b*) photographed by transmitted light

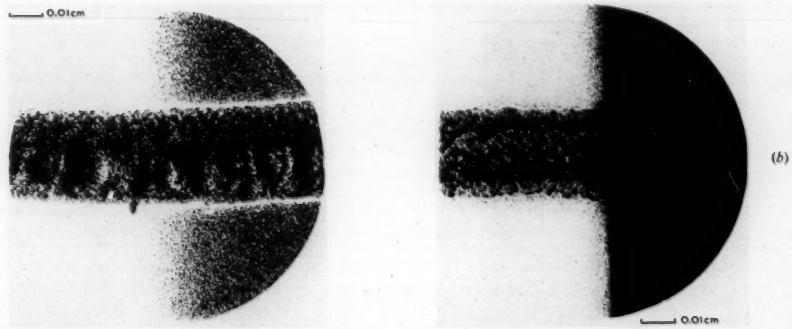

Fig. 3. Track left by a hemispherical glass slider on a photographic plate. Half of each plate was exposed to light after rubbing but before development. Load, 2 kg; speed, 2 cm/sec. Photographed by transmitted light. (*a*) Chemical developer; (*b*) physical developer

The effect of pressure and friction on photographic emulsions

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The production of a latent image in photographic emulsions by friction, and the desensitization of the emulsions to subsequent illumination, both by friction and inhomogeneous pressure, has been investigated. It is suggested that the grains of silver halide are deformed by pressure and form internal faults which act as latent image traps. This causes the desensitizing effect. If the forces are sufficient to shear the grains, then a latent image is formed which can be developed by an ordinary developer. The mechanism whereby this latent image is formed is not clear, but thermal effects probably play the main part.

The effect of pressure and friction on photographic emulsions has been described by various observers since the end of the nineteenth century[1] with some disagreement as to the experimental facts.[2, 3, 4] It is, however, now well established[5] that most substances, when rubbed across a photographic emulsion, produce a latent image which can be developed with an ordinary chemical developer. It is also known that inhomogeneous pressure may produce little or no latent image but causes a marked desensitization of the emulsion to subsequent irradiation by light, and that hydrostatic pressure has little or no photographic effect at all.

It has been usual to treat the effects of friction and of inhomogeneous pressure as two separate phenomena, but it is suggested here that the difference is only one of degree, not of kind. It is found that the latent image produced by friction is also accompanied by a very strong desensitizing action to subsequent exposure to light.

The work of Bowden and Ridler[6] has shown that local high-temperature flashes are very easily produced between two sliding surfaces. The temperature rise is greater for non-conductors than for metals and it has been shown, for instance, that the local temperatures developed by rubbing glass with metals are of the order of 500° C when the speed of sliding is of the order of 30 cm/sec.[7] The highest temperature produced cannot exceed the lower melting point of the two rubbing surfaces.

Mather[8] has attempted to explain the latent image formed by friction on the basis of high temperatures developed at the rubbing interface. This does not, however, account readily for the latent image formed by inhomogeneous pressure, nor does it take note of any desensitizing action of pressure or friction.

The object of this paper is to give a simple explanation of the effect of both friction and pressure on photographic emulsions.

FRICTION APPARATUS

Most of the frictional experiments were carried out on a simple machine comprising a brass bar pivoted horizontally near its centre, mounted on a three-wheeled brass carriage. The bar carries a counter-balance weight at one end and a holder at the other into which can be fitted wheels $\frac{1}{4}$ in wide, with rims of semicircular cross-section, to give rolling friction. These can be locked to give sliding friction and in this way the radius of curvature of the surface can be kept the same for sliding and rolling. Hemispherical sliders of various materials can also be fitted into the holder.

Loads up to 3 kg are placed directly over the rubbing surfaces, and provided the carriage is always moved away from the rubbing point, the effect of the frictional force on the normal reaction between the surfaces can be neglected. If it is moved in the other direction, the rider tends to dig into the photographic plate which serves as the lower surface. Both the carriage and the plate can move between two pairs of guide rails at right angles to each other and the plate's position between its rails can be set in the dark by means of a screw. The carriage is moved by hand at a fairly constant speed of about 2 cm/sec. This arrangement was found to be adequate for a general investigation of the latent image formed by friction.

To determine the effect of speed and the coefficient of friction between the various materials and photographic emulsions the Bowden–Leben friction apparatus was used.[9] As it is not suitable for low loads, 2 kg was used in these experiments.

Another apparatus (Fig. 1) was later set up to investigate the effect of repeated rubbing over the same track, and was also used to check the experiments carried out on the first machine under better defined conditions. In this machine a double hydraulic piston *A* drives a hemispherical slider *B* across the photographic plate *P*; a bellows *I* excludes dust from the cylinder. The load *L* is applied directly over the point of contact of the rubbing surfaces on a brass arm which is pivoted at *C* where the driving force from the piston is applied. The weight of the arm itself is counterbalanced at *D*. The pivot, photographic plate, and the line of movement of the piston lie in the same horizontal plane to eliminate the possibility of the frictional force affecting the normal reaction between the rubbing surfaces. The direction of motion of the piston can be reversed by means of an electro-magnetic valve *E*. The piston is made to reverse its own motion by operating microswitches *G* and *H* at

either end of its traverse and at the same time a mechanical counter F operated electromagnetically records the number of complete cycles which take place. A selector switch S and a holding relay R are also provided as shown in Fig. 1.

CONTROL EXPERIMENTS

Some experiments were first carried out with photographic plates of different sensitivities to light. They were subjected to rolling and sliding friction of wheels of silver, Perspex and brass, under loads of from 50 g to 3 kg. Ilford developer ID11 was used in all cases, but the times of development were varied according to the recommendations of that firm.

Plates used	*Speed Scheiner*
Ilford Process	12
Ordinary	16
Soft Gradation Pan	28
H.P.3	34

The degree of blackening obtained after development was estimated visually. The blackening increased with increasing load; that produced by sliding under a load

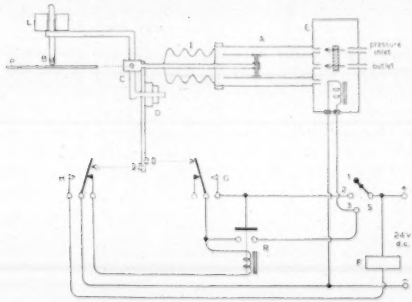

Fig. 1. Diagram and circuit of self-reversing friction apparatus. L, load; P, photographic plate; B, hemispherical slider; C, pivot; I, dust excluder; D, counterbalance weights; A, double piston; E, electro-magnetic valve; H, microswitch; R, holding relay; G, microswitch; S, selector switch; F, counter

of 50 g was just visible. Rolling under a load of 2 kg gave a track of about the same blackness as sliding under a 100 g load. The corresponding tracks of different materials showed minor differences which will be described later. No significant variation was found in the intensities of the black developed lines produced by the same slider or wheel under the same load on different types of plate. All later experiments were therefore carried out on unbacked Ilford Ordinary plates which were developed in Ilford ID11 developer by the light of an Ilford S safelight.

Experiments on the Bowden–Leben apparatus showed that both the blackening and the coefficient of friction of a given substance on a photographic plate were in general independent of the speed of sliding over a wide range of speeds. In the case of silver the speed was varied from 0·15 cm/min to 60 cm/min, the coefficient of friction varied smoothly but erratically from 0·3 to 0·4 and the blackening after development was the same in all cases. This is sufficient justification for driving the simple friction apparatus by hand. A track formed by rubbing a silver rider under a 2 kg load was included on each plate as a standard for comparison.

It is known that many non-radioactive materials are capable of fogging and desensitizing a photographic emulsion by being placed near it.[5,10] This is attributed to the production of ozone or hydrogen peroxide.[11,12] Freshly cut pieces of the various materials used in the friction experiments were therefore left in contact with Ilford Ordinary and HP3 plates for up to one day, but no effect was detected.

RESULTS OF RUBBING PHOTOGRAPHIC PLATES

Tracks were obtained by rubbing hemispherical sliders of $\frac{1}{4}$ in diameter, of various materials, on photographic plates under loads of 200 g and 2 kg, using the first apparatus; and the frictional force for each slider under a 2 kg load was measured on the Bowden–Leben apparatus.

The developed tracks were observed under a microscope with a narrow depth of focus and the blackening was found not to be confined to the surface layer of the emulsion, although the greatest concentration of developed grains was there.

The amount of blackening was found to depend principally on the frictional force but also on the ability of the hemispherical slider to retain its shape. If it was very soft or had a low melting point so that a large "flat" was worn quickly, then little or no blackening was observed. It is possible to divide the kinds of blackening obtained with the 2 kg load into four classes, depending on the hemispherical slider used.

(1) Hard substances with low thermal conductivities and high coefficients of friction, such as stainless steel, develop sufficient heat at the rubbing surfaces to soften the gelatin and cause marked blackening.

(2) Hard substances with high thermal conductivities and medium coefficients of friction, such as copper, produce a narrow black track but do not apparently soften the gelatin. Nevertheless, considerable distortion is present and the tracks present a crinkled appearance under the microscope.

Most of the group 1 substances join this group, if rubbing conditions are made less stringent by reducing the load and speed of sliding.

(3) Substances of low melting point, such as Wood's metal, where the temperature rise on rubbing will melt the slider before softening the gelatin, and soft substances, such as cadmium and lead, which will give way themselves before the gelatin, produce broad tracks with slight

blackening. It is often possible to see particles of metal, torn off the slider, adhering to the gelatin. Owing to the much greater area of contact the pressure on the emulsion is much more homogeneous and the effective pressures are much less than for group 2 materials.

(4) Very soft substances, such as wax, or substances of very low melting point, such as ice, which also have low coefficients of friction, produce no visible blackening. The friction is too low to distort the gelatin and the pressure is nearly homogeneous owing to melting of the sliders.

The experimental results and some physical properties of the sliders are summarized in Table 1, and photomicrographs of some of the tracks are shown in Fig. 2 (see p. 20). The table emphasizes the importance of a high coefficient of friction and hardness of the sliders in producing a latent image. As the melting point of the slider increases no sharp transition in image-forming properties is found; even low melting-point sliders such as Wood's metal produce a latent image.

Table 1. *Experimental results with different sliders*

	Group in order of blackening of plate	Coefficient of friction	M.P. °C	Thermal conductivity cal cm °C^{-1} sec^{-1} cm^{-2}	Vickers hardness 5 kg load
1	Tungsten carbide	0·4	2 800	—	1 600
	Stainless steel	0·5	1 500	0·11	420
	Nickel	0·5	1 450	0·13	208
	Aluminium	0·6	650	0·50	27
	Platinum	0·35	1 773	0·17	85
	Constantan	0·3	1 250	0·05	210
	Glass	0·4	—	0·002	—
	Perspex	0·4	—	0·0001	20
2	Silver	0·35	960	0·97	41
	Copper	0·3	1 080	0·91	70
3	Lead	0·3	327	0·08	2–4
	Cadmium	0·35	320	0·22	20
	Wood's metal	0·2	80	0·03	12
	Polytetrafluorethylene	0·1	—	0·0001	6
4	Paraffin wax	0·15	60	0·00009	—
	Ice	Very low	0	0·004	—

One or two further experimental facts were noted. A silver slider cooled down by solid carbon dioxide gave a less black trace than a similar slider at room temperature. This may be due to water from the atmosphere condensing on the silver and lubricating the slider, but it is possible that a lowering in hot spot temperature was brought about, decreasing the blackening. If the silver slider was deliberately moistened with water, then a low value of the coefficient of friction ($\mu = 0{\cdot}15$) was obtained together with practically no blackening of the plate. A reduction in intensity of blackening under a silver slider was also obtained by flooding the plate with a stearic acid solution in benzene and letting it dry before rubbing. The reduction was not due to the benzene since the intensity of the tracks formed by subsequent rubbings could be restored almost to normal by washing the stearic acid off the plate with more benzene.

Repeated runs over the same track increased the blackening at first. The track formed by two runs in different directions under a load of 1 kg was, however, less black than one run under a load of 2 kg. After the second run little increase in the blackness of the track was observed. Two thousand runs over the same track under a load of 2 kg, using a silver slider, produced no blackening of the emulsion which was visible without development. After development the track did not appear to be very much blacker than that obtained after two runs under a load of 2 kg, and it appears that there is a limit to the amount of blackening which can be obtained.

No track was observed if a thin layer of paper, fixed relative to the plate, was interposed between the rubbing surfaces. Static loading of hemispherical riders on to the plate also produced no blackening, although other workers report that this can occur under sufficiently large loads with suitable emulsions.[8]

EFFECT OF TEMPERATURE ON PHOTOGRAPHIC PLATES

Some pieces of Ilford Ordinary plate were put in a light-tight tin in an oven at a known temperature for a given time and then developed; the results are given in Table 2.

Table 2. *Results of baking photographic plates*

Expt. No.	*Temp. of oven °C*	*Time in oven min*	*Results after development*
1	116	10	Nil.
2	116	60	Slight blackening.
3	135	10	Slight blackening (less than expt. 2).
4	140	10	Black.
5	160	10	Gelatin burnt chocolate-brown, visible without development.

DROPPING BALLS ON TO PHOTOGRAPHIC PLATES

A photographic plate was placed, emulsion upwards, on a horizontal flat lapped brass block. A thin film of glycerine was spread between the plate and the block in order to hold the plate in position and reduce its liability to break. Steel balls of various diameters from $\frac{1}{16}$ in to $\frac{1}{2}$ in were dropped on to the plate from heights increasing from 1 cm until the plate broke with the particular ball. No blackening of the plate was observed after development.

If the plate was tilted then a crescent of black appeared after development on the lower side of the circle of contact. At this place maximum pressure and some shear should occur in the emulsion.

No difference in results was obtained by heating the balls to 100° C before dropping them.

DESENSITIZING EFFECT OF INHOMOGENEOUS PRESSURE AND FRICTION

The application and release of inhomogeneous pressure forms very little latent image, but the emulsion is desensitized to subsequent irradiation by light. Rolling friction, sliding friction with a layer of thin paper interposed between the sliding surfaces, dropping balls, and directly loading hemispherical sliders on to photographic emulsions were all found to cause this desensitization. When a frictional latent image is formed desensitization accompanies it. The appearance of the developed track then depends on the time and intensity of the subsequent exposure to light before development. It was found that the track produced by rubbing a photographic plate with a copper or silver hemisphere under a load of 1 kg appeared black when the plate was developed without exposure to light. If, however, the plate was exposed to bright daylight for one minute after rubbing but before development then the track appeared lighter than the background.

Inhomogeneous pressure did not affect a latent image already present and the latent image produced by rubbing was superposed on any existing in the emulsion.

The desensitizing effect of inhomogeneous pressure and of friction is only observable if the photographic plate is developed with an ordinary chemical developer. This develops those grains of silver halide which have a surface latent image, but not those in which the latent image is trapped internally. By using a physical developer all the latent image can be developed. The first step in using a developer of this type is to dissolve away the silver halide in alkaline "hypo." This leaves behind it all the latent image which is then made visible by immersing the emulsion in a solution containing a soluble silver salt and a mild reducing agent. The reduced silver is deposited preferentially on the latent image. The physical developer used in the next experiment was given by Henney and Dudley[13] and contained silver nitrate, amidol and other substances.

Various sliders were rubbed on a photographic plate of which half was then exposed to light. On development with an ordinary developer desensitization was very marked at the edges of the frictional blackening. When the experiment was repeated but the plate was developed with the physical developer, no desensitization was observed (Fig. 3, see page 20) and the blackening due to rubbing was still present superposed on that of the light.

DISCUSSION

Reychler[2, 3] suggested that the latent image produced by friction arose through the gelatin being rubbed from off the silver halide crystals, leaving them exposed to the action of the developer. This theory is disproved at once by the fact that the frictional latent image can be developed by a physical developer after fixation has removed all the silver halide (Fig. 3*b*, see page 20). Mather[8] explained the effect of friction on the basis of the heat developed at the rubbing interface. This does not, however, explain the production of a latent image by a slider of Wood's metal which melts at 80° C, below the temperature found to be necessary to form a latent image. Nor is the developed image restricted to the surface of the emulsion as Mather's theory might suggest. Neither of the authors considered the desensitizing effect of friction and inhomogeneous pressure on photographic emulsions.

It is probable that the application of inhomogeneous pressure at first causes slight deformation and internal faults in the silver halide crystals. Such faults are known to act as electron traps[14] and any latent image then formed by irradiation by light is trapped inside the crystal and is not affected by ordinary developers. Physical developers will, however, reveal its presence. This then is the desensitizing effect of inhomogeneous pressure to subsequent irradiation by light.

As the magnitude and the inhomogeneity of the pressure are increased the silver halide grains will deform more severely and slip. Rubbing the emulsion will also cause severe deformation and slip of the grains. It is suggested that this slip gives rise to a latent image.

When only slight slip occurs the latent image which is formed may be trapped inside the silver halide crystal, perhaps at the internal fault where it is produced. The internal latent image will then also act as an electron trap, and cause desensitization of the emulsion.[5] It is not certain whether the internal faults or the internal latent image is mainly responsible for the desensitization, but some results of Wood quoted by Berg[5] suggest that internal faults are important. It is said there that a high intensity light flash which desensitizes an emulsion to subsequent exposure by low intensity light through the formation of an internal latent image will not desensitize the emulsion to a subsequent exposure to X-rays. Inhomogeneous pressure is, however, capable of desensitizing an emulsion even to X-rays.

If the slip is sufficiently great then some latent image is obtained on the surface of the crystals and a blackening is obtained after development. The desensitization which accompanies the frictional blackening is occasioned by the presence of slightly deformed grains with internal latent image traps along with the developable grains with surface latent image. At the edges of the friction track only the slightly deformed grains are present, as might be expected (Fig. 3*a*).

All the experiments described in this paper support the view that when shearing movements are caused in the gelatin and hence slip in the crystals, a surface latent image is very easily produced, whereas inhomogeneous pressure gives rise to very much less slip and surface latent image, and in many cases none.

Rolling friction, lubricated sliding, and direct pressure loading cause little shearing in the emulsion, so that only a weak latent image is formed. Lighter loads reduce the inhomogeneity of the pressure, the shearing and hence

the intensity of the latent image. When the inhomogeneity of the pressure is reduced by using top rubbing surfaces of low melting points which grow "flats" very quickly or when the shear is reduced by using very soft materials or ones with low coefficients of friction, then little latent image formation is to be expected (group 3, Table 1) and in extreme cases none (group 4, Table 1).

Under a given load the number of silver halide grains which are made to slip and become developable will be nearly independent of the speed at which sliding takes place and hence the blackening will be nearly independent of the speed of sliding. The results of repeated slidings over the same track can be explained in a similar manner. Sliding over the same track in the reverse direction for the first time makes a fresh set of silver halide grains developable. This adds to the grains already affected. Some of the grains, however, will be affected both times and hence the blackening on development will not be double that obtained after only one run over the track. Further runs after the second will not make many more grains developable as most of those which slip will already be developable. A slight increase may occur, but is not readily detectable by the eye. No print out of metallic silver is expected even after a few thousand runs, nor was any found.

The exact mechanism whereby shearing of a grain of silver halide makes it developable is not certain. At the interface where the slip takes place individual electrons may gain sufficient energy to reach the conduction energy levels of the silver halide. If these electrons occur at isolated points this is equivalent to a direct mechanical decomposition of the silver halide. This phenomenon has never been shown conclusively to exist and in most cases where mechanical work produces a chemical effect, even on the atomic scale as in the detonation of explosives, heat is a necessary intermediary.[15] A mechanical mechanism therefore appears to be unlikely.

If a number of adjacent ions in the silver halide lattice at the interface gain energy we have then a hot spot, and it seems plausible to suppose that when the silver halide grains are sheared temperature flashes up to the melting point of the silver halide may be produced. Such temperatures are far more than those which have been shown to produce a latent image by direct heating of the emulsion.

The mechanism of latent image formation by heat is a separate problem, and will not be discussed here beyond saying it is probably brought about by the heat increasing the size of the sensitivity specks until they become developable in their own right.[16,17]

CONCLUSION

The experimental observations on the effect of friction and inhomogeneous pressure on photographic emulsions can all be explained by one simple process. The grains of silver halide are deformed by the pressure and the internal faults set up act as latent image traps, causing a desensitization of the emulsion of subsequent light. If the forces on the grains are sufficient to shear them then a latent image is formed in the grain.

The manner in which the shearing of a silver halide grain forms a latent image is not certain, but a possible thermal mechanism has been suggested.

ACKNOWLEDGMENTS

I wish to thank Dr. F. P. Bowden for his advice and encouragement during the investigation. I should also like to thank King's College, Cambridge, for a Foundation Scholarship, the Department of Scientific and Industrial Research for a maintenance grant, and the Ministry of Supply (Air) for equipment.

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Correspondence

The determination of the "fast" and "slow" vibration directions of a plate of double refracting material

I recently had occasion to prepare quarter wave plates of mica and to mark upon them the "fast" vibration directions. Fundamentally the determination of the "fast" direction involves the determination of the vibration direction corresponding to the least refractive index. It would appear that in the past the direct determination of the indices in the case of thin sheets of mica has presented difficulties since various methods have been adopted to avoid this. A direct method of measuring the refractive indices of mica was described by Record and Jones,[1] and it occurred to me that this, of course, gave a direct determination of the "fast" and "slow" vibration directions. A sheet of mica (quarter wave) of easily recognized form (see figure) was taken, and the extinction directions

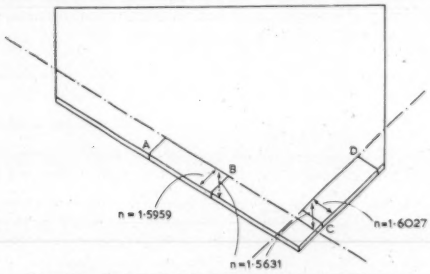

Diagram of mica sheet showing manner of cutting test samples, vibration direction, and refractive indices

marked (chained lines in the figure). Two pieces of mica, AB and CD, of size suitable for the Abbé refractometer were cut from the sheet, the long edges of the pieces being parallel to the extinction directions. Measurement of the refractive indices gave

for piece AB, $n = 1{\cdot}5631$ and $n = 1{\cdot}5959$
for piece CD, $n = 1{\cdot}5631$ and $n = 1{\cdot}6027$

Since the value $n = 1{\cdot}5631$ is common, this must refer to a vibration which is common to both strips, i.e. to the vibration perpendicular to the plane.

Consideration of the direction in which the light traverses the mica in the Abbé refractometer shows that the other refractive index refers to vibrations across the strip, thus the refractive index and the vibration directions for light refracted with these values are as shown in the figure. Since the velocity is proportional to $1/n$ then the "slow" vibration direction is that with the high value of n (i.e. $1{\cdot}6027$) and we have determined the "fast" and "slow" directions. The directions so found were confirmed by other standard methods.[2,3] As a matter of interest and in confirmation of the above, the sheet of mica, split to give a thinner one, was placed in one of the beams in Fresnel's biprism experiment. Examination with a piece of polaroid placed behind the eyepiece gave a movement of the fringes as the polaroid was rotated. The slow direction found by this method corresponded to that found as above.

I am glad to express my thanks to my colleagues at the Nottingham and District Technical College, and to Dr. N. Davy of the University of Nottingham, for their help in discussion and in checking of the observations.

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"Survey of general and applied rheology"

I have read Mr. Macey's letter in your December issue and seek a little of your valuable space to reply.

Mysticism: the doctrine that ultimate reality may be apprehended directly, independently of the ordinary processes of sensation and reasoning. Examples: see Swedenborg, H. P. Blavatsky, Jakob Boehme. The statements of these authors are not beyond human comprehension, but they cannot be verified by normal persons and hence are not scientific. In contrast, rheology is based on observation and experiment, as is any other science.

"Physical properties" only express the behaviour of a material under certain (and limited) conditions of experiment. A truly Hookean solid does not exist. Some rheological systems can be analysed mathematically into combinations of ideal springs and dashpots, but it does not follow that elements corresponding to these abstractions have any physical existence. In any case, quasi-properties *need* not be exalted to the status of physical properties; their use does *not* entail the abandonment of Newtonian time and mass scales; and, if strongly correlated with subjective judgements, they may be as useful in their own way as physical properties, which equally are founded on empiricism.

These problems are at least debatable and the last word has yet to be said on either side. Possibly the idea of quasi-properties may prove unfruitful; but I cannot see how an attempt to explore their possibilities can taint a science or occasion regret.

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NOTES AND NEWS

New Books

Sound. By F. G. Mee, M.A., B.Sc. (London: William Heinemann Ltd.) Pp. ix + 171. Price 8*s.* 6*d.* net.

In this book, the author has attempted to deal with all matters of general interest in sound, and its connexion with other branches of physics. In the preface he says that sound has always been somewhat of a Cinderella among the branches of physics studied in schools, the reason being that it is a poor "examination subject." It would perhaps be

truer to say that unless a student is a good mathematician, or something of a musician, the subject is both difficult and dull. But there is probably no more rewarding branch of physics from the teacher's point of view when one has pupils whose interests can be aroused in both these directions; Mr. Mee has clearly had this good fortune, and it has inspired what the reviewer feels is an excellent book to place in a schoolboy's hands. The author has successfully communicated his own belief that this is an important subject, an interesting subject, and something to be enthusiastic about. The treatment of musical instruments is good, and one would have liked to see more of it; in particular, it would have been interesting to see Mr. Mee dealing with organ stops, on Bonavia–Hunt lines but with the help of the oscilloscope. The mathematical work is thorough, and therefore in places difficult; but since the book is intended to reach at least the highest levels of work done in schools, this is in itself a good feature, and will exercise the talents of those who really wish to master the principles of the subject.

G. R. NOAKES

Wave Motion and Sound. By R. W. B. STEPHENS, Ph.D., A.R.C.S., D.I.C., F.Inst.P., and A. E. BATE, Ph.D., F.Inst.P. (London: Edward Arnold and Co.) Pp. viii + 448. Price 45*s*.

This book is intended for pass and honours students, the main body of the text being designed to cover the requirements of the former, while the thirty appendices, occupying 75 pages, contain supplementary material and mathematical developments required by the latter. The groundwork of classical vibration and wave theory is thoroughly treated, and commendable care has been taken to point out the relevance of the results in all branches of physics. There does not, however, seem to be any logical reason why transverse vibrations of strings should be dealt with in Chapter 4, vibrations of bars, membranes and plates at the end of Chapter 5, resonance, mainly as exemplified by air columns, in Chapter 9, and forced vibrations not until Chapter 13. Modern illustrations of reflexion and refraction are provided by the seismic method of geophysical prospecting and meteorological acoustics, accompanying which is a good account of partial reflexion and the measurement of sound absorption. Due attention is paid, in the chapter on the velocity of sound, to sources of error, and there is an interesting section on sources moving at supersonic speeds. A chapter on physiological acoustics is followed by one on wave analysis and synthesis which includes an account of sound recording and reproduction. The section on acoustical measurements is excellent, as also is the chapter on architectural acoustics in which a new formula for the calculation of reverberation times is proposed. Electrical, mechanical and acoustical analogies are adequately treated, and the final chapter is devoted to a detailed account of ultrasonics. Among the topics dealt with in the appendices are coupled vibrations, relaxation oscillations, Fourier analysis, vibrational modes of 1-, 2- and 3-dimensional systems, spherical waves, acoustic impedance of pipes, theory of the acoustic horn, radiation pressure and the reciprocity theorem. References are confined to texts likely to be available to students and a useful set of examples is provided. The space allotted to music and musical instruments is so restricted as to make over-simplified and misleading statements almost inevitable. In practice the point of impact of the hammer on a pianoforte string is not 1/7th of its length, but varies between 1/8th in the bass and 1/13th in the treble. Freedom of transposition is secured not by mean tone but by equal temperament. The strike tone is not the component of highest pitch in the note of a bell. The node in figure 5.30 (*e*) is at $0{\cdot}216l$ from the free end, not $0{\cdot}226l$. On the other hand, it hardly seems necessary to devote space to the differentiation of a sine and cosine function or to define moduli of elasticity and describe the determination of Young's modulus by the stretching of a wire. More care might have been taken to ensure uniformity of notation; frequency, for instance, is denoted by f, n and ν, the velocity of sound by v, c and C, specific heats by C and S. Apart from such points and the fairly numerous, and mostly obvious, minor misprints, this book provides a comprehensive, reliable and attractive degree course in wave motion and sound.

G. F. HERRENDEN-HARKER

Wave Theory of Aberrations. By H. H. HOPKINS, B.Sc., Ph.D., F.Inst.P. (London: Oxford University Press.) Pp. viii + 167. Price 15*s*. net.

The quality of the image produced by an optical system is determined, on paper, by tracing rays from object points through the system. The direct result, for any given object point, is a set of emergent rays which are approximately parallel for a system like a complete telescope or microscope, or a pencil converging approximately to an image point for a system like a camera lens. These rays are everywhere normal to the emergent wave. It is the deviations of the wave-surface from the ideal spherical or plane form which are important, the deviations of the rays from exact concurrence at an image point being less informative since they depend so much on their degree of convergence. Also, for most people it is simpler to think of a single wave-surface than of a pencil of rays. It is likely, therefore, that the theory of the performance of optical instruments could be better expressed in terms of wave-surfaces than of rays. This is the main thesis of, and reason for, Dr. Hopkins's *Wave Theory of Aberrations*.

Too much must not be claimed for either method of treatment. For some purposes the ray treatment is the better, as, for example, for numerical computation or the theory of the perfect optical instrument, in Clerk Maxwell's sense. For the general theory of Seidel aberrations, and more particularly for the addition and stop-shift theorems, the wave-theory is the more readily understood. Thus the reader will find that Dr. Hopkins depends upon computation of rays for deriving the aberrations, or departures from the ideal spherical form, of the wave surface, in its progress through an optical system.

The results obtained would mostly be recognizable by one familiar with Dennis Taylor's *System of Applied Optics* or, more particularly, Conrady's *Applied Optics and Optical Design*, and the total content approximates to that of the latter, on the theoretical side. Conrady's book, however, is outstanding in examples, and the arguments are fully detailed. Hopkins's book is confined to theory and written with brevity, and one suspects that students (even the post-graduate students to whom it is addressed) might sometimes find difficulty in consequence. For example, in discussing wave-propagation, instead of clearing up the difficulties which are mentioned, the reader is expected to refer to advanced papers and books. A treatment, adequate for the purposes of the book, ought to be possible, which raised no serious difficulties. Another example occurs in Chapter IV on the general theory

of aberration types, in which, apart from the straightforward expansion into series, the critical point is the demonstration that the variables must take certain forms. The "demonstration" given is hardly more than a statement of the forms in question, and inadequate for anyone without a good knowledge of analytical three-coordinate geometry.

Apart from minor defects of this sort the book is a valuable addition to the literature and one would not be surprised to find that students prefer Dr. Hopkins's treatment to that by rays alone. The reviewer certainly does. There may not be many converts among experienced lens designers who use ray calculations, and necessarily understand ray methods so well as to feel little benefit from learning a new dialect for their optics. As one has come to expect from the publishers, the book is very well produced. E. W. H. SELWYN

Photographie Instantanée et Cinematographie Ultra-Rapide. By P. FAYOLLE and P. NASLIN. (Editions de la "Revue d'Optique," 1950.) Pp. 188 + vii. Price 800 fr. net.

This book, by two military engineers, reviews the various techniques of high-speed photography, and would be of interest to readers requiring a general knowledge of the methods now in use. Its value to one working on the subject is more doubtful. There is no mention of photography as such and the treatment of optical phenomena is inadequate. A few errors have been noted, such as two incorrect renderings of the name Boys and an implication that "Polaroid" is manufactured by Eastman Kodak as well as by the Polaroid Corporation. Apart from having paper covers and no index the book is attractive especially in that it has nearly as many illustrations as pages. Most of the high-speed photographs shown are from studies in ballistics. E. W. H. SELWYN

Les Fondements théoriques de l'Électrodynamique. By J. HÉLY. (Editions de Montligeon.) Pp. 72. Price 300 fr. net.

The principles of electromagnetic theory are usually introduced by showing how Maxwell's equations arise as the generalization of experimental results. In this book, the opposite course is adopted. The author starts by stating the equations of electromagnetism in the most general form as fundamental laws and then proceeds to derive results from them. To show the relation of the quantum theory of fields and the theory of nuclear fields to classical electromagnetism, the equations are written in the form

$$4\pi\frac{i}{c} = \Box A + \text{grad}\, L + k^2 A$$

$$4\pi\rho = \Box V - \frac{1}{c}\frac{\partial L}{\partial t} + k^2 V$$

where $L = \frac{1}{c}\frac{\partial V}{\partial t} + \text{div}\, A$; $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$

By writing $k = 0$ we obtain the classical equations of electromagnetism, while for $k = 2\pi\mu c/h$ where μ is the meson mass we obtain equations which can be applied to nuclear fields.

The author is concerned entirely with fundamentals and there are no applications of the theory to special problems. Among the topics discussed are the self-energy of the electron, the distribution of energy in the electromagnetic field, the conception of the electromagnetic field as a photon field and the variation principle of electromagnetism.

The book serves as a brief introduction to more detailed studies of a controversial subject. T. B. RYMER

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SPECIAL ARTICLE

The movement of a cohesionless granular bed by fluid flow over it*

By R. A. BAGNOLD, O.B.E., F.R.S.

Our knowledge of the basic principles underlying "Loose boundary hydraulics" is weak, owing mainly to the difficulties of observation and measurement. The corresponding case of grains moved by wind presents fewer experimental difficulties and is found to be fairly simple. But the behaviour of the grains is here so different that the principles have not been found easy to apply to the water case. These differences are examined, and an attempt is made to explain them in terms of the relative degree to which a free grain can penetrate through the fluid under its own initial momentum. The penetration is defined, and its effects are applied to various grain-fluid phenomena.

1. INTRODUCTION

Very heavy expenditure is incurred annually on the maintenance and improvement of navigable waterways, on river and flood control, on irrigation canals and on coast defences against the sea. Whether this money is spent efficiently or otherwise depends largely on the engineer's knowledge of what may be called "Loose boundary hydraulics." Essentially this subject comprises a study of what happens when water flows over, or when its flow is contained by, a bed or channel composed of loose granular material whose exposed surface grains are set in motion by the flow. The subject is characterized by the fact that the shape of the boundary changes with the flow past it, and the flow in its turn is affected by these changes. Moreover the flow near the boundary is not that of a pure fluid but of a peculiar combination of grain and fluid movement.

The engineer is frequently called upon to predict the ultimate shape which the boundary—the bed—will assume when some proposed artificial change is made in its configuration elsewhere, or when a change is made in the granular load carried by the flow. In the present state of our knowledge these things cannot be calculated by applying any known physical principles. When an experimental model is made, known principles can be applied only to the fluid flow; none are known to apply to the moving grains. The material for the model is selected by past experience of what works best out of the limited range of substances readily available. No model material gives true similarity of behaviour, but again by past experience it has been found that a certain distortion of the model scale gives results that can be interpreted to full scale. This empirical method of approach, which is in the nature of a personal art rather than a transmittable science, though it is surprisingly effective in the hands of a master, is very slow. In the absence of a soundly based insight into the physical laws governing the interaction between grain and fluid movement, it cannot be regarded as satisfactory.

But precise experimental work is very difficult. Our observation of what goes on under water has to be largely indirect, and the dense crowd of moving grains above the bed usually obscures our vision of it. Worse still, the virtual impracticability of both the Pitot tube and the hot-wire leaves us with no handy or reliable way of determining either the velocity profile over a cross-section of the flow or the turbulence characteristics. Again, no sound method has been found of gauging the rate of grain movement along the water bed, or indeed of defining just where the bed surface begins and the flow ceases. Lastly, both the flow pattern and the turbulence characteristics are profoundly affected by the existence of a free air-water surface, and both are further complicated by the fact that since a liquid has to be contained in a channel, the wetted boundary can never be flat.

The engineer, being a busy man, has avoided these difficulties by dealing with and recording only those quantities which are easily measured by conventional methods, regardless of whether they are the simplest for solving the underlying physical problems. He records only the mean values of flow, velocity, bed drag and grain movement over the whole cross-section, in terms of discharge, hydraulic gradient and load. From these quantities, and from a rather alarming number of others, for some of which the physical significance is rather obscure, he is slowly evolving a set of empirical relations.

A complementary approach which I think the physicist would prefer is to examine the behaviour of idealized spherical grains in grain-fluid systems generally, using experimental results from as widely differing conditions as possible. Such an approach seems the likeliest to disclose the underlying principles. Since, however, the flow conditions in the engineer's case of natural grains in water vary over the cross-section of the channel, any quantitative results obtained from this alternative approach would have to be integrated over the cross-section before they could be finally compared with the engineer's records. On the other hand, many useful ideas should emerge from such an approach at a much earlier stage. In particular we should be enabled to decide what are in fact the basically important quantities to measure and record.

In what follows I shall try to summarize what we know or can reasonably deduce about the behaviour of

* Based on a lecture given before the London and Home Counties Branch of The Institute of Physics on 27 September, 1950.

grain-fluid systems in general, confining myself throughout to the condition that grain movement takes place close to a bed consisting wholly of grains of the same kind and that all the grains are of the same size.

The general conditions seem to be definable, to a first approximation at any rate, in terms of the following six variables: fluid density ρ, kinematic viscosity ν, drag force τ per unit area close to the bed, and turbulence coefficient T; here expressed as the ratio of mean eddy velocity to the drag velocity $V_* = \sqrt{(\tau/\rho)}$, grain density σ, and diameter d. In terms of the apparent density ratio $\gamma = (\sigma - \rho)/\rho$ the most practically important case of natural grains in water, for which $\gamma = 1{\cdot}6$, is seen to occupy but a narrow zone intermediate between sand in air ($\gamma = 750$) on the one hand and special artificial grains in water, for which $\gamma =$ (say) $0{\cdot}001$, on the other. We have also a very wide range of possible grain sizes and a range of boundary drag from the threshold needed to start grain movement on the bed up to as many times this value as we please.

The well-known work of Shields[1] has covered the range of grain-water systems between $\gamma = 0{\cdot}06$ and $3{\cdot}2$ for grain diameters between $0{\cdot}38$ and 5 mm under conditions of general channel turbulence, and has shown that the behaviour of the grains on the bed in all cases falls very nicely into a single pattern in terms of $(\tau/\rho)(\gamma g d)^{-1}$ and the Reynolds number Vd/ν on the grain. White,[2] using oil, has continued this same pattern into the condition of wholly viscous channel flow. Owing, however, to the difficulty in measuring slow water speeds at given points in the channel cross-section all values of both τ and V_* were got indirectly from the hydraulic gradient of the whole flow.

The Shields' pattern for grains in water is concerned with the onset or threshold of motion by individual grains, and with the effect on the bed surface of their subsequent movement along or extremely close to it. Two important questions remain uncertain. First we would like to know more about the condition at which the pattern begins to break down owing to the appearance of appreciable cohesion between the grains when they become very small or sticky. This cohesion, which may arise from a variety of causes, tends to prevent initial movement by individual grains. The question enters very largely into the behaviour of fine silts and muds in rivers. Second, just how do grains in general behave while they are off the bed, and what is the effect of these freely moving grains, when in great numbers, on the fluid flow around them. It is with this second question that I propose here to deal.

The reason we know so little about the dynamics of the freely moving grains and of the water around them is that the motion of the free grains—the "saltation" as Gilbert has called it—takes place in water so close to the bed surface that its details have defied investigation. Further, in laboratory water channels, initial movement for all readily available materials occurs under turbulent conditions, and the onset of partial suspension by the turbulence starts so close to the threshold flow condition that the true saltation is hard to distinguish.

In the case of sand in air both these difficulties disappear. The path of the saltating grains, between leaving and regaining the bed surface, rises so much higher into the fluid that the grains' motion can be watched and measured. Moreover the effect of turbulence is not appreciable until far higher fluid flow values are reached.

But unfortunately in jumping from grain-water to grain-air systems—from $\gamma = 4$ to $\gamma = 750$—without any available experimental systems in between, we seem to have lost continuity. Not only have we to measure a quite different quantity from which to deduce the drag τ (the velocity gradient of the wind near the bed, instead of the gravity slope of the whole water surface), but we find that profound changes have taken place in the general behaviour of the grains. Such, however, is the ease of observation and measurement in air as compared to water and such is the simplicity of the saltation mechanism in air, that it seems worth while examining these changes and their cause. If the changes are found to be of degree only we may still be able to apply the principles we have learnt from the simple case of grains in air to the more difficult one of grains in water.

2. THE THRESHOLD OF GRAIN MOTION

We have no reason to doubt that the general principles which Shields and others have applied to the definition of the condition at which the surface grains begin to be disturbed by the fluid flow hold good for all grain-fluid systems. But some experimental discrepancies between the air and water cases are rather to be expected because (*a*) the value of the drag τ has been found in different ways, (*b*) the initial movement of a few grains on a big area of the bed has to be observed by eye and the exact stage at which initial movement may be said to have started is a matter of personal judgment, (*c*) we cannot be sure that the surface packing in the two cases is such that the degree of exposure of the individual grains is quite the same, (*d*) unless the air experiments are made in a totally enclosed wind tunnel, unavoidable variations of wind speed will give lower values of τ_0 than the mean measured speed would suggest.

Shields' experimental values of the dimensionless function

$$\alpha = (\tau_0/\rho)/(\gamma g d)^{-1}$$

in terms of the Reynolds number Vd/ν for grains in water are sketched in Fig. 1. For all ordinary grains in both air and water the curve ceases to be of practical use for values of Vd/ν below about $1{\cdot}0$ owing to the effect of various cohesive forces between very small particles. (And for the same reason it must cease at higher values in the case of mechanically fluffy grains.) Hence, if we restrict ourselves to cohesionless grains of ordinary densities, the value of α is seen to vary between the quite narrow limits of $0{\cdot}1$ and $0{\cdot}03$. It should be noted that the slightly upward right-hand part of the

curve refers to conditions when the fluid flow around the surface bed grains is almost wholly turbulent, whereas the steeper upward part on the left refers to increasingly viscous motion around the grains. The extreme left approaches the condition of wholly viscous flow.

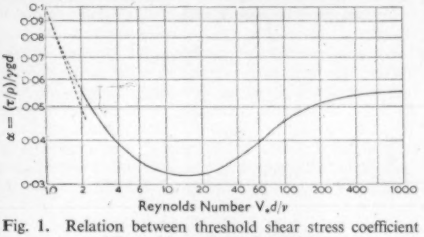

Fig. 1. Relation between threshold shear stress coefficient and Reynolds number (after Shields)

The corresponding values of α found for air gave a curve of the same general shape, but with α reduced to a quarter of its value for water. In terms of fluid velocity this discrepancy would become a half instead of a quarter. Further investigation is needed to clear up this point.

3. BEHAVIOUR OF FREE GRAINS WITHIN THE FLUID—SALTATION

For values of the drag τ above the threshold τ_0, some of the surface bed grains are set in motion—more and more of them as τ increases. In both air and water these grains move over the bed in saltation. They rise from the bed with negligible forward speed, are accelerated and carried forward a certain distance by the fluid flow, and finally, by the action of gravity, they fall back to the bed again, where all their acquired forward momentum is lost.

In water the falling grain, as it passes through slower and slower moving fluid layers, reaches the bed at such low forward speed that its final impact is too feeble even to dislodge any of the bed grains. In fact it is usually seen to settle gently among them. But in air, on the other hand, the initial rise of a grain, once the saltation has started, is caused not directly by the fluid but by the mechanical impact of a previous falling grain. For grains hit the surface at such a high speed that they splash up other grains, making little craters as they do so. The splashed-up grains may rise a hundred or more diameters off the bed before their initial upward momentum is all lost by the combined forces of fluid drag and gravity.

This difference in the mechanism of the saltation has a direct bearing on bed-form. Ripples in general may be regarded as but a manifestation of the inherent instability of most deformable plane surfaces to any kind of deformable movement over them. Small random departures from the plane offer an increased aspect angle to the movement, which causes increased local drag and in turn increased local deformation and increased aspect angle; and so on to a limit; after which the reverse applies. But different kinds of mechanism produce different kinds of ripple. Depending on the flow condition many kinds of ripple form in deep water under steady flow, as opposed to oscillatory movement under surface waves, but none resembles the ripples formed in air. Observation seems to indicate that these water bed ripples are due to local variations which they themselves create in the pattern of both flow and turbulence at the bed surface. In air, on the other hand, it is the violent impact of the descending grains which forms and controls the ripples. They are of but one form regular and continuous in plan and sinusoidal in cross-section. There is evidence to show that the ripple pitch corresponds to the distance between take-off and landing of the mean saltating grain. The evidence is very much against local variation of either flow or turbulence playing any part in ripple formation in air.

4. PENETRATION

What, then, is the cause of this basic difference in the character of the saltation in the two cases, and why is the grain in air so much less sensitive to fluid turbulence? The answer to both questions plainly lies in the differing degree to which a free grain can by its own momentum penetrate through the fluid and can sustain its own absolute motion against the fluid force which resists relative motion.

A free grain is acted on by two kinds of force. There is the vertical body force due to gravity which is proportional to $(\sigma - \rho)gd^3$ and therefore remains constant at all times for all the grains of any system; and there is the fluid force $\frac{1}{8}c\rho v^2\pi d^2$ where v is any instantaneous relative velocity between grain and fluid, and c is the empirical drag coefficient whose value varies with v but is known for a sphere in terms of vd/ν. This force varies throughout a grain's path, both in magnitude and direction. Consideration of the general movement of a grain resulting from these two forces combined is therefore very complicated. But it seems possible to isolate the effect of the fluid force in an approximate way for comparative purposes by an artifice, the validity of which seems to have considerable experimental support.

At the threshold condition, the ratio of the fluid force on the average bed grain to the gravity force is the ordinate $\alpha = \tau_0/(\sigma - \rho)gd$ of Fig. 1. The fluid force on a grain the moment it has left the bed will be different, but probably not far in excess of the threshold drag force under the condition at the extreme right of the curve where the drag is exerted by the fluid surrounding nearly the whole of an individual grain. Hence we can assume that when the bed drag is τ_0 the ratio of the fluid force on a grain at the moment it leaves the bed to the gravity force is nearly constant for all systems. Since the gravity force is constant for any one system

the fluid force at any higher drag $\tau = n^2\tau_0$ for that system will be n^2 times the gravity force And this ratio will be the same also for all systems when n^2 is made the same. Under these conditions, therefore, the behaviour of grains in all systems should be the same (assuming, of course, that the mechanical properties of the grain material are roughly similar) provided all free grains react in the same way to the fluid force opposing relative motion. So by comparing different systems at equal values of $n^2 = \tau/\tau_0$ or of $n = V_*/V_{*0}$, we can examine the relative effect on the grains of their differing reactions to the fluid force alone.

The following simple definition of the penetrating power of a grain through the fluid surrounding it is put forward. The value of c in the expression for the fluid force varies with the relative velocity v. But over a range of v from say v to $v/2$ it will be sufficiently accurate for our purpose to assume a constant value for c corresponding to a mid-velocity $\frac{3}{4}v$. So by equating the fluid force to the grain's mass times its acceleration, writing dv/dt as vdv/ds and integrating with regard to s over the range v to $v/2$ we get

$$s = \frac{4}{3}\log_e 2\,\frac{\sigma}{c\rho}d_e, \text{ or } P = \frac{s}{d} = 0{\cdot}9\frac{\sigma}{c\rho}$$

where P is the characteristic penetration of the system. Except in so far as velocity and viscosity are implicit in c the penetration is independent of both and depends only on the ratio of absolute densities. Physically the penetration is the distance in grain diameters which a grain will travel through a fluid under its own initial momentum before its relative velocity is reduced or increased by a factor of 2.

Take, for example, the case of a grain of dune sand of diameter 0·2 mm at the moment it leaves the bed under a wind which is near the threshold strength (n being just over unity). The value of v at grain level is of the order of 200 cm/sec for which c is about 2. σ/ρ is about 1 200. So the penetration P is just over 500 and the grain will travel over 500 diameters before its velocity relative to the air is reduced to half value. For the same grain in water, and at the same value of n, v is about 8·5, giving $c = 3$, and $\sigma/\rho = 2{\cdot}65$. The penetration is now 0·8, so the grain will travel less than its own diameter.

It will be noted that since P depends only on the absolute density ratio σ/ρ as opposed to the apparent ratio $\gamma = (\sigma - \rho)/\rho$, equality of density, for which $\gamma = 0$, has no special significance. The value of P is not asymptotic to unit density ratio and continues to have just the same physical meaning when $\sigma < \rho$. The bed is merely at the top instead of at the bottom. So the range of possible grain-fluid systems does not end at $\sigma/\rho = 1$ but continues on into such systems as cork grains in bromoform or mercury. (Here we should have a new combination of a very small value for P and a large value for γ.) This explains why, as we experiment with grains whose density approaches nearer and nearer to that of the fluid, we find less and less change in their behaviour. We are merely narrowing the experimental range. It also shows that although various workers have been able, over the seemingly wide range, in terms of apparent density, of 3 to 0·06 in water to fit the bewildering transformations which occur in the small-scale bed features into one neat sequence in terms of ascending values of n, this sequence can only be valid for a comparatively narrow range of absolute density ratios. For even at $\gamma = 3$ ($\sigma/\rho = 4$) the value of P for the same conditions as in the example above is still less than 2. We should not expect any serious change in the character of the system until P was made considerably higher than this. It is a pity that no experimental grain-fluid combination is available which would give values of P intermediate between 2 and 200.

5. LOW GAMMA SYSTEMS AS EXPERIMENTAL MEDIA

The above dynamical ideas seem to go some way in confirming what one can surmise from observation—that grain-water systems for which γ is very small indeed behave in very much the same way as ordinary grains in water, as far as their saltation is concerned, and can therefore be used for experiment. Systems for which γ say 0·001 offer several advantages. The motion is so slow that every detail can be watched by eye. In an ordinary laboratory flume the threshold of grain movement occurs far into the condition of wholly viscous channel flow; so the saltation mechanism can be studied by itself. On the other hand the turbulent condition also occurs at very low speeds of flow, and its exaggerated effects (see Section 6) also can be watched by eye. Further, the difference in the thermal expansion coefficient makes it possible to vary γ at will over quite a big range by temperature control. The present practical difficulty lies in controlling the constancy of γ from grain to grain. For at a mean value of 0·001 an error of 0·1% in the absolute density when the grains are made gives a difference in γ of 100%. The technique is, however, quite new and no doubt this trouble will be overcome.

6. PENETRATION AND TURBULENCE

The general effect of turbulence is to cause statistical scatter, both in range and height, of the mean path of the saltating grains. Depending on the penetration, which is a measure of the distance a grain travels through, as distinct from with, the fluid, bigger eddies of equal velocity may be expected to have a greater effect than small. For a grain should only be appreciably affected when the size of the eddy exceeds something of the order of P times the grain diameter. A grain should pass right through a smaller eddy. This explains another point of difference between the behaviour of grain in water and air.

Disregarding for a moment the effect of penetration, it seems reasonable to assume that the ratio of the mean

upward component of the eddy velocity to the grain's terminal speed of fall through the still fluid provides a sound measure of the influence of turbulence on the saltation. As long as this ratio is less than unity no grain can rise higher than its initial upward momentum at take-off would carry it in a vacuum. Now the terminal speed V_g of a spherical grain is given by

$$V_g^2 = \frac{4}{3}\frac{\gamma g d}{c}$$

Since $V_{*0}^2 = \tau/\rho = \alpha\gamma g d$, when α is the threshold shear stress coefficient of Fig. 1, $\gamma g d = V_{*0}^2/\alpha$, and we have

$$V_g^2 = \frac{4}{3\alpha c}V_{*0}^2$$

or

$$V_g = \sqrt{\left(\frac{4}{3\alpha c}\right)}V_{*0}$$

If T is the ratio of the eddy velocity to the drag velocity V_*, we can substitute TV_* for V_g, so that

$$V_* = \frac{V_{*0}}{T}\cdot\sqrt{\frac{4}{3\alpha c}}$$

The value of T for undisturbed parallel flow has been found not to exceed 7, and for quartz sands of diameter 0·2 mm in both water and air $\sqrt{(4/3\alpha c)}$ is of the order of 4. So under general flow conditions such that turbulence is already fully developed at the threshold of grain motion, i.e. at $n = 1$, we might expect the dispersive effects of turbulence to be appreciable even at the earliest stage of the grain saltation. With quartz grains in water the effect is indeed noticeable when, or very soon after, they first begin to move. Very light grains are much more easily affected, because the value of c for their terminal speed of fall is considerably greater. Saltating quartz grains in air, however, for the same values of c and of $\sqrt{(4/3\alpha c)}$ as for quartz in water, are not appreciably affected by turbulence until we push the flow intensity up to 4 or 5 times its threshold value. For at lesser speeds no eddies operate near the bed whose upward currents are extensive enough compared with the far bigger penetration distance Pd.

In considering the effect of turbulence, however, we must be careful to distinguish between the condition governing the size an eddy must be in order to affect the saltation path of a grain, and the behaviour of a grain in eddies of comparable size as regards penetration. For in defining penetration we dealt only with fluid forces, to the exclusion of body forces. The acceleration within an eddy gives rise to body forces on both grain and fluid. So a grain with negligible inertial velocity through the fluid will react to an eddy as if its density were $(\sigma - \rho)$ instead of σ. This distinction is of no account in high-penetration systems where the two values are so nearly equal, but it becomes important when we compare one low penetration system with another. The influence of turbulence becomes greatly exaggerated as we make γ approach zero, whereas the influence of viscous flow does not. Hence a study of the behaviour of systems in which γ is very small enables us to distinguish between the effects.

7. REACTION OF GRAIN MOVEMENT ON THE FLUID FLOW. MECHANISM CONTROLLING THE GRAIN TRANSPORT RATE

No instances are known of fluid currents being able to transport solid cohesionless material at unlimited rates. In fact experience shows that a steady current of given properly defined strength can transport material of a given kind at a certain rate and no more. So we are

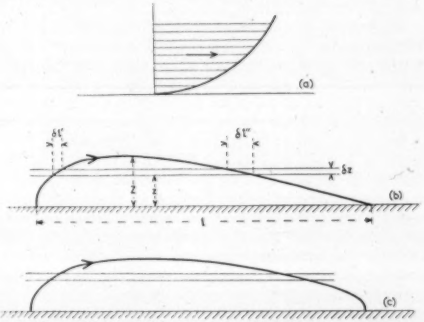

Fig. 2. Saltation path: (*a*) fluid velocity, (*b*) high penetration, (*c*) low penetration, height greatly exaggerated

forced to conclude, as common sense suggests, that in all grain-fluid systems of the sort we are considering some mechanism exists whereby the rate of fluid flow in the neighbourhood of the bed is linked to the rate of grain transport which it causes. In the high penetration case of sand in air this mechanism is apparent and its effects can be measured.

An essential result of saltation, as described at the beginning of Section 3, is the application of a drag to a fluid zone which is not itself in direct contact with the stationary bed grains. Consider any thin layer of fluid of thickness δz (Fig. 2) at a height z above the bed. Owing to the fluid velocity gradient near the bed, a grain will gain momentum from the layer while it is rising through it and will lose momentum to it while descending. The net change of momentum will be $m(\delta u' + \delta u'')$ where $\delta u'$ and $\delta u''$ are the grain's velocity changes relative to the bed. The consequent change in the momentum of the fluid layer will be spread over the small elements $\delta l'$ and $\delta l''$ of the total saltation length l. So the momentum change per unit of distance will be

$$m\left(\frac{\delta u' + \delta u''}{\delta l' + \delta l''}\right) \text{ or } m\frac{\delta u}{\delta l} \text{ for brevity}$$

Similarly if a mass q_s of saltating grains, all pursuing identical paths relative to the bed, are passing unit

width of a fixed transverse vertical plane in unit time, the time rate of change of fluid layer's momentum per unit area parallel to the bed, or in other words the saltation drag $\delta\tau_s$ on the layer must be $q_s\delta u/\delta l$. This may be positive or negative, a drag or an urge. It must be a drag on the top layers, but may be an urge on the bottom layers. If we knew the fluid velocity gradient we could calculate the detailed distribution of the drag at any height z by a laborious ballistic method. But the whole saltation drag on the fluid in the saltation zone must be given by $q_s(u_2 - u_1)/l$ where u_1 and u_2 are the grains' initial and final speeds. We will assume u_1 is zero, so we are left with $\tau_s = q_s u_2/l$.

Depending on the characteristic penetration of the system u_2 may be nearly zero as in the case of water, or nearly the speed of the fluid at the max. height Z as in the case of air. In both cases we have a drag on the fluid at the top of the zone, which is transferred downwards by the grains. In the case of air most of this drag is transferred direct to the bed, and in the case of water most is transferred to the bottom fluid layer. The bottom layer must therefore be speeded up, with a consequent increase in the fluid shear stress between it and the bed. So whether grain impact takes place or not, it seems that the effect of the saltation on the fluid must be practically the same.

In air, owing to the comparatively great height of the mean saltation path, the whole process can be made manifest by measurements of the fluid velocity.(3) Fig. 3 is a sketch of the wind velocities as measured with a fine Pitot tube. The top zone is indicated by the steepening of the velocity profile, which shows that some of the shear stress is carried by the grains. Lower down there is a layer whose velocity remains constant; and still lower we can even see something of a feeble exchange of momentum from the falling grains to the fluid before the grains finally hit the bed. In water the grains' very small penetration makes the saltation take place so close to the bed that the same details may never be observable directly. But there seems little doubt that they must be there.

The existence of zone of constant finite fluid velocity equal at that height to the threshold velocity gives, I think, a clear answer as to what is the mechanism linking the grain movement q with the strength of the fluid flow. Any increase in the fluid speed and shear stress above the saltation causes an increase in the saltation such that its increased drag reduces the fluid flow in it until the flow is brought back to the threshold value. Since no extra bed grains can now be set in motion, any further increase in the saltation is prevented.

The peculiar velocity distribution pattern in which all the velocity rays for greater shear stresses come to a focus not on the ordinate of zero velocity according to the well-known Göttingen equation $u_z = 5{\cdot}75V_* \log z/k$ where k characterizes the fixed roughness, but somewhere on the ray representing the threshold condition, may not be confined to fluid motion over a loose granular

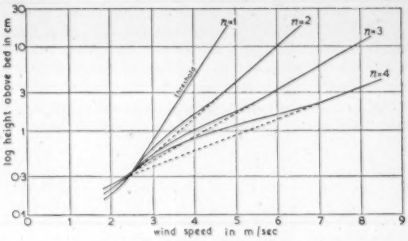

Fig. 3. Experimental velocity profiles showing effect of the saltation of sand in air

bed. In a paper shortly to be published, J. R. D. Francis, of Imperial College, London, has produced experimental evidence that the same kind of distribution occurs in the wind when it is creating sea waves. In both cases the velocity distribution of the activating fluid seems to satisfy the modified equation

$$u_z - u_{k'} = a V_* \log z/k'$$

where k' probably represents the effective control height of the active disturbed bed above the still bed, and $u_{k'}$ is the constant velocity at that height. For Francis' k' is not the height of the waves themselves, which grow bigger with the fetch, but seems to correspond to the more constant height of the much smaller wind disturbances on their surface.

It may be that the same kind of fluid distribution and the same mechanism for controlling of the magnitude of the surface movement applies not only to grain saltation but to all cases of fluid flow past deformable boundaries.

The practical importance of a study of the saltation mechanism lies in two directions. How far and in what way does it affect the engineer's problem of the relation between discharge, hydraulic gradient, cross-sectional shape and type of bed surface irregularity; and on what does the rate of bed load movement depend and how can it be calculated? Since the conditions under flowing water vary markedly over the cross-section of its natural channel, whereas the corresponding conditions under a wind do not, it seems urgent that we evolve instruments capable of the ready measurement, at any given point in the water cross-section, of both the velocity of the fluid and the intensity of grain movement. As soon as, and not before, comparable data are made available, what can be observed in air can begin to be applied to what cannot be observed in water.

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The application of factor analysis to tests on cheese

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The basic principles of factor analysis are discussed. The technique is applied to the study of the relationship between a number of rheological tests and essential subjective judgments on cheese. The stability of the various "factors" is demonstrated by a comparison of two studies on groups of about 200 cheese made in successive seasons. The first two factors are shown to be closely related to measures of surface hardness (indentation by a loaded sphere) and subjectively assessed crumbliness, respectively. The remaining factors seem to be complex in origin, depending on arbitrary grouping in the tests and assessments selected.

An earlier contribution to this Journal (Harper, Kent, and Scott-Blair[1]) outlined the application of factor analysis to industrial test data. In view of the relative unfamiliarity of the technique it is perhaps desirable to make a formal statement of the fundamental assumptions. If a number of objects undergo a number of tests A, B, C, . . . in which scores or measurements a_i, b_i, c_i, . . . are allotted to the i-th object, correlations may be found to exist between the scores in various pairs of tests. A single correlation gives a standardized measure of the extent of the common variation in each set of measurements which are paired. Thus it may be considered that the correlation expresses the extent to which any two tests measure the same "factor." When the intercorrelations of several tests are considered the systematic relationships may require the postulation of several "common factors." The variation involved in any test may be split up into three components, that common to other tests, that due to random error, and the remainder which represents the essentially individual aspects of each test and is usually termed the "specific variance." Thus the information contained in a battery of tests A, B, C, . . . may, through the table of correlation coefficients, be represented by fewer factors F_1, F_2, . . ., thereby effecting further useful condensation. (When large masses of data have to be analysed some form of condensation is essential for the mental assimilation of the material concerned, and for the effective communication to others of the salient points of an investigation.) The factors could further be orthogonal in the sense that if tests could be devised to measure these factors alone, the scores allotted to the objects in any two factors might be uncorrelated. Thus orthogonal factors can be represented in a model by mutually perpendicular vectors. There is, however, no *a priori* reason why "oblique" or correlated factors should not be employed, although the present paper is restricted to the consideration of orthogonal factors.

The score allotted to an object in a test can be *estimated* by a linear function of the scores it would have received in idealized tests designed to measure each factor separately.

Thus $$a_i' = w_{1a}f_{1i} + w_{2a}f_{2i} + \ldots \quad (1)$$

and $$b_i' = w_{1b}f_{1i} + w_{2b}f_{2i} + \ldots \quad (2)$$

The estimates a_i' and b_i' do not take into account the inherent random errors in the original scoring or measuring procedures or the specific variance of each test. Thus it will not usually be possible for the estimates to be identical with the original scores, but the W's (which are the *factor loadings*, independent of the individual object) and the *factor measures* f_i are chosen to reproduce between a' and b' the observed correlation (r_{ab}) between a and b. Again, the reproduction will not be perfect in any actual investigation, but sufficient factors are extracted to reproduce the observed correlations within the limits of their own sampling errors. It is assumed that the scores in equations (1) and (2) are expressed in standard measure (i.e. as deviations from the mean expressed as a multiple of the standard deviation). With this proviso, the original correlations may be estimated by the equation:—

$$r_{ab}' = w_{1a}w_{1b} + w_{2a}w_{2b} + \ldots \quad (3)$$

Since the original scores are available in any case, no useful purpose is served by estimating them at this stage. Since the present paper is primarily concerned with *relationships* and their probable origin, equation (3) is of greater interest than equations (1) and (2). This brief statement, which must be regarded merely as an outline to give some orientation to what follows, must suffice at this stage.

It will be realized that if there is a set of orthogonal factors by which the original tests may be replaced, then by linear transformation there is an infinite number of such sets. By this process of transformation, or rotation of axes, a set of factors may be chosen of which at least some of the members correspond to realizable tests or to already recognized physical properties.

In fact the table of "factor loadings" enables certain inferences to be made about the qualitative nature of the tests. The relative values of the factor loadings for each

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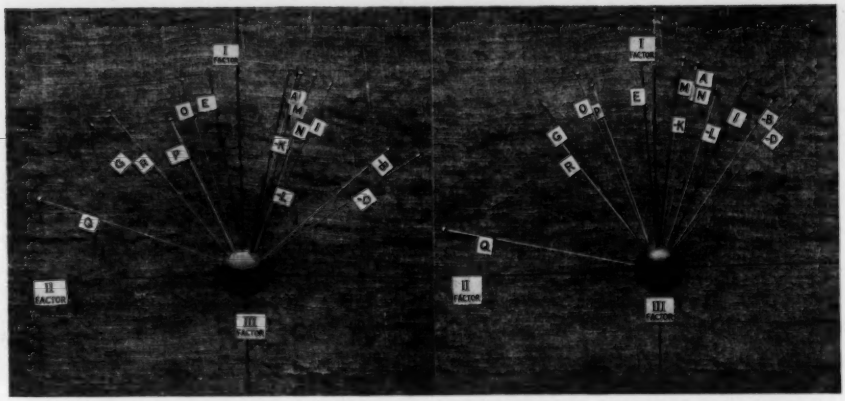

Fig. 1. Three-dimensional models indicating the main relationships between various tests and assessments.

(*left*) 1948 data; (*right*) 1949 data.

Each test is represented by a labelled vector. The distance of the label from the origin expressed as a fraction of the total length is the square root of the corresponding communality. The correlation between any pair of tests is given approximately by the scalar product of the corresponding vectors.

factor similarly allow inferences to be made as to the origin and nature of the factors, as contributory sources of variance to the numerical measurements and scores. Linear transformation, or rotation of axes, may lead to more clearly defined factors which may be selected for the economical description of the material tested. The selection and definition of the main "dimensions" of a given field of observation is one of the functions of factor analysis, although after the first few preliminary studies specific hypotheses should be formulated and tested.

EXPERIMENTAL PROCEDURE

The experimental data cover two separate investigations in the same factory in successive years. By this means it is hoped to show that the technique gives reasonably stable results, although systematic differences call for interpretation. The data obtained in 1948 have already been described elsewhere[2] and the main conclusions outlined. The present study will be concerned mainly with appraising the data obtained in 1949. No apology is needed for the inclusion of subjective data (in numerical form) in this study. A number of the mechanical tests have been developed by copying the actions of the professional cheese-grader and, no matter how interesting the rheological properties of cheese may be, from an industrial point of view overall quality and flavour (which are closely related) are the most important characteristics. In spite of the essential subjective nature of flavour and overall quality, the average assessment by a small group of individuals is quite stable and the rheological properties, in particular surface hardness, are partially predictive of both the immediate quality and its future development.[3]

In 1949, 185 Cheshire cheese, manufactured over a period of two months, were systematically examined by means of the mechanical tests and the subjective assessments. Five cheese were selected each day from the same vat, stored under controlled conditions, and tested when 2 weeks and when 2 months old. The present paper deals solely with the two-months-old cheese.

Details of the tests and the subjective assessments

Objective tests.—The ball compressor, as the name implies, is a type of sphere indenter in which a loaded sphere 1 in in diameter is pressed into the cheese, the depth of indentation being measured on a dial gauge. *A*, the amount of indentation after loading for 30 sec, and *B*, the percentage recovered elastically 30 sec after removal of the load are recorded.

The Meyer figure *D* is given by the slope of the curve of the logarithm of the deformation against the logarithm of the time when a steadily increasing load is applied to the indenting sphere. For this test, the ball compressor was modified by replacing the fixed load by a beam along which a weight was drawn at a steady rate. The instrument was thus designed to measure hardening rather than hardness.

E is a measure of the depth of penetration of a standard cork borer, 2 cm diameter, 60 sec after applying a load of 4·5 kg, recorded by means of a scale and vernier attached to the vertical parts of the apparatus.

The needle penetrometer, which is based on the principle of the standard bitumen penetrometer, consists of three needles attached at equal distances round the circumference of a circular disk, 10 cm in diameter, which slides over a central vertical column to which a scale and vernier are attached. The depth of penetration 60 sec after application of a load of 600 g was measured. These penetrations, through the rind and with rind removed, are represented by *I* and *G* respectively.

In tests *E*, *G*, and *I*, the penetrating device was initially inserted in the cheese to a fixed depth before the run was started.

K measures the resistance of the cheese to penetration of a hand-operated skewer, of traditional design, recorded by the compression of a strong spring attached to the handle.

The Breaker is a type of Izod tester in which a three-pronged rake attached to a pendulum arm is made to swing through an angle of 90° and then to break a standard cylindrical boring of cheese clamped horizontally. The angles through which the pendulum swings are measured both with and without a sample in position. The index *L* is the difference between these two angles expressed as a percentage of the latter, thus representing the percentage energy used in the fracture.

S, the approximate density in g/c.c. of each cheese, was calculated from its weight and dimensions.

A measure of the particle size distribution was given by test *T*. Cylinders of cheese 2 cm high, 1·5 cm diameter, were compressed slowly between parallel plates through a distance of 0·75 cm. A short period of shaking separated the particles which were graded in 10 size categories with the aid of a template. *T*, the "index of crumbliness," is given by the difference between the number of particles below and above a certain size.

The depth of penetration of a long needle under an impact force due to a mass of 395 g falling through a distance of 18 cm, is represented by *U*. The principle is that of a pile driver, with one impact instead of several successive impacts.

For each test, four runs were performed on every sample, usually two on the top and two on the bottom of the cheese and a simple average taken.

Subjective assessments.—A series of subjective assessments was made on every cheese on the day of testing by a panel of three judges, one of whom was a retired professional cheese grader, the other two being engaged in carrying out the experimental work. The assessments of the various attributes were made on a 5 or 7 point scale. In all, seven separate characteristics were scored, the judges being provided with special recording and instruction sheets. The cheese were thumbed on the outside and scores for firmness and springiness (*M* and *N*) recorded. Ratings for similar "properties" of a boring are represented by *O* and *P* respectively. *Q*, the crumbliness of the sample, was assessed by breaking down a small piece of the boring between the finger and thumb, while the texture, *V*, was examined visually.

R, the overall quality score, is, by definition, an integrated assessment of physical appearance, body, texture, and flavour. The flavour was the main factor taken into account by the judges.

Replicate sets of ratings were made on each cheese by each of the three judges and a simple average of the six scores taken as a measure of each characteristic.

Table 1 gives the necessary details of the tests and assessments.

Table 1. *Key to tests*

	Objective test	*High values correspond to:*
A	Ball compressor total	Soft, easily indented.
B	Ball compressor percentage elastic	Appreciable recovery.
D	Meyer log deformation/log time slope	
E	Borer 60 sec reading	Soft, easily penetrated.
G	Needle penetrometer (rind removed)	Soft, easily penetrated.
I	Needle penetrometer (through rind)	Soft, easily penetrated.
K	Skewer	High resistance (to penetration).
L	Breaker percentage energy used in breaking	High resistance (to breaking).
S	Density	
T	Crusher	Large excess of small particles over large particles.
U	Falling needle	Easily penetrated.

	Subjective judgment	*Range of score*	*High values correspond to:*
M	Firmness (whole cheese)	1–7	Soft.
N	Springiness (whole cheese)	1–5	Lack of springiness.
O	Firmness (plug)	1–7	Soft.
P	Springiness (plug)	1–5	Lack of springiness.
Q	Crumbliness	1–5	Lack of crumbliness.
R	Overall quality	1–7	Poor quality.
V	Texture	1–5	Close texture.

EXPERIMENTAL DATA

Correlation matrix—Table 2

Although the correlation matrix represents an appreciable condensation of the raw data, the number of useful observations which may be drawn is very restricted. In order to save duplication, the complete correlation matrix involving 18 tests or assessments has been partitioned into a sub-cell containing only 14. These 14 tests or assessments are those used in the earlier study (1948 data), and have been analysed as a separate group. The additional four tests were added to the 1949 investigation as a result of the experience derived earlier. In particular, tests *S* and *T* were included to give an objective assessment of crumbliness which had not been adequately assessed by the breaker (test *L*).

It has already been indicated that the information provided by the correlation coefficients, although a considerable condensation of the original data, is still too diffuse for useful interpretation. However, two specific items should be noted:—

(i) The high correlation between the assessments of firmness and springiness, both on the outside (*M* and *N*)

and on the boring (*O* and *P*). This is supported by later observations.

(ii) Most of the correlations obtained in 1949 are higher than those in 1948, but examination fails to suggest that the increase can be attributed to systematic changes in the range of the measurements.

Factor analysis: 14 *test data* (1949)—*Table* 3

Five factors appear to be necessary to reproduce the observed correlations, within the limits of error. Extracting factors in such a way that each, in turn, accounts for the maximum amount of variance in the tests as a group, the contributions of factors I, II, III, IV, and V are 58·2%, 12·9%, 6·9%, 2·7%, and 2·0%; total, 80·7%. In the present form the variance accounted for by factors IV and V is extremely small and there is reason to believe that part of the high residual variance after factors I, II, and III is due to the fact that the pairs of assessments (*M* and *N*) and (*O* and *P*) are qualitatively indistinguishable and thus inflate the correlations r_{MN} and r_{OP} from the values that would be expected on the basis of "common factors" alone. The qualitative similarity of the pairs *M* and *N*, and *O* and *P* appears in the general analysis (see below). This leads directly to the suggestion that it might be valuable to repeat the analysis, including only one member of each pair.

In the original form, factor I is closely related to the Ball Indentation Test (*A*), $r_{AI} = 0{\cdot}949$. With a small rotation of factor I it might be possible to use this test for identification purposes, thus establishing an important "dimension" for the effective description of Cheshire cheese. (Both this and the various borer tests are believed to give measurements closely related to what the grader terms "body.") Factor II predominantly contrasts the effect of tests carried out on or through the rind of the cheese with those carried out within the cheese. This can best be substantiated by comparing the entries in Table 3 for tests *G* and *I*, and for assessments *O* and *M*. The main difference is that the first member of each pair represents a test or an assessment within the cheese, the second is a similar test or assessment on the outside of the cheese, i.e. through the rind. The most marked change in both instances is in factor II. However, factor II is most closely related to *Q*, the assessment of crumbliness $r_{QII} = 0{\cdot}817$, an assessment which by its very nature can have no counterpart on the outside of

Table 3. *Factor analysis*, 14 *test data*

	Factor					
Test	I	II	III	IV	V	h_3^2
E	+0·850	+0·078	+0·376	+0·210	−0·041	0·916
G	+0·714	+0·401	+0·314	+0·307	−0·034	0·865
Q	+0·190	+0·817	+0·164	−0·235	+0·205	0·828
O	+0·853	+0·219	−0·225	−0·092	−0·249	0·896
P	+0·840	+0·210	−0·300	−0·093	−0·137	0·866
R	+0·596	+0·400	−0·329	−0·102	+0·242	0·692
−*K*	+0·694	−0·088	+0·294	−0·183	−0·148	0·631
I	+0·746	−0·336	+0·284	+0·056	+0·064	0·758
−*D*	+0·646	−0·506	+0·174	−0·207	−0·040	0·748
A	+0·949	−0·227	+0·130	+0·045	−0·078	0·976
M	+0·924	−0·141	−0·120	+0·161	+0·106	0·924
N	+0·897	−0·153	−0·176	+0·099	+0·142	0·888
−*B*	+0·822	−0·437	−0·201	−0·122	+0·048	0·923
−*L*	+0·635	−0·238	−0·385	+0·149	−0·174	0·660
$\Sigma\lambda^2$	8·146	1·806	0·963	0·375	0·281	$\Sigma\Sigma = 11{\cdot}571$
% variance	58·19	12·89	6·88	2·68	2·01	82·65

Table 2. *Correlation coefficients*, 1949 *data*

Differences (1949–1948) in italics

	A	*B*	*D*	*E*	*G*	*I*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*	*S*	*T*	*U*	*V*
A	—																	
B	−0·846	—																
	−0·096																	
D	−0·730	0·737	—															
	−0·188	*0·081*																
E	0·838	−0·563	−0·534	—														
	0·048	*−0·089*	*−0·150*															
G	0·654	−0·304	−0·240	0·819	—													
	0·181	*−0·353*	*−0·233*	*0·111*														
I	0·821	−0·710	−0·710	0·733	0·501	—												
	0·073	*−0·055*	*−0·204*	*0·041*	*0·123*													
K	−0·723	0·561	0·583	−0·649	−0·503	−0·590	—											
	−0·204	*0·180*	*0·199*	*−0·066*	*−0·113*	*0·007*												
L	−0·636	0·723	0·420	−0·443	−0·282	−0·466	0·329	—										
	−0·142	*0·281*	*0·327*	*−0·148*	*−0·159*	*−0·143*	*0·109*											
M	0·887	−0·800	−0·623	0·751	0·616	0·696	−0·593	−0·661	—									
	0·126	*−0·239*	*−0·150*	*0·062*	*0·153*	*0·032*	*0·085*	*−0·397*										
N	0·852	−0·807	−0·609	0·688	0·556	0·676	−0·566	−0·647	0·951	—								
	0·150	*−0·221*	*−0·187*	*0·071*	*0·185*	*0·065*	*−0·049*	*−0·340*	*0·161*									
O	0·733	−0·636	−0·404	0·635	0·615	0·481	−0·579	−0·605	0·748	0·732	—							
	0·084	*−0·300*	*−0·154*	*−0·060*	*−0·035*	*−0·049*	*0·040*	*−0·277*	*0·050*	*0·206*								
P	0·712	−0·645	−0·426	0·609	0·558	0·450	−0·502	−0·593	0·759	0·765	0·892	—						
	0·185	*−0·357*	*−0·309*	*0·110*	*0·041*	*0·069*	*−0·165*	*−0·213*	*0·264*	*0·331*	*0·174*							
Q	−0·003	0·196	0·213	0·248	0·456	−0·091	−0·151	0·237	0·029	0·021	0·277	0·272	—					
	−0·085	*−0·197*	*−0·074*	*−0·072*	*−0·225*	*−0·079*	*−0·072*	*0·163*	*−0·154*	*−0·032*	*−0·114*	*−0·080*						
R	0·419	−0·441	−0·144	0·379	0·443	0·228	−0·232	−0·375	0·531	0·538	0·647	0·655	0·492	—				
	0·074	*−0·352*	*−0·095*	*−0·151*	*−0·163*	*−0·034*	*−0·022*	*−0·266*	*0·147*	*0·244*	*0·084*	*0·167*	*−0·055*					
S	−0·407	0·398	0·302	−0·295	−0·212	−0·285	0·112	0·475	−0·446	−0·365	−0·223	−0·290	0·294	−0·151	—			
T	0·105	−0·384	−0·228	−0·162	−0·379	0·127	0·025	−0·421	0·109	0·129	−0·005	0·002	−0·638	−0·107	−0·263	—		
U	0·853	−0·654	−0·597	0·806	0·691	0·758	−0·613	−0·484	0·779	0·743	0·603	0·563	0·000	0·316	−0·377	0·033	—	
V	−0·440	0·501	0·377	−0·190	0·005	−0·327	−0·166	0·611	−0·445	−0·405	−0·259	−0·314	0·623	0·131	0·516	−0·531	−0·379	—

the cheese. (It will be indicated later that tests *A* and *Q* may be used to define important properties of the cheese.)

None of the loadings after those of the second factor is high and any attempts to identify the subsequent factors will have to be tentative. In fact, there is evidence to suggest that the main dimensions of variation after that indicated by tests *A* and *Q* do not correspond to any realizable test. However, the factors are not *necessarily* to be identified as properties of the material. Some of them may represent the effect of multiple and overlapping groupings in the conditions of testing. If attention is restricted to orthogonal (independent) factors, the methods of analysis may *constrain* some of the factors to lie in a region not adequately covered by any realizable test.

With these reservations, factor III differentiates mainly tests *E*, *G*, *I* and $-K$ from $-B$, *P* and $-L$. The first group represent tests involving penetration, the second group may be expected to represent the main aspects of recoverable deformation, or its counterparts. To what extent this is concerned with amount of recovery as contrasted with rate of recovery requires further consideration. The qualitative interpretations of the "factor loadings" for factors I, II and III are similar to those put forward for factors I, II and III in the previous study. The loadings in the subsequent factors and the small total contribution makes it doubtful whether these can usefully be identified at this stage. Nor is the agreement between factors IV or V of the present investigation with factor IV in the previous one, at all good. However, the errors of estimation increase as the total contribution of a given factor decreases.

One further observation requires to be made on the 1949 analysis. Inspection of the "factor loadings" for the pairs of assessments (*O* and *P*) and (*M* and *N*) shows that the differences between the members of each pair are small and may probably be ignored (a precise test is not possible here). That (*O* and *P*) and (*M* and *N*) were to be regarded as qualitatively indistinguishable has been mentioned earlier and this is in agreement with observation made in the previous analysis.[2] It is interesting to note that professional cheese graders usually refuse to differentiate between the firmness and springiness of Cheshire cheese.

Factor analysis. 18 *test data* (1949)—*Table* 4

The extension from 14 to 18 tests shows up certain interesting facets of the computational procedure and its dependence on the particular selection of tests and materials used.

At each stage of the analysis appropriate tests are reversed in sign so as to make entirely positive the totals of each column of the correlation matrix, or of the corresponding sum of the residuals, left at each stage. The residual after extraction of the first factor would be of the type:—

$$\delta_{ab} = r_{ab} - a_1 b_1 \qquad (3)$$

When the four additional tests are added to the analysis, right from the first, the computational procedure requires that the sign of test *Q* shall be reversed. Allowances must be made for this before making a direct comparison between the 14 and 18 test analysis. Furthermore, in all factors after the first, since only relative differences between tests are of importance, the signs of the complete set of "factor loadings" may be reversed without any effect on the reconstructed correlation coefficients. This again stresses the need for care in comparing one analysis with another, for allowance must be made for the *possibility* of a complete reversal in signs.

Table 4. *Factor analysis*, 18 *test data*

	Factor					
Test	I	II	III	IV	V	h_3^2
$-Q$	+0·087	+0·842	+0·245	+0·091	+0·057	0·787
$-D$	+0·709	+0·111	+0·222	+0·353	−0·125	0·704
$-S$	+0·481	+0·281	+0·034	−0·266	−0·036	0·383
$-V$	+0·545	+0·687	+0·014	−0·394	−0·078	0·930
$-B$	+0·897	+0·179	−0·129	+0·258	−0·084	0·926
T	+0·237	+0·703	−0·159	+0·142	+0·134	0·608
$-L$	+0·762	+0·281	−0·243	−0·153	+0·204	0·783
U	+0·834	−0·187	+0·328	−0·057	−0·046	0·843
I	+0·778	−0·048	+0·317	+0·206	−0·091	0·759
E	+0·778	−0·415	+0·319	−0·115	+0·030	0·893
G	+0·581	−0·638	+0·201	−0·311	+0·032	0·883
A	+0·962	−0·134	+0·177	+0·076	−0·016	0·982
$-K$	+0·619	−0·344	+0·107	+0·445	+0·310	0·806
M	+0·934	−0·136	−0·060	−0·007	−0·126	0·910
N	+0·903	−0·135	−0·146	+0·056	−0·136	0·876
O	+0·774	−0·336	−0·325	−0·086	+0·209	0·869
P	+0·770	−0·281	−0·364	−0·139	+0·052	0·825
R	+0·450	−0·420	−0·579	−0·070	−0·200	0·758
Σk^2	9·113	3·032	1·196	0·863	0·322	$\Sigma\Sigma = 14·525$
% variance	50·63	16·85	6·64	4·79	1·79	80·70

The number of successive approximations used in this case is only two, previous studies having shown that the changes from one approximation to another are much less than the changes from one year to another. The main function of the present analysis is to assess the nature of the additional tests *S* (density), *T* (crusher), *U* (falling needle), and the subjective assessment *V* (texture), in relation to the information provided by the background of the other, more familiar tests. The main features to note about the new tests are the following:—

S, density.—Within the present setting, this test has a very low communality,[1] which measures that part of the variability (variance) of the test scores (for one test), which may be accounted for by the suggested factors, as distinct from the part which is to be regarded as peculiar to the test itself (error variance plus specific variance). Since, as a measurement its value must be fairly precise, all that can be inferred is that characteristics covered by the other tests are only slightly affected by or related to density.

T, crusher.—This test, in which the difference between the number of small and large particles is calculated, was designed to give an objective measure of crumbliness. Comparison of the "factor loadings" in *T* and $-Q$ show a considerable degree of correspondence, although the crusher test is not completely independent of the

main borer/indentation characteristics (factor I). Rather unexpectedly, $-Q$ and T differ in sign in factor III.

U, falling needle.—The main contribution of this test is covered by factor I, and it seems to add little information not provided by existing tests.

V, texture.—The rating of textures, which may perhaps be termed the visual assessment of structure, shows an extraordinarily high communality, the highest of all the subjective assessments. In the present setting its main "components" are represented by factors II, I and IV. Factors II and I cover respectively the effects of crumbliness and the general cohesion of the material. Factor IV essentially contrasts $-K$ and $-D$ with G and $-V$, but this does not assist the processes of identification or description.

Diagonal method of factoring

Direct comparison, factor by factor, between two separate studies using the same tests but different samples of materials, is really based upon the implicit assumption that the two sets of samples of material are equivalent, random groups. Until an investigation has been carried out the information required to decide whether the sets of samples are equivalent is not available. The fact that four factors, extracted successively, in two separate studies are labelled I, II, III, and IV gives no guarantee that they will be even approximately equivalent in pairs. If one analysis includes additional tests then the successive factors of a purely statistical nature will most certainly not correspond precisely.

There are several methods of meeting this difficulty. One would be to rotate the axes into positions which may be expected to correspond in the separate studies. This presupposes an adequate objective criterion to decide when the rotation of axes is satisfactory. Even when this exists, the computational labour involved is considerable. Where one or more of the independent factors may satisfactorily be passed through a selected test then it is possible to use a method described by Thurstone,[4] which he has called the *Diagonal Method of Factoring*. Since, at each stage, the method makes use solely of the correlations with the one selected test it is really only applicable to what may be termed "infallible data" or data without error. To get over this difficulty the method has been applied not to the experimentally determined correlations but to the self-consistent set derived by reconstruction from the previous "centroid" or "simple summation" analysis. By this means in the 1948 and 1949 studies two factors may be "passed through" tests A and Q, respectively. The factors concerned correspond closely to the actual tests and are almost orthogonal. The residuals left in each case, after using these two factors to account for the correlations, are too small to identify any subsequent factors with a realizable test and one is inclined to regard the remaining systematic patterning of the correlations as due to "multiple and overlapping classification in the conditions of testing." In addition, the size of the residuals for specific cells in the correlations differs considerably from one year to the next.

It is not intended to reproduce here the whole analysis carried out by this technique. However, Table 5 gives the corresponding "loadings" for 1948 and 1949 for each test in factors which are defined efficiently by tests A (surface hardness) and Q (crumbliness rating). (In order to define a factor satisfactorily by a given test a correlation between factor and test of not less than 0·80 is arbitrarily selected.) Although the changes in factor A for tests Q, R, O, E, A, I and $-B$ are small, apart from Q, there is a preponderance of positive entries throughout the table, indicating that surface hardness, as measured by test A, makes a larger contribution to the remaining tests in 1949 than in 1948. The most marked increases are connected with tests G (needle penetrometer with rind removed) and $-L$ (pendulum breaker). The changes in factor Q are more evenly balanced, the most marked changes being a substantial reduction in the absolute size of the "loadings" for tests G and $-B$. Reiterating, the changes from 1948 to 1949 were as set out below.

(1) An overall increase in the contribution of surface hardness to all tests except Q (crumbliness). The increases are most pronounced for tests G and $-L$, although it is doubtful whether the changes are individually significant.

(2) The changes in the effects of crumbliness, as assessed subjectively, are generally more evenly distributed, with emphasis on a reduction of the absolute contribution to tests G (needle penetrometer, rind removed) and $-B$ (ball compressor, percentage elastic recovery).

Table 5. *Factors A and Q by diagonal method*

	Factor *A*			Factor *Q*		
Test	1948	1949	*Difference* (1949–1948)	1948	1949	*Difference* (1949–1948)
Q	+0·038	−0·011	−0·049	+0·814	+0·910	+0·096
R	+0·405	+0·413	+0·008	+0·631	+0·510	−0·121
G	+0·451	+0·652	+0·201	+0·806	+0·486	−0·320
O	+0·686	+0·755	+0·069	+0·438	+0·310	−0·128
P	+0·560	+0·725	+0·165	+0·346	+0·311	−0·035
E	+0·823	+0·861	+0·038	+0·377	+0·262	−0·115
−K	+0·582	+0·728	+0·146	+0·110	+0·141	+0·031
M	+0·795	+0·902	+0·107	+0·123	+0·048	−0·075
N	+0·731	+0·866	+0·135	+0·005	+0·035	+0·030
A	+0·956	+0·988	+0·032	0·000	0·000	0·000
I	+0·806	+0·829	+0·023	−0·045	−0·085	−0·040
−B	+0·835	+0·853	+0·018	−0·502	−0·205	+0·297
−D	+0·579	+0·747	+0·168	−0·323	−0·218	+0·105
−L	+0·438	+0·634	+0·196	−0·108	−0·221	−0·113

Further analysis of the possible reasons for these changes is required.

After extracting factors A and Q the predominant diagonal residuals without reference to sign, in order of magnitude, were:—

1948: K (0·234), M (0·220), P (0·219), O (0·184).

1949: R (0·261), P (0·244), O (0·231), L (0·209), G (0·204), B (0·153).

Since the main point of this particular analysis is to

compare most effectively the two successive years, the relative indeterminacy and lack of stability of the subsequent factors after *A* and *Q*, as defined by specific tests, make it undesirable to continue further. The presence of the pair *M* and *N* and also of *O* and *P* in the relatively high residuals may be related to their qualitative similarity, in pairs. This point has been discussed above. In 1949, the presence of *R* (overall quality) as the highest residual suggests that certain essential characteristics of the cheese can only be covered effectively by the inclusion of this particular subjective assessment. It would also be of interest to know its reliability, for if this is greater than the observed communality the assessment *R* may also cover stable characteristics not directly related to the rheological properties. Rheological measurements form the main contribution of the particular group of tests which have been studied.

Representation of factor analysis by 3-dimensional models

In discussing the comparison of two separate studies the phrase, "two factors may be passed through tests *A* and *Q* respectively," was used. This refers to the representation of the relationship between the various tests by means of the geometry of hyper-space. If only three factors are required to do this, or if the effects of the subsequent factors are small and may be ignored, then a solid model, such as that shown in the earlier paper,[1] may be made to represent approximately the relationship between the tests. The length of the vector representing any test is the square root of its calculated communality and the spatial distribution is such that within the limits of error (and also subject to the condition stated above) the correlation between any two tests is given by the scalar product of the corresponding test vectors. The figure shows two such models, one derived from the 1948 analysis and one from the 1949 analysis of the 14 test data. The degree of correspondence and the minor differences between the two studies can readily be seen.

Essential conclusions

On the basis of two independent studies of the relationships between a number of empirical tests and subjective assessments of the essential characteristics of Cheshire cheese, at least three independent "sources of variation" have been detected. Of these, the first two (in order of magnitude of their total contribution to this particular investigation) are closely related to the measurement of surface hardness (indentation under a constant load in a fixed time), and the subjective assessment of crumbliness. The first of these altogether accounts for over 50% of the observed variation between the various samples. In these studies no objective counterpart of the crumbliness rating was used. The third important "source of variation" cannot be identified with any realizable test, but seems to contrast the effects of tests involving penetration with those involving recovery of the material, in so far as these contribute to the pattern of relationships expressed by the correlation coefficients.

An analysis which included four additional tests or assessments gives useful information about them. In particular the excess of the number of small particles over the number of large particles obtained by breaking down a sample of the cheese in a standard manner is closely related to the subjective assessment of crumbliness. However, this purely objective index is not independent of surface hardness and, so far, no objective test has been designed which can adequately replace the subjective assessment of crumbliness. The predominantly visual assessment of texture has two interesting characteristics. First, it is related to both crumbliness and surface hardness. Second, since 93% of the variance of this assessment can be accounted for in terms of "common factors," the assessment itself must be fairly precise. Not more than 7% is left to be accounted for by errors of estimate and other minor contributory sources of variation. This compares well with the best of the objective tests.

Finally, although the evidence is not entirely conclusive from the present investigation alone, it is becoming increasingly apparent that purely objective measurements cannot be used alone to predict efficiently some of the important subjective assessments. Other aspects not adequately dealt with by the objective tests appear to play a part in some of the subjective assessments. With many important industrial products it is precisely these subjective characteristics which represent the final criterion against which the usefulness or effectiveness of the product is to be judged.

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Characteristics of some further films suitable for use in industrial radiography*

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The characteristics of two screen-type films, in combination with both lead and salt screens, one non-screen X-ray film and two ordinary single-coated photographic films are presented. The results show that Eastman M film has the highest contrast and finest grain of any non-screen film investigated, whilst the slow speed photographic films Ilford Line and Process allow the range of speed to be increased. Although the general use of salt-screen combinations is not recommended, they do increase the speed range very considerably by virtue of their high speed. For the conditions specified, the total speed range available is about 3 000 : 1 for standard development time, and some 100 000 : 1 if development is varied.

INTRODUCTION

In a previous paper† the characteristics of a number of films suitable for industrial radiography were described, particular attention being paid to effects due to variation of development time; the practical applications of the results were also discussed. This paper contains similar information concerning some additional films and film-screen combinations which have been found useful for various applications.

The following films have been investigated: Eastman M X-ray, Ilford Line, Kodak Industrex S X-ray, Ilford Industrial A X-ray, and Ilford Process.

Eastman M film is a fine-grain X-ray film used in America but not at present available in this country. Its characteristics render it very useful where the highest contrast with good resolution are necessary. Ilford Line and Process single-coated films have been included

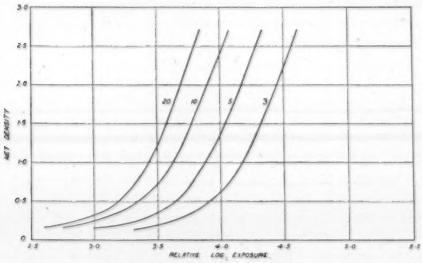

Fig. 1. Ilford Line film

since, although not X-ray films, they have been found to be of value where very high resolution is required.

The characteristics of all these films when exposed with lead intensifying screens were investigated, whilst Ilford Industrial A and Kodak Industrex S films were also exposed with salt intensifying screens, for use with which they are intended. 200 kV radiation from an X-ray unit by Newton-Victor Ltd. was used after filtration

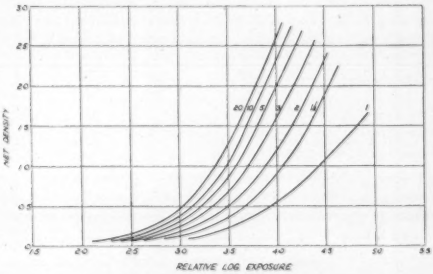

Fig. 2. Ilford Process film

through a steel plate 0·75 in thick for the non-screen exposures, and 1·5 in thick for the salt screen exposures. These thicknesses were chosen so that exposure times would be comparable with those occurring in practical radiography, i.e. in the range 10 sec to 15 min. The lead screens consisted of 0·003 in lead in front of the film, and 0·05 in behind the film. The focus-film distance was 36 in. The salt screen exposures were made with constant tube current and voltage corresponding with constant intensity to obviate possible complications due to reciprocity failure in the films. The films were developed in standard X-ray developer maintained at 68° F.

DISCUSSION OF RESULTS

The shape of the characteristic curves.—The characteristic curves for the various film-screen combinations are collected in Figs. 1 to 9 (the numerals on the curves are development times in minutes). The exposures are in milliampere-seconds.

It will be seen from the curves that Eastman M and Ilford Line films have slow speed and high contrast properties appertaining to films previously classed as group (i). Ilford Process film yields a characteristic curve similar in general shape except for its relatively low contrast.

Kodak Industrex S and Ilford Industrial A films, when exposed between lead screens, are relatively fast, coarse-grained, and of only moderate contrast; these are

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† DURANT. *J. Sci. Instrum.*, **25**, p. 105 (1948).

properties of films which have been classed as group (ii). When used with salt screens they have characteristics which justify their being placed in a separate category, namely group (iii). Thus the salt screen combinations have very high speed and their contrast is slightly higher

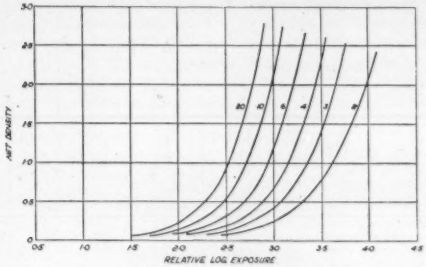

Fig. 3. Eastman M film

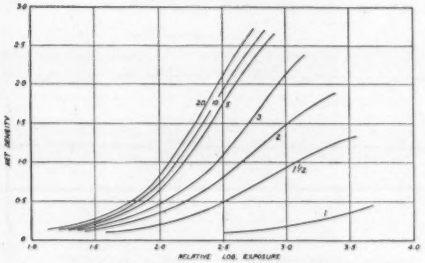

Fig. 4. Industrial A film and lead screens

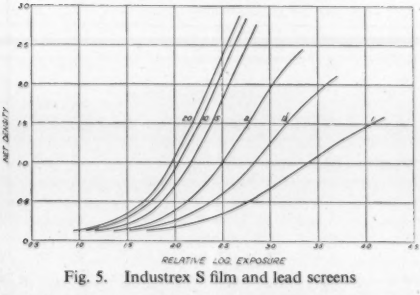

Fig. 5. Industrex S film and lead screens

than that of group (ii) films. Their curves also have very short toes, so that the contrast at low densities is comparable to that of group (i) films at corresponding densities. The general shape of the curves, however, is similar to those of Ilford Fine Grain Ordinary film in that the "gamma" is maintained over only a short density range, the shoulder of the curves commencing at a density of about 2·5. (The "gamma" of a film is the maximum contrast of which it is capable under specified developing conditions.)

Contrast characteristics.—Figs. 10 and 11 have been obtained by plotting the slopes of the characteristic curves at density 3·0 for groups (i) and (ii) films and on the straight line portion for group (iii) films. In Table 1 some derived values are given.

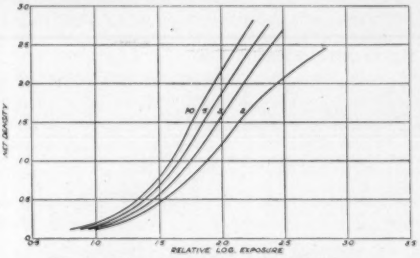

Fig. 6. Industrial A film and fast salt screens

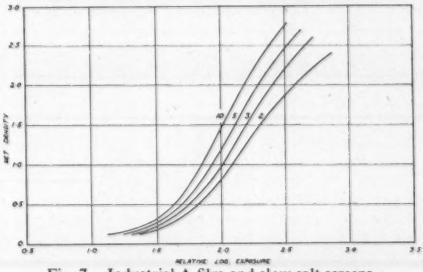

Fig. 7. Industrial A film and slow salt screens

Table 1. *Variation of contrast with development time*

Film-screen combination	*Contrast* (5 *min. dev.*)	*Maximum contrast* (20 *min. dev.*)	*Development time (min to reach* 90% *max.*	95% *max*
Films used with lead screens:				
Eastman M	5·2	5·8	5·0	11·5
Ilford Line	4·6	4·7	1·8	3·3
Ilford Process	2·8	2·9	2·0	4·6
Kodak Industrex S	2·6	2·9	4·4	11·1
Ilford Industrial A	2·4	2·7	5·3	11·3
Films used with fast salt screens:				
Kodak Industrex S	3·2	3·6	5·5	7·4
Ilford Industrial A	2·8	3·1	5·9	7·8
Films used with slow salt screens:				
Kodak Industrex S	3·4	3·7	4·8	6·7
Ilford Industrial A	3·0	3·3	5·3	6·4

Eastman M and Ilford Line films have contrast characteristics similar to Industrial C and Crystallex films respectively, the contrast at density 3·0 for 5 min development being 5·2 and 4·6 for the former, compared with 5·1 and 4·3 for the latter. The contrast of Eastman M film is high even for short development times, e.g. it is 5·0 at density 3·0 for 2 min development, and continues to increase with increasing time. On the other hand, the contrast of Ilford Process film is relatively low, being only 2·8 at a density of 3·0 for 5 min development; this contrast increases only slightly with time of development from about 2·5 min upwards, as seen in Fig. 10 and implied in Table 1.

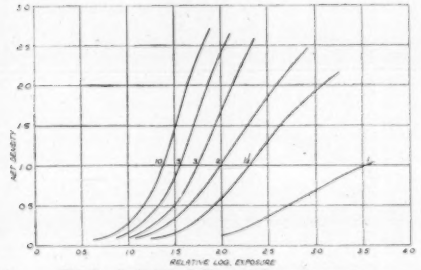

Fig. 8. Industrex S film and fast salt screens

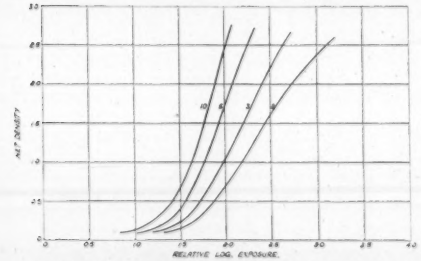

Fig. 9. Industrex S film and slow salt screens

Ilford Industrial A and Kodak Industrex S films with lead intensifying screens have similar contrast characteristics to group (ii) films; their "gamma" for 5 min development is about 2·5, the optimum being reached after approximately 4 min development.

When used with salt intensifying screens the contrast is appreciably higher; thus for 5 min development their "gammas" are respectively 2·9 and 3·2 with fast screens and 3·0 and 3·4 with slow screens. The small differences between the values for fast and slow screens used with the same film are of little significance and are considered to lie within the limits of experimental error. The results show that the standard practice of developing these films for 5 min is well founded in that optimum contrast is thereby attained. Although the film contrast obtained with salt screen techniques is not very high, this is counterbalanced by their high speed, which permits the use of considerably lower tube voltages than those required for high contrast types of film.

Speed characteristics.—The relative exposures required to give a net density of 1·5 for different development times are plotted in Figs. 12 and 13. Table 2 shows some values derived from the curves.

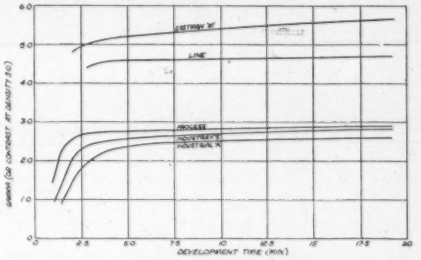

Fig. 10. Variation of contrast with development time for lead screen combinations

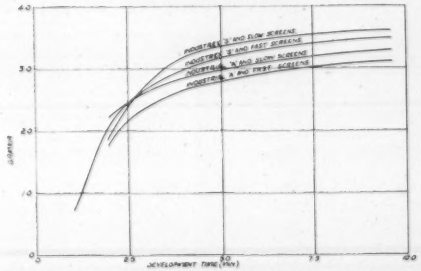
INDUSTREX 'S' AND SLOW SCREENS
INDUSTREX 'S' AND FAST SCREENS
INDUSTRIAL 'A' AND SLOW SCREENS
INDUSTRIAL 'A' AND FAST SCREENS
DEVELOPMENT TIME (MIN.)

Fig. 11. Variation of gamma with development time for salt screen combinations

Ilford Line and Process films are approximately ten times slower than the average group (i) film and would only be used where very high resolution is required and where the available tube voltage is sufficiently high to enable exposure times to be kept within reasonable limits. The speed of Line film increases considerably with prolonged development, and therefore fuller development may be used to compensate for its low speed. Process film, on the other hand, gains little in speed when development is prolonged, and the optimum time of development from the standpoint of both contrast and speed is about 3·5 min.

Eastman M film has approximately half the speed of Ilford Industrial C film for normal development, though its relative speed increases considerably with development time; since its contrast also increases with development

time, as seen from Fig. 12 and Table 2, without undue increase in graininess, prolonged development is essential for realizing the full merits of this film.

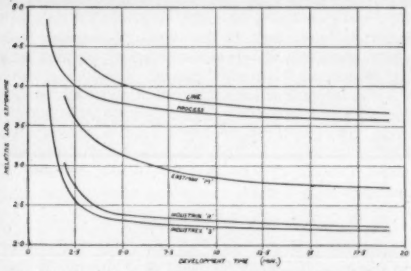

Fig. 12. Variation of exposure with development time for lead screen combinations

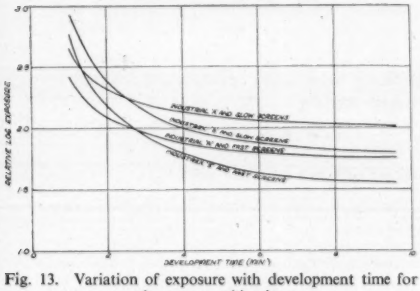
INDUSTREX 'A' AND SLOW SCREENS
INDUSTREX 'S' AND SLOW SCREENS
INDUSTREX 'A' AND FAST SCREENS
INDUSTREX 'S' AND FAST SCREENS
DEVELOPMENT TIME (min.)

Fig. 13. Variation of exposure with development time for salt screen combinations

Table 2. *Variation of speed with development time*

Film-screen combination	*Relative speed* (5 min dev.)	*Maximum relative speed* (20 min dev.)	*Development time (min.) to reach* 50% max.	80% max.
Films used with lead screens:				
Eastman M	8·6	10·4	6·5	11·2
Ilford Line	1·0	1·0	6·2	9·9
Ilford Process	1·6	1·3	3·4	7·9
Kodak Industrex S	50·0	34·7	4·1	8·3
Ilford Industrial A	39·4	31·2	3·5	13·3
Films used with fast salt screens:				
Kodak Industrex S	3 600	3 250	4·1	7·4
Ilford Industrial A	2 800	2 020	2·6	6·5
Films used with slow salt screens:				
Kodak Industrex S	2 200	1 910	3·4	6·9
Ilford Industrial A	1 400	1 120	2·4	7·0

Kodak Industrex S and Ilford Industrial A films, when used with lead screens, have speed characteristics similar to group (ii) films, but they are about 30% slower than Kodak Industrex D and Ilford Industrial B respectively. When exposed with salt screens their speed characteristics vary with tube voltage and current, focus-film distance, thickness of metal and other factors, and the following values are valid only for the stated exposure conditions. For the examination of 1·5 in of steel with a tube voltage of 200 kV and current of 3 mA, the focus-film distance being 36 in, the exposure with a slow film-screen combination would be reduced by a factor of approximately 40 from that required with group (ii) films used with lead screens. For a fast film-screen combination this factor would be approximately doubled, in agreement with published data obtained with widely varying exposure conditions.

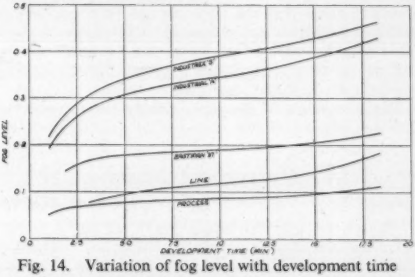

Fig. 14. Variation of fog level with development time

Prolonged development of group (iii) films produces a slight increase in contrast and speed, which is offset by increased graininess. There is therefore little advantage in exceeding a development time of 5 min.

Fog level and grain characteristics (*Fig.* 14).—In general, the faster films have a higher fog level which increases more rapidly as development is prolonged. The values vary considerably with the age of the film, especially for the faster types; the films used in these experiments were not more than a few months' old from the date of purchase.

The graininess of non-screen films increases with increasing speed, as noted in the previous report. Screen-type films used with lead screens, although slower, appear nevertheless to be a little grainier than group (ii) films, but when used with salt screens the graininess of the combination does not appear to deteriorate further. The salt screens do, however, result in poorer resolution mainly owing to scattering of the fluorescent light from individual crystals of the intensifying salt.

APPLICATIONS

Ordinary photographic films such as Ilford Line and Process are of only limited application in industrial radiography, being used where the highest practicable resolution is the principal requirement. One or other of these films will produce a radiograph of the required quality in a moderate exposure time, if sufficiently high voltage is available.

Eastman M film has the highest contrast of any film so far investigated for normal development times, and would be valuable for this characteristic alone. When, however, the development is prolonged, both contrast and speed increase considerably without undue increase in graininess. If finer grain is required shorter development times may be given without seriously reducing the contrast.

Kodak Industrex S and Ilford Industrial A films are rarely used with lead screens since better characteristics are obtainable with shorter exposures, using the ordinary non-screen films, but they are widely used with salt screens in industrial radiography owing to the very high speed of the combinations. The resulting resolution is, however, so poor compared with that of group (i) films that the latter should be considered as standard for all normal industrial radiography, in which allowance for their slow speed can be made by an increase in tube voltage; the required increase is often smaller than might be supposed.

The advantages of the group (iii) combinations occur when the thickness of metal under examination is too great for the available X-ray equipment, so that exposure times required with other films would be prohibitive. In conditions where the use of salt screens is necessary, the slow types should invariably be used since they give higher resolution than the fast types. If necessary their slower speed may be counterbalanced by working at a slightly reduced focus-film distance.

A disadvantage of the salt screen combinations is that the optimum contrast is maintained over a short density range. This useful range is only about 0·8, as compared with 2·0 for non-screen films, hence the range of specimen thickness which can be recorded in one exposure is correspondingly limited.

ACKNOWLEDGMENTS

Acknowledgment is made to the Chief Scientist of the Ministry of Supply for permission to publish this paper; to Mr. C. Croxson for his continued interest and assistance in its preparation; and to Mr. T. J. Shephard, who assisted in the experimental work.

The physical basis of the residual vacuum characteristic of a thermionic valve

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The physical basis of the form of the residual vacuum factor characteristic of a thermionic valve is investigated over the anode voltage range 50–250 V. The characteristic is found to be almost wholly due to soft X-ray radiation from the bombarded anode surface. The paper describes a method of separating X-ray photoelectric and ionic effects, and uses this separation to undertake low pressure measurements in the range 10^{-6}–10^{-9} mm of mercury.

1. INTRODUCTION

In a previous paper[1] the form of the residual vacuum factor k_0 of a thermionic valve was shown to be a direct function of the anode voltage V_a. The magnitude of the characteristic, as distinct from its form, was found to be concerned only with the geometry of a particular valve structure and the nature of its anode surface. Both magnitude and form were furthermore found to be independent of the nature of the cathode. Typical characteristics are shown in Fig. 1. Curve *A* is appropriate to a common form of pentode valve with oxide cathode, and curves *B* and *C* refer to a bright-emitter valve. The curves are typical of over a hundred cases examined and are characterized by a sharp discontinuity occurring when V_a is between 80 and 100 V. An explanation of the physical basis of the characteristic was sought in the works of Richardson and Chalklin, Thomas, Bell, Davies, and Gosling.[2,3,4,5] According to these works the impact of electrons on the anode gives rise to soft X-rays which irradiate the negative electrode and cause it to emit photoelectrons. The effect of this emission on an external circuit is indistinguishable from the reception of positive ions on the negative electrode and the commonly observed "reverse-grid-current" in valves is normally due to a superposition of both effects. Using anodes of nickel, tantalum, and thorium the author has always found the discontinuity to occur between 80 and 100 V, but the careful and detailed work of Richardson on nickel shows no such discontinuity of comparable magnitude. It appears therefore that either Richardson has missed the effect or that it is due to some other cause. The upper part of the characteristic shows close quantitative relationship with Richardson's work and it is therefore most probably due to X-ray irradiation. The lower part, however, shows some similarity to a positive ion characteristic and might be due to positive ions arising from a "ground" pressure in the valve or from sputtered atoms from the bombarded anode. It could equally well be explained as an X-ray effect of lesser intensity to that occurring above the discontinuity. One object of this paper is to investigate these alternatives.

2. SEPARATION OF EFFECTS

Means are required for separating the radiation and ion effects. The classical method employed by Richardson

and others was to employ a parallel plate condenser which filtered out the ions in a transverse electric field, whilst allowing the X-radiations to pass through without attenuation. The author has adopted the alternative scheme of "filtering" out the radiation effect leaving the positive ions for direct measurement. The reason for using the alternative method lay in a desire to improve the performance of triode ionization gauges. This point will be considered in Section 5.

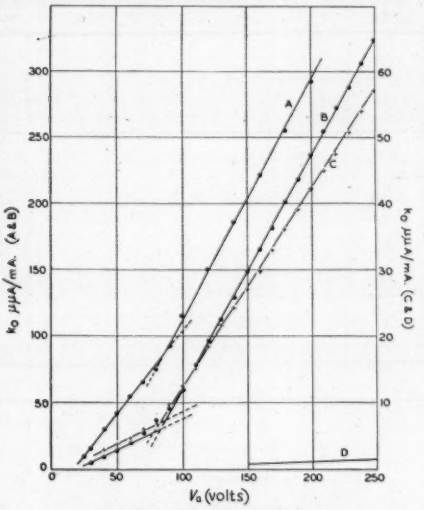

Fig. 1. Residual vacuum factor characteristics

3. TECHNIQUE OF PHOTOELECTRIC SUPPRESSION

A cross-section of the essential electrodes of an experimental valve for suppressing the radiation effect is shown in Fig. 2. The valve is a tetrode with a cathode *C*, an anode *A*, a cylindrical suppressor shield *S*, and a coaxial cylindrical collector *K*. The cathode is a thoriated tungsten spiral situated just above a slit in the suppressor shield. To determine the relative magnitudes of radiation and ion effects the valve is operated in the following manner. The anode voltage V_a is set at the value, say 200 V, at which the measurement is to be made and the anode current adjusted to suitable magnitude by controlling the temperature of the cathode. The suppressor shield and collector are both maintained at a negative potential of − 40 V. Under these conditions, a superposed stream of positive ions and X-radiations passes down through the slit in the shield and falls on the collector. The galvanometer *G* in the collector circuit then measures the totality of positive ions gained by the collector and photoelectrons lost by the collector due to X-radiation. The potential of the collector *K* is next raised from − 40 V to − 2 V and the photoelectron emission suppressed by the retarding field of − 38 V within the suppressor shield. The galvanometer now measures only the positive ion flow. Difference of the two measurements is the magnitude of the X-radiation effect. Similar measurements are carried out over an appropriate range of anode voltage V_a.

Experimental results are set out in Fig. 1. The curve *B* is a k_0/V_a characteristic using the suppressor shield and collector connected together as a common negative electrode. Curve *C* is obtained when the current is measured in the collector circuit alone, but with both collector and suppressor shield at − 40 V. Curve *D* is a measure of the ion current in the collector circuit after the radiation effect has been suppressed by a retarding field of − 38 V. These measurements were taken with the tube evacuated to a constant k_0 condition. Under such a condition it is concluded that approximately 97% of the current in the negative collector electrode is due to X-radiation and less than 3% to positive ion flow. Since the k_0/V_a characteristic is independent of the form of cathode it follows that the reverse-grid-current measured in oxide-cathode valves is almost wholly due to X-radiation when measured in the k_0 state. Measurements made on calibrated pentode valves at this station have shown that the actual gas pressure in the k_0 state must be less than 1×10^{-9} mm of mercury. Actual measurement of the pressure in the k_0 state has not yet been achieved, but the author and Dr. S. Wagener are attempting to make the measurement in a special tube combining an oxide-cathode with the photoelectron suppression technique.

The 80–100 V discontinuity noted as a general phenomenon in all types of thermionic valves examined is probably due to generation of X-rays from deeper levels in the atoms of the anode material. Such a discontinuity would be expected at about 90 V in the cases of nickel, tantalum and thorium.

4. VALIDITY OF TECHNIQUE

The validity of the suppression technique depends upon two assumptions—that no X-radiation reaches the inner surface of the suppressor shield, and that all photoelectrons generated from the collector are suppressed by the retarding field within the shield. It will be apparent from Fig. 2 that the geometric layout of the electrodes makes it impossible for X-radiation from any part of the anode surface to reach the inner surface of the shield except by reflection from the collector. Search has been made for evidence of such reflexion, but it has not yet been found. The intensity of the retarding field within the suppressor shield necessary for the suppression of the photoelectrons has been investigated and the results are shown in Fig. 3. With $V_a = 200$ V, $I_A = 38$ mA, $V_g = -40$ V it appears that 85% of the photoelectrons have an energy of less than 10 eV and

that a 30 V retarding field is sufficient for total suppression within the limit of experimental sensitivity employed. Similar measurements carried out at anode voltages ranging from 150 V to 250 V showed no appreciable variation of the energy distribution with anode voltage. It will be apparent from these results that the energy of the photoelectrons is much lower than might be expected from the Einstein equation. This is, however, in accordance with the observations of earlier workers on the energy distribution of photoelectrons ejected by soft X-rays. Rudberg[6] and Kirchner[7] suggest that a soft X-ray photon penetrates the metal surface and then transfers its energy to an electron which proceeds to eject a number of lower energy electrons from the metal surface. The energy of these "secondary" electrons is commonly observed to be of the order of a few electron volts.

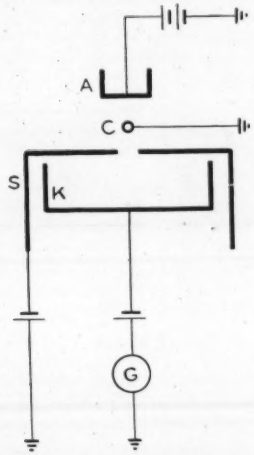

Fig. 2. Electrode layout of experimental tube

5. USE OF EXPERIMENTAL TUBE AS AN IONIZATION GAUGE

All triode ionization gauges have a lower limit of measurement set by soft X-ray irradiation of the positive ion collector. This limitation appears to be fundamental, although it may be mitigated to some extent by lowering the anode voltage V_a or by Baynard and Alpert's[8] device of decreasing the irradiated area of the collector. The lower limit of commercial ionization gauges available in this country appears to be about 1×10^{-6} mm of mercury for accurate measurement. At lower pressures the radiation effect becomes increasingly comparable in magnitude with the true ion effect and accuracy progressively decreases. A four-element tube of the suppressor type appears to avoid the inherent limitation of the triode gauge. Providing suppression is complete and no electron emission occurs on the inner surface of the suppressor shield, there appears to be no obvious lower limit to measurement. Practical tubes, known as suppressor ionization gauges, have now been in use at this station for over a year and have given good service in the range 1×10^{-4} mm of mercury to 1×10^{-9} mm of mercury. Details of this gauge and its control unit will be published in due course.

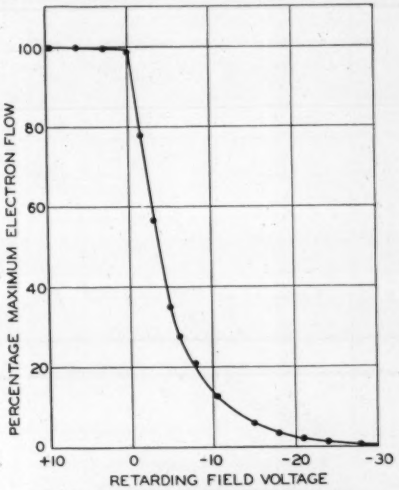

Fig. 3. Energy distribution of photoelectrons

ACKNOWLEDGMENTS

Acknowledgment is made to the Engineer-in-Chief of the General Post Office for permission to make use of the information contained in this paper. The author also wishes to record his appreciation of the skilled technical assistance of Mr. D. J. Sargent.

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Factors limiting the accuracy of the electrolytic plotting tanks

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Experiments are described to investigate the sources of error which occur in plotting tanks. The effects due to a.c. polarization were measured in a test cell. Other errors due to misalinement and surface tension (meniscus) were investigated by means of a special experimental tank consisting of two concentric cylinders. Potentials in this differed by not more than 0·2% from the calculated potentials at any given point on the surface, provided the necessary precautions were taken. It appears possible, but difficult, to reduce this figure of 0·2%. An inferior figure can be expected for the more usual type of test apparatus.

In the study of magnetic or electrostatic electron lenses, and for many other purposes, accurate knowledge of the potential field distribution produced by the pole-pieces or electrode system is required. In the absence of space charge the potential field is Laplacian.

With the exception of special cases the potential field distribution can be obtained only by relaxation or analogue methods. The former can be carried out to any required accuracy but involves considerable labour. The latter are much easier to apply but the results are of limited accuracy. This paper describes an investigation into the limits to the accuracy of the most common of these analogue methods, the electrolytic plotting tank.

It is a generally accepted opinion that an accuracy of 1–2% only is obtainable for field plots by the tank method.[3, 10, 18] The inaccuracy is ascribed to the "polarization" effect in the electrolyte which results in the disturbance of the desired conditions near the electrode surface. Electrically the effect is considered as being due to the inclusion of a reactive component with an associated loss component in the tank impedance.

Since for computation of lens errors in electron optics it is generally necessary to know the potential distribution with an error of considerably less than 1% it was decided to ascertain the limiting accuracy which could be obtained by the tank method.

An initial consideration of the problem and past work leads to the conclusion that the chief sources of error would be the mechanical inaccuracies, the polarization effect and the distortion of the liquid surface by surface tension. Although it appears that the importance of screening of electrical circuit components and balancing of stray fields has not always been realized, the circuit technique should present no difficulty; the bridge can always be checked by substitution of standard resistances for the tank.

Under mechanical inaccuracies are classed inaccuracies in the shapes of the electrodes, misalinement of the electrodes, false positioning of the probe and errors arising from too great a depth of immersion of the probe in the case of three-dimensional systems. It is difficult to formulate a general statement with regard to these errors since circumstances vary greatly for different electrode systems. Local inaccuracies in the electrode shape and/or alinement may produce a purely local effect on the field. On the other hand, since the probe position can generally be determined very accurately, it appears advisable in cases where the entire field is to be plotted, to ensure that the geometry and alinement of the electrodes conform to the same order of accuracy as that to which the probe position can be determined. Using recognized engineering methods, an accuracy of at least a thousandth of an inch in the probe motion should be attainable. The probe position error can be estimated by multiplying the probe shift by the corresponding potential gradient at the probe. This also applies in a vertical direction in the case of three-dimensional systems and leads to the requirement of a very small depth of immersion of the probe.

Surface tension gives rise to the formation of a meniscus both at the probe and at the electrodes. The latter causes a distortion of the field near the electrode surface and in addition appears to exert a small force on the probe in the vicinity of the electrode. Since the probe is preferably made very thin, this force may cause it to bend, resulting in an additional positional error shown as an apparent increase in backlash of the probe micrometer mechanism. Measurements are described later.

THE POLARIZATION EFFECT

Much has been written on the subject of polarization in electrolysis,[11, 12, 19] and yet it is not easy to abstract from the literature the factors relevant to the present problem. The magnitude and properties of the polarization effect were studied here in some initial experiments.

A simple electrolytic cell consisting of polished copper electrodes (1 cm^2) and an $M/4$ solution of copper sulphate was connected in a Wheatstone bridge circuit energized with a variable frequency a.c. voltage (Fig. 1). The out-of-balance voltage was plotted as ordinate against the input voltage as abscissa on a cathode-ray oscillograph. If the cell acted as a pure resistance the bridge could be balanced and a straight line would be observed on the screen, being horizontal at balance. In practice, however, two effects were noticed. At low frequency an ellipse, distorted into a "hysteresis-like" loop, was obtained, while at higher frequencies (greater than a few hundred cycles a second) the figure approached an ellipse. The figure could be tilted by adjusting the bridge balance. These effects were fairly general.

The former figure demonstrates that the cell introduces a non-linear circuit element in addition to a

reactance. The non-linear element may be resistive, reactive or both. At higher frequencies the non-linearity disappears but a reactive component remains.

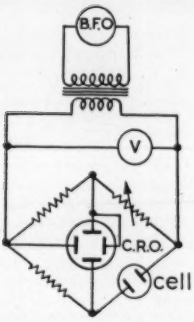

Fig. 1. Bridge circuit used to determine nature of polarization effect

A variable capacitor was next connected across the resistance arm in series with the cell, and it was found possible to balance out the reactive component completely at higher frequencies to give a straight horizontal line showing the reactive component to be capacitive.

A further test in which the voltage across the bridge was varied showed that at lower frequencies the magnitude of the reactance and the linearity were dependent on the current passing through the cell, whereas at higher frequencies balance remained unaffected by this variation. The maximum current density in these tests was 5 mA r.m.s. per cm^2 of electrode.

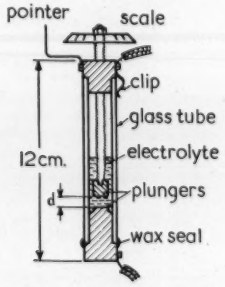

Fig. 2. Impedance test cell

Since the electrolytic tank analogy only truly holds for a tank which is only resistive, it is of importance to investigate more closely this polarization reactance and to determine the conditions under which it can be reduced to a minimum.

DETERMINATION OF THE ELECTRODE SURFACE IMPEDANCE

Further tests showed that the equivalent circuit of the cell could be envisaged as consisting of a resistance R, corresponding to the main body of the electrolyte, in series with a parallel combination of resistance and capacitance (r, c) corresponding to the surface impedance of the electrodes. This was demonstrated using a cell (Fig. 2) similar to that employed by Jones and Christian,[11] consisting of various metal plungers whose separation was continuously variable, fitting closely into a glass tube. The cell was connected in the bridge circuit of Fig. 3.

By setting the separation of the plungers to a maximum of 1 cm and decreasing it in small steps down to a value just prior to contact, it was found that only adjustment of the resistance R in series with the cell was required to maintain the bridge balanced, thus demonstrating the main body of the electrolyte under

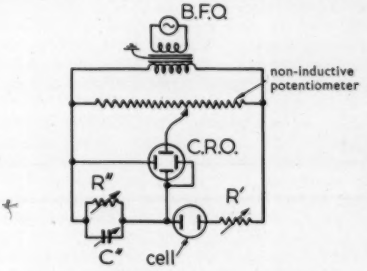

Fig. 3. Bridge circuit used with tests on impedance test cell

test to be resistive and the capacitive component to be present only very near the electrodes. When the plungers were brought into contact a definite jump of the trace on the oscillograph indicated a sudden change in the cell impedance owing to the short circuiting of the electrode surface impedance.

A value F was chosen as a figure of merit for any electrode-electrolyte combination. It is defined as the ratio of the resistance of one centimetre cube of the electrolyte (m_a) to the surface impedance (z) across one square centimetre of one electrode and gives a general idea of the effective electrode distortion produced by the polarization effect.

The determination of the cell impedance, and hence F, was carried out in a substitution bridge (Fig. 4) using the test cell of Fig. 2. The bridge consisted of the two ratio arms (the potentiometer) balancing against a parallel combination of resistance and capacitance (R_1, C_1) and the cell, the latter being replaceable by the equivalent combination of a resistance R_2, corresponding to the electrolyte, in series with the parallel resistance

and capacitance elements R_3, C_3 corresponding to the surface impedance. The conductance of the electrolyte and hence the value R corresponding to a given electrode separation was first determined by noting the change in R_2 required to maintain balance for a given change in electrode separation. A balance was now obtained with S_2 closed; with S_2 opened, S_1 closed and R_2 set equal to R, the values of R_3 and C_3 required to rebalance the bridge then corresponded to the surface components of the cell. The procedure was repeated for several values of the electrode separation and the mean values for R_3 and C_3 taken. This was desirable since a small error in the value of R_2 had a large effect on the measured value of the electrode surface impedance. A maximum spread of 8% in the value of z resulted.

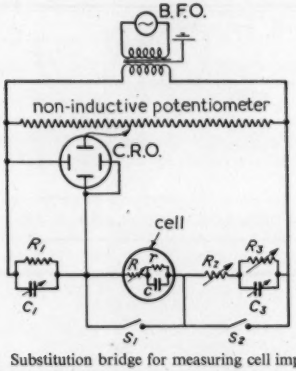

Fig. 4. Substitution bridge for measuring cell impedance

The method was applied to various electrode surfaces and electrolytes. The results are summarized in the table, together with the results abstracted from the paper of Jones and Bollinger.[12] The latter results have been amended to correspond with the rest of the data of the table.

Substantial reductions of the surface impedance can be brought about by graphite or platinum black coatings.

In the tank the error due to the voltage drop across the electrode surface impedance can be considered equivalent to a shrinkage of the electrode of the order of $1/F$ cm in a direction away from the electrolyte. This is not strictly correct since the surface impedance is complex; the shrinkage is, however, never greater than $1/F$ cm. For more practical electrode systems it is reasonable to assume that a positional (i.e. mechanical) accuracy of the order of 0·004–0·001 in is usually obtainable, depending on the particular geometry. In order that the error due to polarization should be no greater, $1/F$ in should be less than this, i.e. a value of F of the order of 100–400 is required. The table shows that such values can be obtained by using a graphite coating, this being generally adequate and easily applied, though in special cases platinization may be desirable.

VARIATION OF SURFACE IMPEDANCE WITH ELECTROLYTE CONCENTRATION, TEMPERATURE AND FREQUENCY

Polished and cleaned copper electrodes immersed in copper sulphate solution were used. In the first experiment the cell temperature was kept at 0° C and the strength of the copper sulphate solution was varied from $M/2$ to $M/30$, whilst in the second case the concentration of the solution remained fixed at $M/2$ and the temperature of the cell was changed from 1° C to 70° C. The applied frequency in both cases was 1 500 c/s. The variations of F with temperature and concentration are shown in Fig. 5. It is observed that the value of F increases rapidly with decreasing concentration of solution, varying by a factor of 5 for a change in strength of $M/2$ to $M/30$. Similarly, as the temperature is raised from 1°–70° C the value of F increases by a factor of 8. The latter results indicate that it may be advantageous to run an electrolytic tank at higher temperatures. It would, however, no doubt be extremely difficult to maintain stable conditions at other than room temperatures.

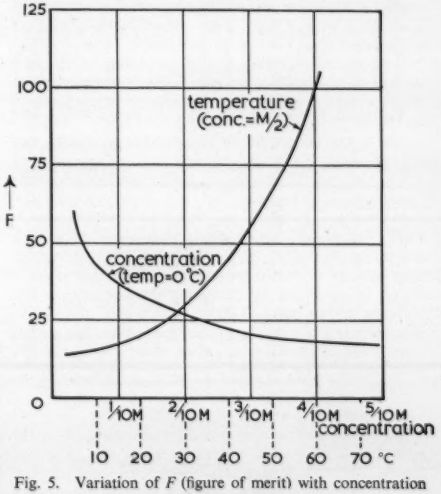

Fig. 5. Variation of F (figure of merit) with concentration and temperature
Copper electrodes, copper sulphate solution, frequency 1 500 c/s

Another method of measuring the surface impedance components with fair accuracy consisted of pushing the electrodes so close together that the electrolyte resistance

became negligible. This was used to determine the variation of surface impedance with frequency. Since now the equivalent circuit of the cell consisted simply of a resistance and capacitance in parallel, these values could be measured directly without substitution. The frequency was varied over a range of 200–4 000 c/s, and the values of r and C as well as $\tan\phi = \omega Cr$ plotted as a function of frequency in Fig. 6.

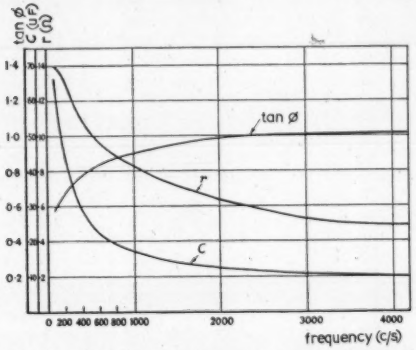

Fig. 6. Variation of electrode surface components with frequency
Polished copper electrodes, $M/2$ copper sulphate solution

The value of the surface impedance falls with increasing frequency, the power factor angle varying only slightly at higher frequencies. Thus the value of F can be increased by increasing the frequency. The increase is slow above 1 000–2 000 c/s and the relative disadvantages of working the circuits at high frequencies soon outweigh the resulting reduction of polarization. A suitable operating frequency would be around 1 500 c/s.

MEASUREMENTS IN A TEST TANK

The remainder of the investigations was concerned with measurements of potentials in a test tank. A simple electrode system was constructed consisting of two concentric cylinders. Such a system was chosen because of ease of construction to a high degree of accuracy; the field distribution is known and of a simple form and no errors are introduced by approximation at field boundaries, such as is the case with models immersed in an insulating tank of finite dimensions. In addition, the electrodes have unequal surface areas and curvatures and thus afford an opportunity for comparing the extent of polarization and surface-tension errors at two electrodes at which different fields exist.

The tank is shown in Figs. 7 and 8. The inner cylinder consisted of copper and the outer cylinder was made from a brass tube subsequently copper plated. The probe block was traversed along a rectangular bar by a micrometer screw. The bar was attached to a rotating rim, the probe thus being enabled to explore the electrolyte surface along any radius.

The probe consisted of a portion of polythene cable whose central copper conductor was etched to a diameter of 0·0009 in over a length of $\frac{3}{8}$ in. The probe fitted through an insulator in the probe block and its height was adjustable. Grooves were cut into the insulator to increase the leakage path to the probe. The tank was provided with three levelling screws and mounted on a solid foundation to avoid vibrations.

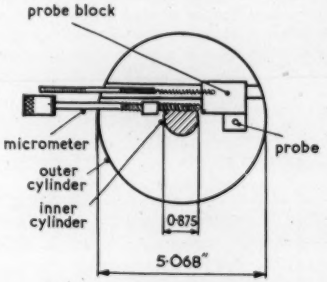

Fig. 7. Test tank (plan); depth 5 in

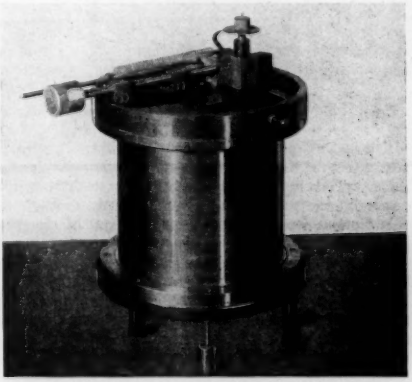

Fig. 8. Photograph of tank shown in Fig. 7

The electrode surfaces were cleaned with fine emery paper and ether, and the tank levelled and filled with singly distilled water. The resistance between inner and outer cylinders measured 250 Ω.

The tank was now filled until no meniscus existed at the electrodes. This was achieved by bringing the electrolyte surface coincident with the plane of the top

of the electrodes, the latter having been machined flat. The disappearance of the meniscus could be detected by observing the reflection of a lamp in the electrolyte surface. It will be shown that the removal of the meniscus is necessary since the error in the measured field distribution due to the meniscus is relatively large in the case where the probe only skims the electrolyte surface. The meniscus formed at the tip of the very fine probe was observed to be negligibly small.

The tank was connected as a potentiometer with the probe as centre tap, and together with a 0·1% decade potentiometer (H. W. Sullivan Ltd.) formed a potentiometer-bridge circuit as described elsewhere.[4] The detector was a cathode-ray oscillograph whose *Y*- and *X*-amplifier inputs were connected to the probe and potentiometer slide respectively. It was found essential to include a Wagner earth to balance out stray impedances. The effect of the latter was particularly marked, even with the electrolyte resistance between inner and outer cylinders as low as 250 Ω, since there was found to exist a very large impedance between the probe and the electrolyte, comparable in magnitude to the stray impedances. When the probe was close to one of the electrodes (within 0·1 in, i.e. 5% of traverse), a point balance could not be obtained owing to the effect of the electrode surface reactance; instead a small circle of minimum diameter was taken as the balance condition. This effect was, however, negligible at the applied frequency of 1 500 c/s.

It is probable that the difficulty encountered in balancing the bridge with an electrolytic plotting tank, which is usually ascribed to polarization, may in many cases be caused by the effect of stray capacitances, this effect being greatly enhanced by the high value of the probe-electrolyte impedance.

The bridge was checked against two 0·1% decade resistance boxes (Muirhead and Co. Ltd.) in series substituted for the tank. The potentiometer readings were found to agree to within 0·1% with the ratio of the resistance boxes over its entire range. In this test the effect of the probe-electrolyte impedance in the tank was allowed for by including a 1 MΩ resistance in the *Y*-amplifier input lead, serving also to check the Wagner earth. The bridge was sensitive to a change of the potentiometer ratio of 0·0002.

1·25 V was applied between the inner and outer electrodes. Potential measurements were carried out by setting the probe to the required position in the tank and balancing the bridge by reducing the ellipse on the oscillograph to a point;[4] under this condition both probe and potentiometer slide were at earth potential and no current was drawn by the probe.

The zero position of the probe was determined by moving it backwards (i.e. towards the inner electrode) until a jump on the oscillograph indicated contact with the electrode. The micrometer motion was then reversed, overcoming first the backlash and slight bending of the probe until the jump occurred once more, giving the zero position of the probe. At the outer electrode the micrometer reading was noted at the occurrence of a similar jump and generally found to be within 0·002 in of the expected value over its 2 in travel.

The probe was moved along four radii at 90° intervals and the measured voltage distribution compared with the theoretical values calculated from the expression

$$V = \frac{100 \log_{10} r/R_1}{\log_{10} R_2/R_1} \%$$

the total error at a given point being the difference between the theoretical and measured potential. Error distributions were drawn and the maximum error for each orientation observed to be less than 0·2% of the applied voltage, whilst by averaging the results of the four runs the random errors were smoothed out, reducing the error to about half this value.

The average observed values of the polarization drops at the inner and outer electrodes were 0·17% and − 0·05% respectively. This is in good agreement with the values 0·36% and − 0·075% estimated on the basis of an electrode shrinkage of $1/F$ cm. F for the experimental tank was taken as 120 cm^{-1} giving an equivalent shrinkage of the electrodes of $1/(120 \times 2{\cdot}54) \simeq 0{\cdot}003$ in. The surface drops are given by multiplying this equivalent shrinkage by the corresponding potential gradients of 0·12% and 0·025% per 0·001 in at the inner and outer electrodes.

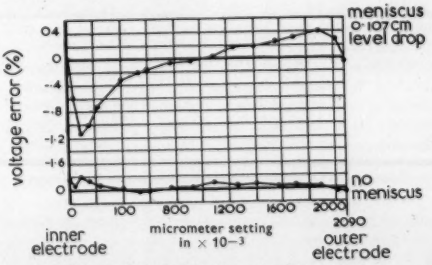

Fig. 9. Effect of meniscus in the test tank

The random probe position error was of the order of 0·001–0·002 in corresponding to a potential error of 0·12%–0·24% and 0·025%–0·05% near the inner and outer electrodes. This error is of the same order as the error due to polarization.

EFFECT OF THE MENISCUS AT THE ELECTRODES

The potential distribution was now measured with a water level reduction (0·107 cm), giving rise to a meniscus at the electrodes, and the error distribution together with the distribution for no meniscus plotted in Fig. 9. With a meniscus at the electrodes the error voltage was observed to have increased by a large

amount, rising at one point to an average value of 1·2%, i.e. to six times the value with a flat electrolyte. It was assumed that the meniscus at the tip of the very fine probe could be neglected.

One fact observed was that the apparent backlash on reversing the micrometer at the inner electrode had increased from 0·001–0·002 in to 0·005–0·006 in. in the presence of the meniscus. This increase in backlash is probably due to the probe being drawn towards the electrode when close to it. The increase in the potential error can be attributed jointly to this additional mechanical error, and to the meniscus in whose proximity the tank analogy no longer holds; furthermore, near and in the meniscus the portion of the probe immersed increases as the probe moves towards the electrode.

BOUNDARIES

The distortion of the field due to the proximity of the tank walls varies a great deal for different systems. In general, however, the error due to the presence of the tank walls should be made not greater than the mechanical and polarization errors. For a given tank size the scale of the model should be chosen to fulfil this condition. The effect due to the tank walls can easily be assessed experimentally by observing the change in the field due to an alteration of the tank boundary (e.g. by effectively moving the boundary towards the electrode system by introducing an insulator into the tank). In many cases the model under investigation possesses some surrounding electrode, in which case the latter may be made to form part of the tank boundary.

Boothroyd, Cherry and Makar[1] have described a method for eliminating the effect of the boundary in the case of two-dimensional systems by using a circular double layer tank with the electrodes placed on the top section.

GENERAL OBSERVATIONS AND CONCLUSIONS

When designing an electrolytic plotting tank, the effect of each of the main sources of error should be considered in the light of the desired overall accuracy. It is, of course, difficult to calculate exactly the error resulting from any particular fault in the tank since this error may differ widely over the area (or volume) to be explored. It should, however, be possible to estimate the maximum errors.

For instance, the effect of mechanical misalinement or deformation depends to a large extent on the local field strength; it is, however, assumed that for an overall desired accuracy of $\Delta\%$ the mechanical parts must be constructed and alined to within $\Delta\%$ of the mean size of the system. This will in general result in an error much less than $\Delta\%$ at most points and no error seriously greater than $\Delta\%$.

The error contributed by polarization may readily be deduced from the table. If graphited electrodes are used, the equivalent positional error (corresponding to $F = 400\ \text{cm}^{-1}$) is less than 0·1% for mean electrode dimensions in excess of a few centimetres. Hence, it appears that the present limits are mainly mechanical or circuital. Even so, a further decrease in the effects of polarization could be achieved by further scaling up of the model or by platinizing the electrodes.

The experimental results show that the meniscus at the electrodes causes a comparatively large error which is not confined only to regions close to the electrodes but extends a considerable distance towards the centre of the tank. The meniscus fortunately can be eliminated by adjusting the electrolyte level, provided the electrode sides are made smooth and clean and form a sharp corner with the machined top of the electrodes.

Performance of electrolytic plotting tanks

1 500 c/s were used in all cases except the last three with 1 490 c/s; current density in all cases except last three was 4·2 mA r.m.s. per cm²

Electrode material	*Electrolyte*	*F* (cm⁻¹)	*Footnote*
Copper (polished)	*M*/2 copper sulphate sol.	12·4	
Brass (polished)	Tap water	105	(1)
Brass (etched)	Tap water	111	(2)
Brass (etched and graphited)	Tap water	232	(3)
Brass (etched and graphited)	*M*/30 copper sulphate sol.	41·8	
Brass (etched and graphited)	Singly distilled water	480	
Brass (platinized)	*M*/1000 copper sulphate sol.	534	(4)
Brass (platinized)	Singly distilled water	940	(5)
Platinum (solid)	Platinizing solution (0·025 *N* hydrochloric acid, 0·3% platinum chloride, 0·025% lead acetate)	6·06	(6)
Platinum (platinized lightly)	Ditto	532	(7)
Platinum (platinized heavily)	Ditto	6000	(8)

(1) Conductivity of electrolyte at 0° C = 4·28 × 10⁻⁴ ohms⁻¹ cm⁻¹.

(2) Conductivity of electrolyte at 0° C = 4·88 × 10⁻⁴ ohms⁻¹ cm⁻¹.

(3) Conductivity of electrolyte at 0° C = 4·53 × 10⁻⁴ ohms⁻¹ cm⁻¹, graphite applied in form of Aquadag.

(4) Platinizing solution consisted of 0·025 *N* hydrochloric acid + 0·3% platinum chloride + 0·025% lead acetate. Platinization: 15·6 coulombs/cm² for each electrode.

(5) Platinization carried out in similar solution at 21·8 coulombs/cm² for each electrode.

(6) These following three results have been abstracted from p. 282 of reference (12), and are given in the same form as the rest of the data of the table.

(7) Electrodes platinized by passing 3 coulombs to each electrode (3 cm diameter).

(8) Further platinization up to 90 coulombs per electrode.

It is necessary to use a very fine probe in order to reduce the meniscus error at its tip. The depth of immersion also requires to be very small in the case of three-dimensional problems (of the order of a few thousandths of an inch). As a result of the small penetration and size a large impedance is introduced into the probe electrolyte circuit. A null method is most essential for making the potential measurements. No special precautions need be taken against d.c. contact potentials since the balancing circuit does not

respond to these. The probe may thus be made from any suitable metal, provided this does not react with the electrolyte. The probe lead should be well screened to decrease its stray capacity to the electrodes and minimize the stray voltages induced from the mains.

It is clearly not of advantage to attempt to gain an improvement in the value of F by running the tank at higher temperatures. It should be ensured that a uniform temperature exists throughout the electrolyte so that the conductivity does not vary locally.

Provided due attention is paid to the points outlined, it appears to be practicable to obtain an accuracy of 0·1%–0·2% in the potential fields obtained through electrolytic tank methods.

ACKNOWLEDGMENTS

The author wishes to express his appreciation of the valuable help and criticism afforded him by Mr. M. E. Haine of these laboratories, and to thank Dr. T. E. Allibone for permission to publish this paper.

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SPECIAL REPORT

Report of a colloquium on spectroscopy—Strasbourg, 1950

A colloquium organized by the Groupement pour l'Avancement des Méthodes d'Analyse Spectrographiques des Produits Métallurgiques (usually known as the G.A.M.S.) took place in Strasbourg during 12–14 October, 1950. In this report the Honorary Secretary of The Institute of Physics' Industrial Spectroscopic Group briefly outlines the principal papers presented.

At the first session of the colloquium standard samples and electrodes were the subject for discussion, and the work done by the Groupement pour l'Avancement des Méthodes d'Analyse Spectrographiques des Produits Métallurgiques (G.A.M.S.) in collecting eleven standard steels, check analysis of which is not yet complete, was reported. The question of how the history of the metal, and its dissolved gas content, affect the spectra is still a controversial question. A paper from Belgium on the comparison of the results of chemical and spectrographic analysis of slags followed—this will be published in full in an early issue of *Spectrochimica Acta*. The chief point of interest here was to see how spectrographic methods can be used to detect systematic errors which affect chemical analysis of ferrous slags, usually at the lower end of the concentration range. One paper described the preparation of carbon rods sufficiently pure for use as spectroscopic electrodes; these are now on sale at The Société Carbone, Lorraine, France.

At the next session direct reading analysis was discussed, and various subjects affecting the errors of analysis. The first paper was a summary of recent progress with American apparatus, of the Quantometer type. Other papers on direct reading instruments, which were not commercially produced, were of equal interest, and in particular emphasized the value of direct scanning of the radiation from a source for periods of the order of a second in determining the value of the source for routine spectrographic analysis.

In a talk on calibration line drift, the results presented at the Sheffield symposium of the Industrial Spectroscopy Group were summarized and amplified. The conclusion drawn was that the effect of humidity on calibration line drift operated entirely through the effect of moisture on the gelatine of emulsions, not at all by any direct effect on the spark or arc. This conclusion might be proved by any user of a direct-reading apparatus for routine analysis. The next paper was on the construction and use of a cathode-ray tube for displaying the density profiles of lines in microphotometry, and its use in qualitative analysis,* and since the apparatus used was actually demonstrated this aroused great

* *Spectrochim. Acta*, **4**, p. 13 (1950).

interest. A German paper on the properties of sources was presented which gave new information about the carbon arc discharge when mixtures of salts are being vaporized. Mixtures of alkali metal salts with the salts of half a dozen other elements in smaller quantities were studied, and it was shown that if small quantities of chromium (for example) with varying amounts of sodium are vaporized, then as the sodium content increases the chromium line intensity first rises, then falls with further increases of sodium. The sodium concentration needed to give maximum line intensity for various minor metals varies with their ionization potentials, and the temperature of the arc column is considered to be the controlling factor in this.

On the next day spectrum sources were further discussed, and there were three papers on unusual low-pressure sources. The first, from Belgium, described the use of a hollow graphite cathode and tubular water-cooled anode, operating in a rare gas atmosphere, under conditions such that the discharge gradually heated the cathode to 2 500° C and thus vaporized the sample. Fractional distillation takes place, and with steel the gases in the metal can be analysed as well as the metal and its impurities, using only a few milligram of sample. Oxygen can be measured by the intensity of the carbon monoxide bands in the spectrum, or possibly by its absorption in the far ultra-violet. In the discussion it was pointed out that as little as one part per million of chlorine can be detected in certain salts with a source of this type. A German paper on cold-running hollow cathode discharge tubes using much smaller currents was described; here the sample is a stream of gas passing through the system at low pressures. The use of a high-frequency electrodeless discharge for the estimation of the halogens and sulphur, using a high-dispersion glass spectroscope, was discussed; some quantitative work had been done by this method.*

A French contributor described work in continuation of that previously reported to the G.A.M.S. on the use of a Tesla type spark to analyse a single phase of an alloy, or to exploit the uneven sparking properties of an etched metal surface by making use of them for analysis instead of trying to ignore them by averaging the sparks on a large area. These experiments show that the distribution of an impurity between the phases of an alloy can sometimes be traced spectroscopically. Also for steel, the effect of sulphur as iron sulphide on the iron spectrum is quite distinctive. The rest of the session was given to papers in German on the fundamentals of spark spectra, and the methods of evaluation or calculation of analytical results. These papers were translated by the author himself, being presented in three languages. The first dealt with the question of the parameters needed to describe a spark source so that other spectroscopists could get the same spectra. These seem to be principally energy and time; that is, the energy defined as the integrated product of current and voltage during a single train of oscillations of the spark, and the duration of such a train. The point was illustrated by slides of matched spectra of aluminium taken on the same instrument with different values of peak voltage, of capacity and of damping set up in a multi-source unit and one other spark unit.

The same author outlined the principles of plate calibration and described a calculating board which could be used to make an algebraic transformation to straighten out the toe of the photographic calibration curve; he could then work out background corrections for spectrum lines using Gaussian logarithms, finally giving the result of the analysis without any calculations being made by the operator. This apparatus is made in Hamburg. All the technical papers, except those mentioned as being published elsewhere (or those describing work already published), will appear in a report of the colloquium to be published by the G.A.M.S.

At the final session, short reports were read on the way in which spectroscopists organized themselves in Great Britain, Germany, Italy and Switzerland, for the exchange of information and for standardization work. In Germany, as in this country, there has been some attempt to standardize methods of procedure as well as to collect standard samples. The work of the British Standards Institution and of the British Iron and Steel Research Association was specifically mentioned. In Germany they have also tried to devise a standard training for spectroscopic assistants. In Switzerland, as in some parts of Great Britain, small discussion groups of spectroscopists are formed rather than separate societies with larger meetings.

The suggestion that a federation of national spectroscopic groups should be formed was not taken up, but it was proposed that various groups should in turn act as hosts organizing international meetings in their own countries. Dr. Masi announced that such a meeting might be held in Venice in September 1951, when the Institute of Metals and Italian Metallurgical Association were holding meetings there. It was also suggested that a spectroscopic meeting might be arranged during the International Congress on Analytical Chemistry in England in 1952.

Other practical propositions for co-operation were made, and a committee of delegates of each country represented was formed which met during the afternoon to see what could be done. They discussed the problem of standardizing terminology and definitions, and that of selecting a bibliography of papers of fundamental value. The comparison of standard alloys from various countries was also considered, and plans to start this work are already in hand. The secretary of this committee is Dr. R. Schmidt of the Central Institute for Testing Materials, Post Bus 49, Delft, Holland.

E. VAN SOMEREN
Honorary Secretary, Institute of Physics' Industrial Spectroscopy Group.

* GATTERER, A., and FRODL, V. *Ric. Spettroscopiche*, **1**, p. 201 (1946).

New Books

Acceptance Sampling. (Washington: The American Statistical Association.) Pp. iv + 155. Price 12*s*. net.

This record of a series of papers and the associated discussions delivered at a meeting held in Cleveland, Ohio, in January 1946, contains much of importance to those interested in the economics of efficient production of commodities of acceptable quality, and deserves to be widely read.

The papers are of a high standard but conform to the usual pattern, as they contain a considerable amount of statistical algebra, which does so much to discourage those who have the duty of supervising industrial operations from embarking on the task of cultivating a statistical attitude of mind when studying industrial problems. Because of the variability encountered in all industrial products these problems demand the use of statistical methods.

On the other hand the discussions contain many comments which will be readily appreciated by supervisory staff and this feature of the publication should encourage them to seek familiarity with statistical modes of thought. A good example of such discussion centred round the economics of inspection and the economics of trying to use defective material.

The papers are accompanied by ample reference to appropriate literature, including some British publications; there is, however, one serious omission as no reference is made to the British Standards Institution Publication No. 600: 1935 on *The Application of Statistical Methods to Industrial Standardisation and Quality Control*, edited by Professor Pearson, which summarized the results of the work of a committee appointed in 1932. When first issued this publication was acknowledged, both in the U.S.A. and Great Britain, to be a major contribution to the subject of Applied Statistics.

The appearance of this record of a very useful symposium on Acceptance Sampling provokes reference to a much more important aspect of applied statistics. Once a product is manufactured time spent on its inspection is largely waste if the product is satisfactory, and inspection is very costly to carry out efficiently on unsatisfactory products, as it involves not only time of inspection but costs of rectification or scrap. Further, there are problems which defy adequate final testing, an example of which is quoted on page 9. "There remain instances of some importance in which no very practical solution has been suggested, such as e.g. the tensile testing of a 10 000 ft coil of wire." It is in such cases that the use of statistical techniques which have been practised to a limited extent in Great Britain, for over twenty years, have their maximum utility. In these techniques results derived from small groups of observations made regularly on processes or on their products serve to provide a running commentary on the state of the processes. The nature of the observations is determined by a combination of good technical knowledge of the process, practical experience in the use of the product and statistical principles. This combination can provide a high assurance that the resulting product is satisfactory, which can be confirmed by well-planned final testing in many instances; and in examples like that referred to above this has been confirmed over many years by the satisfactory performance of the material produced. However, this is not the only important contribution to industrial efficiency which this type of applied statistics renders. Its greatest contribution is possibly the fostering of good understanding between individuals whose duties readily, and so often do, become a source of dissension.

B. P. DUDDING

Operational Calculus—based on the two-sided Laplace Integral. By BALTH. VAN DER POL and H. BREMMER. (London: Cambridge University Press.) Pp. xiii + 415. Price 55*s*. net.

This book aims at presenting in a modern form the operational calculus originated by Heaviside. None should be better equipped than the authors who, particularly the first, have made important contributions to the subject which have gained well-deserved admiration. The authors truly remark that Heaviside transformed difficult problems with discontinuous functions into the fairly easy manipulation of known continuous functions. He often interpreted his results intuitively, which occasioned his controversy with contemporary pure mathematicians, and more recent ones are not free from prejudice. The authors' quotations show this and the reviewer recalls being told, as an undergraduate, that all problems solved by Heaviside's methods must be reworked backwards to check validity. However, a good deal of modern mathematics has been concerned with showing that Heaviside was right and one object of the present book is to treat the main results with a rigour, hoped to be satisfactory to pure mathematicians. For this purpose, and because it is much more general and exact than the original operational methods, the authors use the two-sided Laplace Integral (or transform) in which the range of integration is extended from $+\infty$ to $-\infty$. This overcomes a number of difficulties: an obvious one is when there is a discontinuity at the origin. In addition the Fourier integral and transformation are used though not to the extent to which Bromwich relied on them. The result is that the book is mainly concerned with these integrals and the operational algebra familiar to the engineer is relatively infrequent.

The book, however, goes a great deal further than the logical and rigorous presentation of principles. The first four chapters study the better-known basic results and relations. The fifth chapter deals with the delta or impulse function, often ascribed to Dirac, but used earlier by Heaviside. The sixth chapter considers the ranges or areas of convergence of the main integrals required, including the summation of integrals and series sometimes divergent. Chapter VII treats asymptotic relations and operational transposition rather similarly and Chapter VIII the fairly classical theory of linear differential equations with constant coefficients. Chapters IX and X, in dealing with simultaneous linear differential equations, including circuit theory, and linear differential equations with variable coefficients will not be so familiar. Again, Chapter XI gives a number of more elaborate methods of manipulation. In the succeeding chapters most readers will find a great deal and all will find some new material, including observations on the theory of numbers applied to function theory, the applications of the operational methods and concepts to difference equations and integral equations,

partial differential equations and the simultaneous transformation of two variables here called the simultaneous operational calculus.

Since the arrangement follows mathematical development rather than application the main rules and relations are listed in the last two chapters for the convenience of those who wish to find quickly the results needed for a specific problem without making a mathematical study at the same time.

This book is likely to become and should become a classic. One believes that pure mathematicians would profit considerably. There is no doubt that those interested in applying mathematics will find it of the greatest value and interest.

S. WHITEHEAD

Heat and Temperature Measurement. By ROBERT L. WEBER. (New York: Prentice-Hall Inc.) Pp. x + 422. Price $6.65 net.

This excellently produced book deals with heat and temperature at about the standard of university second year work. Its approach is distinctly experimental. The book falls into three parts: Part I, covering 271 pages, describes the physical principles underlying the measurement of heat and temperature. Fourier's method is mentioned, but his theory lies outside the scope of the book, although a simple case of periodic heat flow is treated. The fundamental principles of thermodynamics are briefly treated. Each chapter has a summary, a list of references (including original papers) for further reading, and about 25 problems, half of them with answers. This part of the book is undoubtedly good. Some interesting pictures of seventeenth-century thermometers are given.

Part II outlines 29 laboratory class experiments rather on the lines of the usual instruction given to the higher forms in schools. It occupies 187 pages, and the schematic diagrams of apparatus are liberally supplemented by photographs; in this, as in other matters, little is left to the reader's imagination.

The remaining 50 pages of the book consist of advice to the student, together with practical instruction, tables of constants, and other data helpful in his experimental work. The scope of this instruction can be gauged from the chapter titles: How to solve physical problems; Laboratory reports; Visual aids (American film strips); Symbols. In addition the following are given: tables of physical constants, atomic numbers, melting points and other thermal properties of the elements and more common materials, full calibration tables of typical thermoelectric metal combinations, tables of variation of sensitivity with temperature, spectral emissivity of materials and oxides, true temperatures corresponding to various values of total emissivity and various brightness temperatures. Further, rather unnecessarily, three pages of tables are given for converting temperatures in Fahrenheit to Centigrade (and including the misprint of 100° C as equivalent to 18° F).

It will thus be clear that the book is extraordinarily detailed and complete, covering almost every aspect of instruction, and including much that, in this country, is usually left either to the student himself or to the laboratory demonstrator or lecturer. It should be a great boon to an overburdened teacher with large classes in either universities or technical colleges. In particular, this book should be extremely useful to the applied science student, to whom it can be confidently recommended.

The main criticism of the book from the physicist's standpoint lies precisely in this over-completeness. Very little demand is made on the student and little encouragement to exercise independence or physical insight. The student will, perhaps, tend to regard experimental work, not as a form of enquiry and exploration, but as an operation of set apparatus to produce set results. Those, however, who are merely interested in the physics of heat and temperature measurement as a tool, should find this a most helpful book.

F. LLEWELLYN JONES

A Hundred Years of Physics. By W. WILSON, F.R.S., Ph.D., D.Sc. (London: Gerald Duckworth and Co. Ltd.) Pp. 319. Price 21*s.* net.

And what a century of development it has been! It covers roughly the period between the 1851 Crystal Palace Exhibition and the Festival of Britain Exhibition in 1951. It has seen the atom develop from a simple billiard-ball concept to the complex structure envisaged to-day in terms of the properties of sub-atomic particles which obey the laws of quantum mechanics. It has seen the concept of an aether, which at the beginning of our period had the properties of an elastic solid, develop into a more subtle medium to carry Maxwell's electro-magnetic waves, and finally jettisoned altogether. It has witnessed the remarkable developments in applied physics which have culminated in the construction of jet-propelled engines which have made Jules Verne's story of the girdling of the earth in eighty days seem but as the perambulations of a tortoise. And it has seen, alas, the development of that very dysgenic instrument, the atomic bomb. Through all this dazzling story Professor Wilson picks his step with practised ease, discoursing learnedly, and yet lightly, on the developments in optics, relativity (Newtonian special and general), thermo-dynamics, atomic structure, wave-mechanics, quantum theory and astrophysics. The book may be read with profit by candidates for university scholarship examinations and by honours students in the universities. It gives full accounts of experimental work and the mathematics employed is not too difficult for an intelligent layman. The author is master not only of his subject but also of a characteristically pleasant discursive style. The book is well produced and the cost is reasonable. It can be recommended unreservedly.

ALLAN FERGUSON

Interaction entre le Noyau et son Cortège Électronique. By DAUDEL, CHANSON, SURUGUE, RATIER, BENOIST, BOUCHEZ, HAISSINSKY and JEAN. Réunions d'Études et de Mises au Point tenues sous la présidence de LOUIS DE BROGLIE. (Paris: Éditions de la Revue d'Optique.) Pp. 200. Price 800 fr.

No country has made after the war a greater effort in scientific rehabilitation than France. To-day the physicist or mathematician who wants thorough and up-to-date information in fields of great actual interest can turn to few better sources than to the Colloques of the Centre National de la Recherche Scientifique, or to the Réunions d'Études et de Mises au Point, of which this is the seventh volume, and which owe their existence to the almost superhuman working power of Louis de Broglie. This volume, which sums up a lecture series given in April and June of 1948, proves also most convincingly that by that time the French workers had not only made good their war-time handicap, but could make most essential original contributions.

The title might suggest a study of the magnetic hyperfine structure or the like, but in fact this is hardly mentioned.

Instead, one is soon reminded by the lucid introduction of Raymond Daudel, which follows a useful recapitulation of the principles of wave mechanics, that few changes can occur in the nucleus which do not affect the electron cloud, and vice versa. Some of the interaction phenomena are already well known experimentally, such as internal absorption, internal conversion, internal *Bremsstrahlung* and internal Raman and Compton effects. But two other most interesting phenomena have been predicted by Daudel and his collaborators. These are the "e-capture," in which an outer electron is absorbed by the nucleus, and its opposite, the "e-creation," in which the nucleus creates an electron in an empty state in the electron shell, while it emits a neutrino. Another conclusion, closely related to these, is that in some cases the ionization of an atom may affect the radio-active decay times, and even might make a stable nucleus unstable.

The other lectures fill in Daudel's programme in a very complete way. P. Chanson gives a concise but complete discussion of nuclear isomerism; J. Ratier a very full theory of internal conversion. Benoit and Bouchez complete Daudel's theory of the e-capture, and Jean of the e-creation. Haissinsky gives a review of the chemical reactions which follow nuclear transformations, under the heading of the "Szilárd–Chalmers Effect."

Though in nuclear physics a book which is based on lectures two years old is unavoidably somewhat dated, it may be hoped that this volume will be studied, and not only by nuclear specialists.

D. GABOR

Measurement of Mind and Matter. By G. W. SCOTT-BLAIR, D.Sc., Ph.D., F.Inst.P. (London: Dennis Dobson Ltd.) Pp. 115. Price 9*s*. 6*d*. net.

In this book, slight in bulk but attractively produced, the author sets out to discuss a number of present tendencies in science which are perennial subjects of divergence, some indeed the incurable "problem children" of modern thought.

Such are the principles of measurement, causality, Gestalt psychology and several others. The implication that analogy has a major part to play is only to recognize that Dr. Scott-Blair is in good company in facing these formidable hurdles. Whether or not his particular technique is capable of carrying him over them, only time and experience can show.

F. I. G. RAWLINS

Author's Guide for preparing manuscript and handling proof. (New York: John Wiley and Sons Inc., London: Chapman and Hall Ltd.) Pp. xvi + 80. Price 16*s*. net.

This little book is in fact the fourth edition of the American publisher's guide for its own authors. It will be found equally useful by other authors in the preparation of their scripts, indices, title pages, figures and the correction of the proofs. The whole complex process of the production of a technical book from its conception to its publication and even to the production of the later editions is outlined in a way which will help authors to understand the reasons for publishers' demands. While this volume can be recommended to potential and less experienced authors of books, some chapters can also be recommended to authors of papers intended for scientific periodicals.

The volume is expensively bound and produced which makes its price in this country rather high, and we suggest that the London House could perform a valuable service by arranging either for a cheap paper bound edition, or perhaps for the publication of a similar book at a price more in keeping with the austere conditions of this country.

H. R. LANG

Journées Internationales de Science Photographique. (Paris: Éditions de la "Revue d'Optique théorique et Instrumentale.") Pp. 126. Price 600 fr.

Participators in the international meeting, organized by J. Eggert and H. Ammann, at Zurich in the spring of 1949, will retain very pleasing memories of it and be pleased to see this permanent record published by L. P. Clerc. The volume includes 25 papers of which 13 are concerned with emulsion manufacture and the theory of the latent image, and 8 with granularity, graininess, resolving power and related subjects.

E. W. H. SELWYN

One Hundred Years of Submarine Cables. By G. R. M. GARRETT. (London: His Majesty's Stationery Office.) Pp. viii + 59. Price 2*s*. 6*d*. net.

The first telegraph submarine cable was laid across the English Channel on August 28, 1850. In order to mark the centenary of this event, an exhibition was held at the Science Museum and this handbook was written to provide "a convenient and permanent record of the development of a service in which British invention and enterprise have made by far the greatest contribution of any of the nations of the world."

The book opens with a chapter on the beginnings of land telegraphy and then describes the laying of the pioneer cables across the Channel and the Atlantic. The technical development of the submarine cable is traced from the primitive cables with gutta percha dielectric to modern multi-channel coaxial cables with air-spaced polythene insulation and submerged amplifiers. The book is interesting reading and is attractively illustrated with 28 diagrams and photographs.

In these times when history is often distorted, and even the history of scientific development is modified to suit political creeds, it is good to publish factual and human accounts of scientific and engineering achievements. *One Hundred Years of Submarine Cables* fulfils this function admirably.

J. S. FORREST

Department of Scientific and Industrial Research Report for the Year 1948–49. (London: His Majesty's Stationery Office.) Pp. 260. Price 5*s*. 6*d*. net.

The *Report* records that the chief concern of the Department's Advisory Council has been how best to use the limited resources available for the solution of current national problems. The Council has had so many problems of importance brought to its notice that it has been difficult to keep research programmes within limits that will ensure an adequate effort being devoted to each problem accepted. Moreover, the *Report* repeatedly stresses the need for more basic research if the Department is to maintain its efficiency and the "scientific health" of its staff. Summaries of the work, covering an immense field, carried out in the Department's fourteen establishments and laboratories and in the forty Research Associations are given. The gross expenditure of the Department during the year was over £3½ million. The *Report* contains items that will be of interest to nearly all scientists and particularly to those concerned with its applications.

Correspondence

A survey of general and applied rheology

I have no wish to enter into controversy about the merits or demerits of my book *A Survey of General and Applied Rheology*, but Mr. Macey's letter in your December number does call for some comment in connexion with the use of the word "Mysticism."

Personally, I do not normally care to use this word in the pejorative sense allowed as a final alternative by the Shorter Oxford Dictionary; but I will do so, for once, because I think that Mr. Macey's letter contains a perfect illustration of "mysticism" meaning "scientific theories which assume occult qualities or mysterious agencies of which no rational account can be given."

Take his sentence: "Time no longer marches past in a linear and regular manner, but is distorted at the whim or fancy of a piece of rubber or a bit of cheese." (This is supposed to happen when the exponent of a power equation is given the status of a physical property). I have tried in vain to find some "rational account" of the meaning of this sentence. The only mathematical equation which I can imagine to express the peculiar idea of "the linear progress of time" is dt/dt = constant. What is this absolute time supposed to be varying linearly with? Presumably with time! I cannot see anything more "real in physical fact" in this sort of concept of time than in $exp\ \sqrt{t}$!

The fact is, of course, that there is no absolute time scale too sacred to be discussed by those who work with such mundane materials as rubber or cheese—or even clay. Seconds, hours, days, etc., are *defined* as of equal length because they represent convenient repetitive phenomena. The constancy of the rate at which time is supposed to march past is definitional. All this is exceedingly well known to those who have read Poincaré, Cleugh, Dingle, Burniston Brown or any other writer who has discussed the question.

Neither I, nor any other rheologist to my knowledge, has ever questioned the advantages of using the Newtonian time scale (i.e. that defined in terms of the motion of light or a Newtonian body in the absence of gravitational fields) to describe the behaviour of complex materials. What I have suggested is that our entirely proper insistence on the use of this time scale for laboratory purposes has led us to require rather cumbersome mathematics (fractional differential equations) in order to describe certain phenomena in terms of the maximum economy of quantities (Occam's Principle). If we used other time scales, the fractional differentials would not be needed. If we scrap Occam's Principle, we can use what Mr. Macey calls "a logical theory," which I take to mean a theory from which a Newtonian Model of dashpots and springs can be constructed. But to do this, while maintaining the Newtonian time-scale, would generally require far more constants to describe the behaviour of our materials.

Dare I hope that, in spite of your reviewer's disapproval of my earlier book, some of those who accuse us integrative rheologists of "mysticism" will glance at my *Measurements of Mind and Matter*, in which I have allowed myself enough space to give what I believe to be "a rational account" of our viewpoint?

Might I also suggest that it would be worth the while of some of your readers to find out what Mysticism is really about? A reading of Fr. Poulain's famous text-book might discourage the use of "mysticism" simply as a term of abuse. To paraphrase a remark in Professor Dingle's recent broadcast, when we say that we wrestle against Principalities and Powers, we do not refer to any dislike of Nutting's equation!

G. W. SCOTT-BLAIR.

Note.—The Editor regrets that, at the moment, he cannot accept any further correspondence on this subject.

Erratum. In the article *Replica techniques in electron microscopy*, by A. W. Agar and R. S. M. Revell, published in the January issue (p. 10) in the legends of Figs. 6, 7, 8, 9 and 10 *for* gold-platinum *read* gold-palladium.

Journal of Scientific Instruments

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SPECIAL REPORT

The development and application of Fourier methods in crystal structure analysis. By A. J. C. Wilson.

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SPECIAL REPORT

Summarized proceedings of a conference on the development and application of Fourier methods in crystal-structure analysis—London, November 1950

The annual autumn conference of the X-ray analysis group of The Institute of Physics was held in London on 16 and 17 November, 1950. The subject was "The development and application of Fourier methods in crystal-structure analysis." In addition to the evening discourse delivered by Professor D. R. Hartree of the University of Cambridge on "Automatic calculating machines," there were two sessions—one on basic theory at which four papers were presented and one on the handling of the arithmetical problems at which four papers were also presented. A small exhibition was arranged in connexion with the conference. The discourse, papers and the discussions are summarized in this report.

INTRODUCTION

The first session of the conference opened with some introductory remarks by Professor Sir LAWRENCE BRAGG (University of Cambridge). Their purpose was to remind the conference of the principles of Fourier methods and to say something about the history of their development. There were two fundamental ideas in the application of Fourier methods. The first was that the X-rays picked out the harmonic components of the variation in electron density, and hence the intensities of the diffracted beams were determined by the amplitudes of the corresponding components of the electron density inside the crystal. Unfortunately, in the process of recording the intensities of the diffracted beams the phase relations between them were lost and it was impossible to reconstruct the crystal by direct synthesis. It was not until about 1934 that the second fundamental idea was developed by Patterson, the synthesis of a map representing completely the observable information about the crystal structure. Physically this map gave the vector distances between the atoms of the crystal and hence, in principle, made it possible to deduce the crystal structure, though in practice its complexity and consequent lack of resolution prevents its direct use except in certain special cases.

It was extraordinary how slowly the use of Fourier methods in structure analysis had developed. They were first suggested by Sir W. H. Bragg in his Bakerian lecture in 1915,[1] but it was not until about 1925 that any practical use was made of them. In the meantime the theory had been developing, particularly through Darwin's work on the intensity of diffraction by mosaic and perfect crystals,[2] and Ewald's work on the reciprocal lattice.[3] One-dimensional Fourier series were used at first, although three-dimensional Fourier series had been considered in connexion with the quantum theory of X-ray diffraction. Two-dimensional Fourier series were first used about 1929, and it was only recently that three-dimensional ones had been used in practical computation.

Criticism of crystallographers for their slowness in realizing the possibilities of Fourier methods was not entirely justified. The application of these powerful mathematical tools had had to wait until the technique of physical measurement had developed correspondingly. Even now new mathematical methods were being developed. Surprises were always being sprung upon us, and the value of conferences such as this consisted in the interest and stimulation produced through bringing together ideas from different institutions.

SOME APPLICATIONS OF THE (F_o-F_c) SYNTHESIS

Dr. W. COCHRAN (University of Cambridge) then presented his paper entitled "Some applications of the (F_o-F_c) synthesis." He said that the problem of the distribution of electrons in atoms and molecules had always seemed to him a particularly interesting branch of X-ray crystal-structure analysis—a branch which had been rather neglected in the last twenty years. Attention had been centred on the measurement of bond lengths with the result that it had almost been forgotten that the equation for the electron density

$$\rho(x, y, z) = V^{-1}\sum_n F_n \cos \Theta_n \qquad (1)$$

where

$$\Theta_n = 2\pi(hx/a + ky/b + lz/c) \qquad (2)$$

meant what is said, or at least could be made to do so. As he saw it, there were three main reasons for this state of affairs. The first was that so many new and interesting compounds were being discovered that there was always a tendency to press on with the problems connected with their stereochemistry, rather than study in detail well-known substances. Secondly, bond lengths could be calculated by simpler theories than those leading to electron-density distributions, and thirdly, the X-ray measurements and methods of manipulating them had not in the past been good enough to distinguish between bonds in any way except by determination of their lengths. Dr. Cochran proposed to talk about possible improvements in method of deriving electron density and also atomic co-ordinates from experimental measurements.

It had been suggested that greater accuracy in the

determination of atomic co-ordinates might be achieved by methods other than Fourier synthesis, in particular by minimizing

$$\phi \equiv \sum_n w(F_o - F_c)^2 \qquad (3)$$

where F_o and F_c were the observed and the calculated values of the structure amplitude and w was a suitable "weight." It was, however, easy to show that minimizing this with respect to the atomic co-ordinates x_j was equivalent to choosing x_j so that the (F_o-F_c) synthesis,

$$D \equiv V^{-1}\sum_n (F_o - F_c)\cos\Theta \qquad (4)$$

had zero gradient at the centre of each atom, the "weight" w being the reciprocal of the scattering factor of the jth atom. From a trial synthesis corrections to achieve this result could be obtained in the form

$$\Delta x_j = \left[\sum_n \frac{1}{f_j}(F_o - F_c)\frac{\partial F_c}{\partial x_j}\right] \div \left[\sum_n \frac{1}{f_j}\left(\frac{\partial F_c}{\partial x_j}\right)^2\right] \qquad (5)$$

the values of F_o, F_c and $\partial F_c/\partial x_j$ being those at the point x_j. Minimizing ϕ with respect to temperature-factor parameters for the individual atoms was equivalent to making $\partial^2 D/\partial r^2 = 0$ for two perpendicular directions at the centre of each atom, even for non-isotropic thermal vibrations, and the scale of the f_j's could be adjusted to make ϕ a minimum by making $D = 0$ at the centre of each atom. The (F_o-F_c) synthesis could thus be used for all minimizations possible with ϕ, and in addition it gave information about the electron density at points other than the atomic centres. When all minimizations had been carried out D showed a zero "saddle-point" at the centre of each atom.

If the hydrogen atoms were not included in the calculation of F_c, peaks corresponding to them remained in the (F_o-F_c) synthesis, which could thus be used to locate them if the values of F_o were sufficiently accurate. Differences between the true electron density and that assumed in calculating f_j were also evident.

Dr. Cochran had applied these techniques to obtain the electron distribution and in particular the positions of hydrogen atoms in adenine hydrochloride. The structure of this compound had been worked out by Dr. June Broomhead from photographic measurements, but he had made more accurate ones with a Geiger-counter spectrometer before applying the synthesis.[4] The final F_o-F_c synthesis showed clearly the positions of the seven hydrogen atoms and also to which atoms of the molecule they were covalently bound. For four out of the seven the binding could not have been predicted in advance. There were no discrete maxima at the centres of interatomic bonds, though the average electron density (about 0·2 e/A^2) was significantly different from zero in these positions. The F_o-F_c synthesis had also been used by Finbak and Norman on oxalic acid dihydrate, and by Brill on diamond. Brill's results showed electron-density maxima of about 2·0 e/A^2 at the centres of the covalent bonds. The discrepancy was probably due to a difference in the treatment of the temperature factor; when this was determined empirically deviations of the Hartree f curves from those of the actual atoms could be partially compensated by choice of temperature factor. The F_o-F_c synthesis was well adapted to showing up any deviations of the atoms from spherical symmetry, but not so well to discovering deviations between the theoretical and the actual distribution of electron densities.

Dr. Cochran summarized by saying that so far the results of the application of this technique showed that hydrogen atoms could be located with fair accuracy by X-rays, and that both covalently-bound hydrogen and hydrogen in hydrogen bonds had an electron density very similar to that of isolated hydrogen atoms. The departure from spherical symmetry of atoms in organic compounds was very slight and the excess electron density in a projected covalent bond was about 0·4 $\pm$ 0·2 e/A^2. In order to go further and examine the connexion between electron density and bond character or to verify directly electron theories of chemical reactivity it would be necessary to have much more accurate experimental measurements, and better values for the atomic scattering factors of isolated atoms would be a help.

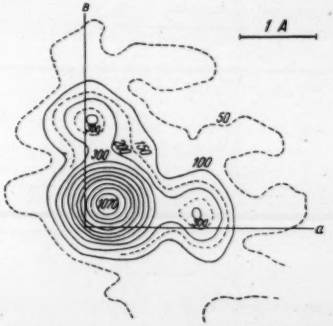

Fig. 1. Section of three-dimensional synthesis for paraffin wax (Vainshtein and Pinsker)[6], showing one carbon and two hydrogen atoms. The potential is in arbitrary units

In the discussion of Dr. Cochran's paper, Mr. J. S. ROLLETT (University of Leeds) said that he had used the F_o-F_c synthesis on dimethyltriacetylene. This compound had eight carbon atoms in a line and the hydrogen atoms on the end two showed up clearly. Dr. A. J. C. WILSON (University College, Cardiff) drew attention to recent Russian work using electron-diffraction techniques.[5,6] The Fourier synthesis used gave the variation of potential inside the crystal, instead of variation of electron density, and it appeared that hydrogen atoms were as clearly visible in this synthesis

as carbon atoms are in normal Fourier syntheses (Fig. 1). Professor J. M. ROBERTSON (University of Glasgow) emphasized the importance of the temperature effect. He thought that this was now the limiting factor in the determination of fine details of the structure and in the location of the hydrogen atoms. It would be useful to measure the structure amplitudes at different temperatures. One would thus be able to make syntheses for the same fundamental variation of electron density but with different amplitudes of thermal vibration. In the same way, the substitution of deuterium for hydrogen should provide useful information on the temperature effect. He also suggested that it appeared easier to observe hydrogen atoms in aliphatic compounds, although they showed up in three-dimensional syntheses of both aliphatic and aromatic compounds. This was in agreement with the idea that in aliphatic compounds the hydrogen atoms were slightly more negative, relative to the carbon atoms, than was the case with aromatic compounds. In the former case, therefore, there would be a slightly higher electron density near the hydrogen nucleus and it would be easier to observe. Dr. COCHRAN, in his reply, said that he thought there were too few examples worked out so far to judge whether these differences in the electron density of hydrogen were real or not.

THE INTERPRETATION OF THE PATTERSON FUNCTION

Dr. C. A. BEEVERS (University of Edinburgh) then presented his paper, "The interpretation of the Patterson function." There were four methods of attack that he had found useful. The first was the observation of peaks near the origin. Vectors within 1·5 A of the origin must be entirely within the molecule, and hence if interpretable would give the orientation of molecules whose structure was already approximately known. Two examples of its successful use were in tartaric acid and glucose. The second method was based on the fact that if a molecule or a part of a molecule had an actual or apparent centre of symmetry there were many coincident Patterson vectors (for example 8 in glucose) and hence the position of the centre could be found. The third method which was frequently useful was the analysis of Patterson–Harker sections, but he had found that chance peaks (which had twice the weighting of the true Harker peaks) were frequently troublesome and their existence had constantly to be kept in mind. If there were a set of known atomic sites, for example those of a heavy atom, a fourth method was possible which consisted of adding the Patterson function to itself several times over with its origin on all the known atomic sites. Dr. Beevers called the resulting array, which might be in two or three dimensions, the vector-coincidence function.[7] This function had peaks at the remaining atomic sites and possessed the symmetry of the known atoms. It was closely related to the Fourier series obtained by using the observed structure amplitudes with signs determined only from the heavy atoms. This method had been successfully used in interpreting a three-dimensional Patterson of strychnine hydrobromide and it had also been shown that rubidium benzylpenicillin gave a useful vector-coincidence function based on the rubidium positions.[8]

STRUCTURE DETERMINATION BY MEANS OF FOURIER TRANSFORMS

Dr. H. LIPSON (College of Technology, Manchester) spoke on "Structure determination by means of optically derived Fourier transforms." The work he described was part of a general plan to take the tedium out of crystal structure determination. The principle was largely due to Sir Lawrence Bragg, and it had been put into effect by Mr. C. A. Taylor. The essential idea was well known, and could be illustrated by an experiment in physical optics. Suppose that the diffraction pattern of a single aperture—for example, a small rectangular hole—were observed; the diffraction pattern of two similar holes side by side would be similar but crossed with a set of fringes, and with two pairs of holes, corresponding to a single unit cell, there would be two crossed sets of fringes showing the beginnings of a reciprocal lattice. The positions of the peaks of these crossed fringes (i.e. the positions of points in the reciprocal lattice) determined the relative positions of the units of pattern, but their intensities were determined entirely by the form of the single unit. Conversely, if a reciprocal lattice were observed and the diffraction pattern of a unit, for example a chemical molecule, were known, it should be possible to orientate one with respect to the other until the two matched. Ideally this should be done with three dimensions; work in Manchester had been an investigation of how far it would be possible to go in two dimensions.

The method was easiest to apply when the projected unit cell contained only a single unit of pattern. This was punched in a sheet of stiff black paper, the holes representing the atomic positions, and its diffraction pattern (optical Fourier transform) observed. Tilting of the molecule within the cell was reproduced by tilting the sheet of black paper, causing the expansion of the Fourier transform in one direction. It was quickly possible to obtain the approximate orientation and tilt of the molecule; if the tilt were large it was better to punch a new mask after the tilt had been found approximately. The Fourier-transform method made it possible to dispense with structure-factor calculation for centrosymmetric structures; the signs of the structure amplitudes could be obtained either by observing the lines of zero intensity in the Fourier transform, or preferably by adding a hole at the centre of gravity of the molecule. This second procedure increased the intensity of regions with positive structure amplitude and decreased that of regions of negative structure amplitude. The Fourier synthesis could then be done immediately. The method had been tested on durene

and naphthalene, and he had hoped to be able to say that it had been used successfully on an unknown structure. It had been tried on one of the forms of acridine,[9] and had been successful in locating eight atoms out of the fourteen. As it happened these eight were sufficient to give the correct signs, though wrong magnitudes, to the large structure amplitudes, and so the correct structure would have been found by a single stage of Fourier refinement.

The structures worked out so far had all contained molecules in parallel orientation in the unit cell, and all molecules were planar. It would be more difficult for non-planar molecules not in parallel orientation, but the difficulties should not be insurmountable. It was hoped to develop a method for superimposing the Fourier transform and the reciprocal lattice optically, so that photography would be unnecessary. The method would then be considerably quicker than the "fly's-eye" method and much more versatile.

In the discussion Dr. H. P. Stadler (King's College, Newcastle-upon-Tyne) said that he had used Fourier transforms to determine the crystal structure of flavanthrone. By assuming a bond length of 1·40 A throughout and neglecting the difference between carbon, nitrogen and oxygen atoms the expression for the transform could be reduced to

$$T = (\cos 4x + \cos 2x)(\cos 5y + \cos 3y + 2\cos y) + (\cos 5x + \cos x)(\cos 4y + \cos 2y + 1) + (\cos 7x + \cos x)\cos 2y - (\sin 4x + \sin 2x)(\sin 5y + \sin 3y) - (\sin 5x + \sin x)\sin 4y - (\sin 7x + \sin 5x)\sin 2y,$$

with repeat distances of 2·2 and 1·27 reciprocal-lattice units in x and y respectively. In this form T was easily evaluated with Beevers–Lipson strips; the result was shown in Fig. 2. Its superposition on the reciprocal lattice determined the signs of sixty-seven reflexions of the (*h*0*l*) zone. A projection using these signs showed all the atoms fully resolved, and yielded co-ordinates which led to a reliability index of 0·27 for the (*h*0*l*) reflexions.

Sir Lawrence Bragg suggested that one of the difficulties of optical methods was the inability of photographic methods to record the full range of light intensity on one film. This difficulty might be overcome, and something equivalent to contour lines obtained, by making a series of photographs with different exposure times. Mr. C. A. Taylor (College of Technology, Manchester) made some further remarks on the trial with acridine, and concluded that one of the virtues of the Fourier-transform method was that it could give information about parts of a molecule, as well as the complete unit. This had been shown some time ago by Knott,[10] but had been forgotten. A large part of the acridine transform was distinguished from the benzene transform only by the shape (not the position) of its peaks.

THE DETERMINATION OF SYMMETRY ELEMENTS BY X-RAY METHODS

Dr. A. J. C. Wilson (University College, Cardiff) presented a paper "The determination of symmetry elements by X-ray methods," by Dr. D. Rogers and himself. For a long time it had been thought that only the symmetry elements containing a translation component could be determined by means of X-rays; Buerger[11] had concluded that only sixty of the 219 effectively distinct space groups were unambiguously distinguishable without a knowledge of the crystal class from other observations. (By making use of a further clue in the Laue symmetry this number might be raised

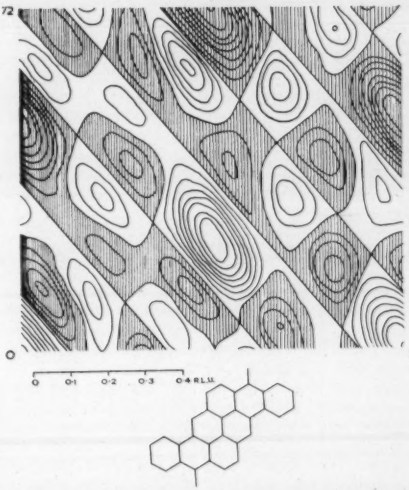

Fig. 2. Fourier transform for flavanthrone ($C_{28}H_{12}O_2N_2$). The shaded parts represent negative areas. Only half the repeat unit is shown in the x direction (vertical); the rest is related by a centre of symmetry at $(\frac{1}{2}, \frac{1}{2})$

to sixty-two.) The symmetry elements not involving translation components, however, had recognizable statistical effects on the intensities of X-ray reflexions. Methods based on these statistical effects might be divided into two groups, called average-multiplier methods and distribution-function methods. Both had developed from a method suggested several years ago for determining absolute from relative X-ray intensity data.[12]

This method was based, roughly, on the principle of conservation of energy. The atoms forming the crystal, if arranged randomly, would diffract a certain fraction of the incident X-rays into a given range of $\sin^2\theta$. If

they were now arranged systematically, as in the crystal, the fraction diffracted into the same range of $\sin^2\theta$, averaged over all possible positions of the crystal, would be the same as before, provided that the range were wide enough for the averaging to even out random fluctuations. This meant that the average of the intensities of all reflexions having approximately the same value of $\sin^2\theta$ was proportional to the sum of the squares of the scattering factors of the atoms within the unit cell, and if the intensities were on the absolute scale and the corrections for polarization, temperature and so forth had been made was actually equal to it:

$$\langle I_{hkl} \rangle \equiv \Sigma \tag{6}$$

where

$$\Sigma = \sum_{j=1}^{N} f_j^2 \tag{7}$$

was the sum of the squares of the scattering factors of the N atoms in the unit cell. This result referred to ideal intensities on the absolute scale. The value of Σ could, however, be calculated merely from a knowledge of the cell content, and hence the factor required to place relative intensities on the absolute scale was obtained by comparing the calculated value of Σ with the average value of the observed (relative) intensities.

The "average-multiplier" methods depended on the fact that when mirror planes or rotation axes were present there were groups of atoms which differed only in one or two parameters, not all three. For example, if the crystal possessed a mirror plane perpendicular to c, there would be pairs of atoms with the same values of x and y, but different values of z. These coincided in the projection on to the ab plane, so that for the $hk0$ reflexions the crystal behaved as if it contained $N/2$ atoms of scattering factor $2f_j$, giving an apparent value of Σ for these reflexions of

$$S = \sum_{j=1}^{N/2} (2f_j)^2 = 2\Sigma, \tag{8}$$

and the average intensity of the $hk0$ reflexions would be twice the general average, giving an "average multiplier" of two. By similar considerations the average multipliers for other symmetry elements could be obtained.[13] By the interaction of two or more symmetry elements multipliers as high as twelve were generated in certain crystal classes.

The intensities of the reflexions on an X-ray photograph were not all the same, but varied from zero up to a theoretical maximum proportional to the square of the number of electrons in the unit cell. There were degrees in the amount of variation, however, as might be seen on examining photographs of a non-centrosymmetric and of a centrosymmetric crystal. It was fairly obvious that the amount of variation was much less in the photograph of the non-centrosymmetric crystal, and that there were fewer very weak spots. These differences in the photographs could be expressed by plotting the percentage $N(z)$ of the reflexions that had intensities less than or equal to $z\%$ of the local average intensity. The theoretical curves took the form[14]

$$(1)N(z) = 1 - \exp(-z) \tag{9}$$

for non-centrosymmetric crystals, and

$$(\bar{1})N(z) = \text{erf}\,(z/2)^{\frac{1}{2}} \tag{10}$$

These were completely verified experimentally for crystals that were sufficiently complex to justify the use of statistical arguments. They were valid not only for the general reflexions but also for centrosymmetric or non-centrosymmetric projections. The presence or absence of a centre of symmetry in a crystal or projection, therefore, could be detected by plotting the experimentally observed fraction of reflexions which are equal to or less than $z\%$ of the local average intensity and comparing with the theoretical curves (Fig. 3).

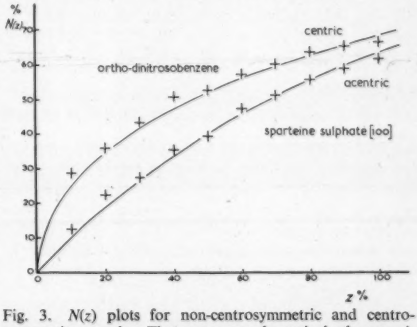

Fig. 3. $N(z)$ plots for non-centrosymmetric and centrosymmetric crystals. The curves are theoretical; the experimental points for the [100] projection of sparteine sulphate (thus found to be non-centrosymmetric, fixing space group as $P2_1$) are included through the courtesy of Mr. D. C. Phillips. Those for *o*-dinitrosobenzene (thus found to be centrosymmetric, fixing the space group as $P\bar{1}$) are included through the courtesy of Mr. R. Hulme

Dr. Wilson concluded with a discussion of some particular substances, and some considerations on the applicability or otherwise of the methods. By their use the thirty-two crystal classes, and hence 215 out of the 219 effectively distinct space groups, could be unambiguously determined. Atoms in special positions were a difficulty with both methods, though they could be allowed for in the average-multiplier method. Heavy atoms in general positions caused no trouble to the average-multiplier methods, though they might make the distribution-function methods entirely inapplicable. He emphasized that statistical methods were only applicable when the structure was reasonably random, and that one had to use a certain amount of intelligence in deciding whether or not they might legitimately be applied. Great caution was necessary for long-chain and layer compounds.

In the exhibition held in connexion with the conference Dr. D. ROGERS exhibited a part, capable of handling six atoms, of an instrument for calculating structure amplitudes. In addition to its primary purpose, this machine showed very clearly how centrosymmetric structures produced a larger fraction of weak reflexions than did non-centrosymmetric. He also showed models illustrating the statistical properties of the three monoclinic and the three orthorhombic crystal classes.

In the general discussion on the above papers Dr. D. SAYRE (University Museum, Oxford) presented a new set of relations between the structure amplitudes (possibly complex) of a substance consisting of known, resolved, identical atoms. If $\rho(x, y, z)$ were the electron density and $F(h, k, l)$ the structure amplitudes, two expressions could be written for the structure amplitudes $F_2(h, k, l)$ of a structure with electron density $\rho^2(x, y, z)$. This structure was very similar to the actual one, but with "atoms" of a different, more concentrated, shape, and hence with different atomic scattering factor f_2. If $g(h, k, l)$ were the ratio of f_2 to the actual atomic scattering factor f,

$$F_2(h, k, l) = g(h, k, l)F(h, k, l) \qquad (11)$$

for all hkl; for spherically symmetrical atoms g was in fact a readily calculable function of $\sin\theta/\lambda$ only. But since

$$\rho^2(x, y, z) = V^{-2} \sum_{h_1k_1l_1} \sum_{h_2k_2l_2} F(h_1, k_1, l_1)F(h_2, k_2, l_2) \exp\{2\pi i[(h_1 + h_2)x + (k_1 + k_2)y + (l_1 + l_2)z]\}$$
$$= V^{-2} \sum_{hkl} \Big[\sum_{pqr} F(p, q, r)F(h-p, k-q, l-r)\Big] \exp\{2\pi i(hx + ky + lz)\}, \qquad (12)$$

$$F_2(h, k, l) = V^{-1} \sum_{pqr} F(p, q, r)F(h-p, k-q, l-r), \qquad (13)$$

and the structure amplitudes for the real crystal had to satisfy the relation

$$V^{-1} \sum_{pqr} F(p, q, r)F(h-p, k-q, l-r) = g(h, k, l)F(h, k, l) \qquad (14)$$

for all hkl. The method had been successfully used to determine the signs of nineteen terms for an unknown structure, hydroxyproline.

AUTOMATIC CALCULATING MACHINES

On the evening of 16 November Professor D. R. HARTREE (University of Cambridge) delivered a discourse entitled "Automatic calculating machines." He distinguished between analogue computers or "instruments" and digital computers, or "machines." The former replaced the calculation by the measurement of some physical magnitude, and had thus an inherent limit on their accuracy; the latter counted in integral steps only, and were limited in accuracy only by the number of digits that could be carried. Analogue computers were ordinarily single-purpose instruments, and designed to give sufficient accuracy for the purpose; digital machines were ordinarily capable of a large number of different computations (summation of series, evaluation of integrals, solution of differential or simultaneous equations, inversion of matrices), depending on the operating instructions fed into them. Professor Hartree gave an outline of the development of digital computers from Babbage's proposed difference engine to "EDSAC" of the Cambridge Mathematical Laboratory, and discussed the organization of calculations for the latter. As an account of a similar discourse has already been published[15] this brief summary must suffice here.

A REVIEW OF PUNCHED-CARD METHODS

The second session opened with "A review of punched-card methods," by Professor E. G. Cox (University of Leeds).[16] The early "Hollerith" methods had consisted in transferring the Beevers–Lipson strips on to punched cards without any fundamental changes in the method of Fourier summation; in particular hand picking of cards according to amplitude had been retained. Because of the ten-fold increase in accuracy aimed at and the consequent ten-fold increase in the number of amplitudes to be used, however, methods of building up any required amplitude from a small number of basic values had been introduced so as to reduce the amount of hand picking. Nevertheless, hand picking had remained as a comparatively slow step in the process of Fourier summation, and one liable to produce errors. In the earlier days these could not altogether be avoided, so that methods of checking, requiring more time, had been indispensable.

Since, in general, more time was consumed and more errors occured in handling the cards than in their passage through the machines, it was desirable to plan methods of synthesis so as to involve the fewest hand operations on them (other than merely the lifting of a pack from the output of one machine to the input of another). In a three-dimensional synthesis, for example, even if hand picking were used for the first stage, errors at subsequent stages could be minimized by summary punching or similar means.

Hand picking could be avoided altogether either by selecting amplitudes by means of a sorter or by punching the experimental F's directly on to cards at the outset. There were many ways of using either procedure.[20] In the first case amplitudes were chosen (e.g. powers of three or powers of minus two) and a large part of each card was punched in a suitable pattern for the selection of amplitudes on the sorter; on the remainder of the cards was a reproduction of a part of the appropriate Beevers–Lipson strip. (A third of a strip, i.e. five or six entries, was convenient because British tabulators would not normally print from more than six counters simultaneously; a complete strip with entries at sixtieths was then represented by three cards.) In the early methods separate cards had been used for positive and negative

amplitudes, but this could be avoided by suitable double punching in each column and putting the cards through the machine upside down when negative values were needed.

Methods such as the preceding were valuable particularly when rolling-total tabulators were not available, but they had the disadvantage of requiring mechanical sorting of rather large numbers of small packs of cards. For many purposes, therefore, methods involving the direct punching of the experimental amplitudes on to cards were to be preferred. This procedure was used in differential synthesis;[19] on British equipment it was necessary at present to carry out the necessary multiplication of the *F*'s by the trigonometrical functions indirectly by "progressive digiting," but where multiplying punches were available it could be done directly. For ordinary syntheses Greenhalgh had described an ingenious method in which the trigonometrical functions were expressed in binary scale so that in effect each binary digit conveyed an instruction to the tabulator to carry out an arithmetical operation on the amplitude; this was possible because of the simple way in which the contents of a counter in a rolling-total tabulator could be doubled.

The particular methods to be adopted depended upon the equipment available; certain things, e.g. calculation of structure factors, required elaborate equipment, but three-dimensional syntheses could, if necessary, be done with only a sorter and a simple tabulator. Two-dimensional projections were usually not worth doing with punched-card equipment, and at the present time in Leeds the punched-card work was almost entirely with three-dimensional and differential syntheses and structure-factor calculations. The computation of a synthesis at $60 \times 30 \times 30$ points (with say 1 000 *F*'s) required fifty to seventy man-hours, of which about two-fifths was spent on the tabulator, and the rest in sorting and reproducing. It was interesting to compare this with the figure of thirty-two hours given by Dr. F. Ordway at the Pennsylvania State College Conference (Easter 1950) for the time required by the NBSAC I (modelled on EDVAC) for calculating the electron density at $60 \times 30 \times 30$ points with 600 complex *F*'s; the apparent slowness of the electronic machine was due to its *serial* output. It seemed likely that British electronic machines now under construction, if fitted with *parallel* output of the Hollerith type, would be able to compute three-dimensional syntheses about ten times as fast as Hollerith equipment of the type now available.

In reply to questions Professor Cox said that the only point in the tenary system of calculation was that it could use amplitudes up to about 300 with a saving of cards. An accuracy of one in 100 was ordinarily too little, one in 1 000 too much, and one in 300 a satisfactory compromise. The amplitude was printed on the cards, but it was not easily seen, and in order to avoid mistakes it was better to use a mechanical sorter. Handling and particularly shuffling of the cards was inadvisable, as any deformation might cause the card to stick in the machine.

THE USE OF THE PEPINSKY ANALOGUE COMPUTER IN CRYSTAL-STRUCTURE DETERMINATION

Mr. C. W. BUNN spoke on "The use of the Pepinsky analogue computer in crystal-structure determination." This computer showed the results of a two-dimensional Fourier synthesis as a contour map on a cathode-ray oscilloscope screen. The basic principle by which this was achieved was the superposition of sets of bands having a sinusoidal distribution of intensity.[21] This procedure produced a distribution of light intensity on the screen, the atoms showing as diffuse maxima, and the earliest form of the Pepinsky machine gave this result. It had now been altered to give a contour map, which had an immense advantage over the earlier form of image.

Some idea of the size and complexity of the computer might be gathered from the following details. There was a master oscillator controlling twenty generators which give frequencies in multiples up to twenty; indices (of crystal planes) up to twenty in each direction could be dealt with. The circuits contained 1 200 valves. The ninety feet of control panel contained 840 potentiometers on which amplitudes of cosine terms were set in by turning dials, and the same number for sine terms. For centrosymmetric projections only the cosine dials were used, the phase of any term being reversed at will by a switch. There were facilities for adjusting the shape and size of the unit cell on the oscilloscope screen; for switching in contours one by one; for using a variable contour level; for showing four contiguous unit cells instead of one; and for magnifying a section of the unit cell for closer study.

The uses of the machine might be divided into two groups: first, computation of definitive syntheses, in which the phases of all terms were known (Patterson syntheses, and the production of final electron-density maps), and secondly, tactical uses in which the facility of phase reversal was utilized in attempts to discover the correct phases of terms.

In judging the performance of a computer one had to take into account both speed and accuracy. For two-dimensional work the machine could hardly be more rapid; the time taken was simply that of setting in the terms by turning dials. For an average projection this could be done in something like fifteen minutes, and the result was on the screen within one second after setting in the last term. The accuracy was, in the nature of things, not great, and could not be compared with that of digital machines. It was probably adequate for Patterson syntheses, and also for electron densities where the interest was in the chemical structure or the general stereochemistry of molecules, but it might not

suffice for electron densities when bond lengths were required. A three-dimensional synthesis was handled as a succession of two-dimensional ones, the time taken being a few hours after the preparation of data for each level. The contour maps could be photographed rapidly; the time-consuming operation of drawing many diagrams was thus avoided.

The outstanding novel feature was the possibility of seeing the result of a phase change instantly on the screen, and this had led to the development of a new trial method of refinement of approximate structures. It had, so far, been applied only to centrosymmetric projections for which all phases are necessarily 0° or 180° and phase reversal was effected by flicking a switch. The principle was to reverse the phase of each term in turn, and to judge from the map which phase was likely to be correct. The power of the method depended on the effectiveness of the criteria which could be used in judging the maps. Two objective criteria, proved to be effective, were (i) the phase that gave the fewer and shallower negative electron-density regions was more probable, and (ii) the phase that gave the fewer and smaller spurious maxima between molecules was more probable. This method had been found to lead to very rapid refinement of structures whose main outlines had already been settled either by the heavy-atom method or by earlier trial methods based on structure factors.[22] Mr. Bunn had had an opportunity of testing the method successfully on sodium benzylpenicillin.

The success of this procedure had led to speculation on the possibility of using such tactics at earlier stages. Was it possible to start with say half a dozen of the strongest low-order terms, ring the changes on the phase combinations, select the most promising map, and then add further terms? Whether this approach would be effective or not depended on the criteria used in choosing between alternative maps. The criteria already mentioned, although effective in the refinement stage, were useless at the beginning. It was possible to look for an expected chemical feature (such as a chain or a ring of atoms), and select the map which showed it most clearly. But this method had to be regarded with great scepticism; it was only too easy to produce maps which looked plausible in a rough way. It should not, however, be concluded that this procedure was useless; rather the plausible rough maps (which were produced extremely rapidly) should be regarded as tentative solutions to be tested by whatever methods were practicable. The straightforward way of dealing with a plausible map was to place atoms (the number and diffracting powers of which were, of course, known) in the map in such a way as to satisfy roughly the electron densities, to calculate structure factors, and then to consider what changes were necessary to rectify the errors in the structure factors. It was no longer necessary to consider each crystal plane separately in a qualitative way; the quantitative systematic way of dealing with this problem was to perform an "error synthesis" (or F_o–F_c synthesis). The error synthesis could be done extremely rapidly on the Pepinsky machine.

Mr. Bunn concluded by saying that the extreme facility with which Fourier syntheses could be performed made it possible to devise new tactics. Fourier syntheses alone were not enough, but with the addition of a rapid structure-factor machine, one could contemplate programmes in which there was a rapid sequence of tentative electron-density maps, structure-factor calculations and error syntheses. Whether such methods would facilitate the surmounting of the main "potential barrier" in crystal-structure determination—the finding of the rough structure—remained to be seen. If they did, the effect might well revolutionize the determination of crystal structures.

SOME CRYSTALLOGRAPHIC CALCULATIONS WITH EDSAC

Dr. J. C. KENDREW (Cavendish Laboratory, Cambridge) then presented the paper "Some crystallographic calculations with EDSAC," by himself and Mr. J. M. BENNETT. EDSAC was a digital computer, working in the binary scale, and its present "memory" had about 500 storage positions. The smallness of its store was a considerable disadvantage for crystallographic computations, and in planning the computing programme considerable ingenuity had to be used in order to get all the necessary information into the memory simultaneously. Doubling the number of storage positions could make it adequate for two-dimensional syntheses, but about 8 000 positions would be necessary in order to do a three-dimensional synthesis without breaking it into parts. It was intended shortly to increase the store of EDSAC from 512 to 1 024 positions, and later to add a magnetic store with a capacity of about 4 000 positions.

The programme for a one-dimensional synthesis

$$\rho(x) = \sum_{h=0}^{N} F_h \cos(2\pi hx/a) \qquad (15)$$

consisted of feeding the values of F_h and $\cos(2\pi hx/a)$ into storage positions, and instructing the machine to operate on them in two cycles, an inner h cycle and an outer x cycle. Since the machine worked on the binary scale, the most convenient subdivision of the cell-edge was into 128 (i.e. 2^7) parts. There were then thirty-three values of the cosine to be stored. The inner (h cycle) took place as follows:

(1) $\cos(2\pi hx/a)$ was put in the multiplying register,
(2) F_h was found and multiplied by $\cos(2\pi hx/a)$,
(3) the result was added into the progressive total,
(4) the instruction "find F_h" was automatically altered to "find F_{h+1},"
(5) x/a was added to hx/a, giving $(h+1)x/a$,
(6) $h+1$ was compared with N, and if found to be less than or equal to N the cycle was automatically repeated.

If at operation (6) in this inner cycle $h + 1$ was found to be greater than N it was an indication that the sum in equation (11) had been completed for a particular value of x, and the machine left off the repetition of h cycles and embarked on an x cycle, as follows:

(1) The progressive total was printed,
(2) the progressive total was set to zero,
(3) x/a was increased by one division of the cell edge and compared with unity; if not greater,
(4) hx/a was set to zero,
(5) the instruction "find F_N" was altered to "find F_o,"

and the h cycles began again. After 128 x cycles, involving $128(N + 1)$ h cycles, the 128 values of $\rho(x)$ had all been printed and the machine came to a stop.

For a two-dimensional synthesis the straightforward process involved four cycles, each enclosing those preceding it: h cycle, k cycle, x cycle, y cycle, but time and storage space could be saved by an apparently more complex sequence: h cycle within a k cycle, result to intermediate store, k cycle within a y cycle, x cycle enclosing the four preceding cycles. The former sequence took about 10·5 sec, the latter 2·5 sec, most of which was printing time.

Dr. Kendrew then discussed measures to economize storage space, such as using a logarithmic scale of F's (which might be all that was warranted by the experimental accuracy) and putting three items of information into two storage spaces. He also described methods for performing three-dimensional syntheses, involving three sets of interacting double cycles. The time required would be much less than that required for Hollerith methods. As examples of the present speed of EDSAC he mentioned that a two-dimensional Patterson synthesis containing 260 terms and evaluated at 561 points had taken twenty-nine minutes in all, and that a three-dimensional synthesis of about 2 000 terms evaluated at 9 537 points had taken nine hours.*

STRUCTURE-FACTOR AND ELECTRON-DENSITY BALANCES

Dr. V. VAND (University of Glasgow) described some "Structure-factor and electron-density balances." For structure-factor calculations the balance consisted of a Meccano frame supported on knife-edges. Each atom was represented by a gear with an eccentric weight. As the gear turned from zero the movement of the weight disturbed the balance, creating a moment proportional to the product of the weight and the cosine of the angle through which the gear had turned. The setting of a counterbalancing rider was then proportional to $\Sigma f \cos \Theta$.

With suitable gearing such a machine could calculate electron densities also. One already constructed had 100 gears, each driven through a differential from one of ten horizontal "slow" shafts, which turned through 1, 2, . . . , 9, 10 revolutions per cycle of the machine, and one of ten vertical "rapid" shafts, which turned through 0, 10, 20, . . . , 90 revolutions per cycle of the machine. The pivoted frame carried a mirror reflecting a beam of light on to a photocell controlling a servo motor, and this automatically restored the balance by moving a rider. Refinements to give better control of the motion included dash pots, photocell amplifier, and velocity feed-back to slow the servo motor as balance was approached. About 300 points in an electron-density synthesis could be calculated per hour, excluding setting time. It would be possible to run the machine so that it stopped automatically when it reached certain values of electron density, thus avoiding labour and the possibility of mistakes in interpolating contours.

Two-dimensional syntheses were performed as one-dimensional syntheses along a line with slope of, for example, one in twenty-four. The method was similar to that of the "one-dimensional crystal" of Donnay and Donnay,[23] but, in order to get a sufficient density of points with only 100 gears, about four parallel lines with displaced origins were used. The machine calculated

$$\sum_n F(n) \cos [2\pi(nt + \alpha)] \qquad (16)$$

For the present purpose this could be

$$\Sigma F(h0l) \cos [2\pi(hx/a + lz/c)] \qquad (17)$$

where

$$z/c = x/ra + m, \qquad n = rh + l$$
$$t = x/ra + m, \qquad \alpha = -hrm$$

r = slope of line, m = displacement of origin, and x and z were co-ordinates in the unit cell.

In the discussion Sir LAWRENCE BRAGG exhibited a simple structure-factor calculator, designed after a conversation with Dr. Vand. In this the atoms were represented by dials with offset weights. The offset was set to $2\pi ky/b$, and the dial to $2\pi hx/a$, so that the moment produced was $F^{\cos}_{\sin}(2\pi hx/a)^{\cos}_{\sin}(2\pi ky/b)$. Balance of the platform carrying the dials was restored by a torsion wire controlled by a micrometer head. The time required to calculate an F was not much less than with tables, but no thought was required and the chance of errors much reduced.

Dr. D. SAYRE (University Museum, Oxford) described a method of calculating structure factors by Fourier synthesis. He considered the electron-density function of a structure of hypothetical atoms for which the atomic scattering factors f were equal to those of the actual atoms within the limiting sphere in reciprocal space, and zero outside. The electron density of such atoms could readily be found, and the electron density $\rho'(x, y, z)$ of

* This summary does not do full justice to the latter part of Dr. Kendrew's paper. It seemed better to use the available space for an intelligible account of the arrangement of a simple calculation, rather than to report all parts of the paper with equal unintelligibility.

the structure sampled at the points of a subdivided crystal lattice. The subdivision could be anything convenient, provided that the sampling points were less than $\lambda/4$ apart. Then the ordinary Fourier sum

$$V' \sum_{x,y,z} \rho'(x, y, z) \exp\{-2\pi i(hx + ky + lz)\} \quad (18)$$

where the summation was over the points of the sub-lattice and V' was the volume of the subcell, gave the structure factors of all reflexions within the limiting sphere, without approximation. The method was in use in both Oxford and Cambridge, and for moderately complex structures required about one-third of the usual time. Dr. Cochran had modified it somewhat to make three-dimensional applications more convenient.

Dr. H. LIPSON exhibited a device, made by Mr. M. M. Woolfson, in which sinusoidal fringes of the correct spacing and orientation were projected on to a set of photocells placed at the atomic positions in a square unit cell. The accuracy was as good as was required for testing trial structures, and the work involved was mainly in the preparation of the master film of fringes. He hoped to make copies of this film generally available at a cost of a few shillings. Mr. BUNN said that the apparatus described by Dr. Lipson was the sort of thing he had in mind when speaking of a rapid structure-factor machine to work side by side with the Pepinsky machine; it would give structure factors as fast as one could change masks and take galvanometer readings—several a minute, or a whole zone in 15 min. This matched the rapidity of the Pepinsky machine for Fourier synthesis. But whereas Lipson's structure-factor machine was very simple and inexpensive, the Pepinsky machine was very costly. Crystallographers really needed a Pepinsky-type machine in their own laboratories, and an inexpensive way of achieving the results of the Pepinsky machine would be very welcome. One possible way was to use a bank of Bragg–Huggins masks, all casting their images simultaneously on to the same screen; this arrangement (which has been used by D. McLachlan and others) gave an image of the crystal structure in the form of a distribution of light intensity; phases could be changed simply by shifting masks. A contour map, which was really necessary for the tactical methods developed in connexion with the Pepinsky machine, could be obtained by throwing the light on to the screen of a television camera (iconoscope) and producing a contour map on the screen of a cathode-ray tube; the amount of circuitry necessary for contouring was quite moderate. This was a scheme for a "utility-model" Pepinsky machine which might be worth following up; it had been evolved during a discussion between D. McLachlan, Dr. Lipson and himself at the conference on computing methods at Pennsylvania State College in April 1950.

Dr. VAND replied that he had a rather similar apparatus under construction. The fringes were the shadows of a set of cardboard strips illuminated by a diffuse source shaped so that the intensity variation in the shadows was sinusoidal. No lenses were needed. There would be 100 boxes projecting fringes on to a screen; as Mr. Bunn had explained, the screen could be replaced by an iconoscope and contour lines exhibited by means of a cathode-ray tube. The arrangement could be simplified by reversing the path of the light; that is by replacing the iconoscope by a cathode-ray tube and the lamp by a photocell.

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ORIGINAL CONTRIBUTIONS

The hardness and strain-hardening of Plasticine

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When Plasticine is loaded with a sphere indenter at a series of increasing loads, time being allowed for the indentation virtually to cease, the yield-values increase markedly. Meyer coefficients, while apparently increasing from values of less than 2·0 at small loads, soon reach a constant high value of about 3·5 and, except perhaps after very long times when they may increase still further, remain approximately constant. But in such experiments, stress, strain and time are all varying. When stress is maintained constant, the rate of indentation (i.e. increase in projected area of indent) progressively slows down with time; but the Meyer exponent obtained under these conditions is found to be less than 2·0. Contrary to the views of earlier workers, it is concluded that Plasticine does work-harden considerably in most accepted senses of the term but the definition of "work-hardening" is too vague for a categorical statement to be made. Plasticine also shows a very slow elastic recovery of considerable magnitude.

As early as 1923, O'Neill[1] showed that for small indentations produced by a sphere indenter, the Meyer coefficient (n)* for pitch and for Plasticine approximated to slightly less than 2·0. n is defined as the slope of the curve plotting $\log W/\log d$ where W is the load and d is the diameter of indent. It has therefore been inferred that not only true fluids such as pitch, but also plastic materials, do not strain-harden (see Finniston, Jones and Madsen[2]). Few further experiments appear to have been done on soft plastic materials. Hind[3] claims that many clays "stress-harden" and actually experimented with Plasticine; but does not appear to have drawn any conclusions as to whether this material hardens. Boor and his co-workers[4,5] are by no means the only authors to describe experiments on high polymers but their work is noteworthy in that they emphasize the importance of clarity of definition of terms. They point out that at least five only partly correlated properties of materials are referred to in the literature as "hardness": (1) Indentation while under load (Rockwell test); (2) Ditto after release; (3) Instantaneous recovery of indent (Shore Scleroscope); (4) Optical properties as changed by abrasive wear; and (5) Loss of weight by abrasion. Though not quantitative, one might add: (6) Mohs' Hardness Scale, which depends on the relative capacity of materials to scratch one another and does not seem to fall within category (4).

In the case of metals, the hardness (Brinell) is determined from the size of the indent following elastic recovery, thus measuring plastic deformation; but in the case of rubber-like materials, from the indent while under load, which is mainly an elastic displacement.[6]

The terms "work-hardening" and "strain-hardening" generally refer to the increases in yield-value which result from working or straining the material, but are also taken to mean the increase in hardness, usually in accordance with (1) above, as expressed by n. See also Burgers and Scott Blair,[7] who propose that the wider use of this term to include any increase in consistency ascribable to working or straining, be discontinued (cf. Schofield and Scott Blair[8]). Since Plasticine is a fairly typical soft plastic material and is now widely used for research purposes by industrial physicists, it seemed of interest to examine with some care its behaviour when indented by a spherical "hardness" tester. Since the behaviour is much more complex than that of metals it is important to control carefully the conditions of the experiments.

* The Meyer coefficient which defines *hardening* is not to be confused with the Meyer number which defines *hardness* (see (4)).

EXPERIMENTS AND DISCUSSION

The Plasticine was purchased in 1 lb blocks, each of which was built up of a large number of cylindrical rods. A preliminary experiment on this material, which could be gently pulled apart after the loading, showed that the displacements of material below the indenting sphere were so limited in extent that a slab of Plasticine some 2·5 cm thick was adequate for the experiments. Plasticine markedly changes its consistency with comparatively small changes of temperature. The experiments were therefore carried out in a constant temperature room. The working of the sample is itself liable to cause a temperature rise. Observations were made on a thermometer embedded in the sample, small unavoidable changes in temperature being noted. Where these exceeded 0·5° C the data were discarded. From replicated experiments, those were selected for which the temperature remained most nearly constant. In studying the effects of increasing load on indentation, the increments of load were made without allowing any time for partial elastic recovery. To allow full recovery between loadings would have been impracticable.

Two types of experiment were carried out. The first presents no new features as an experimental method. A sphere hardness tester was used in which a ball of diameter 2·54 cm is pressed into the surface of the sample, the depth of penetration (Δ cm) being read on

a dial gauge. The magnitude of the load was controlled by adjusting the position of a weight along a horizontal lever arm. The sample was compressed at a series of loads (W) of increasing magnitude, the rate of increase of Δ with time being recorded during a period of 30 min. In order that the experiment should start at a well-defined finite stress, a pre-straining lasting for 5 min preceded the experiment. In order to obtain values of $d\Delta/dt$ it was convenient to plot Δ against $\log t$. The values of $d\Delta/dt$ were then plotted against stress (load/projected area of contact). These curves were not easy to extrapolate to zero rate of indentation and it was therefore decided to follow the not unusual procedure of defining the yield-value as the stress required to cause a very small arbitrary rate of deformation, in this case $0{\cdot}5 \times 10^{-5}$ cm/sec. These yield-values were then read off by interpolation or very slight extrapolation and plotted against deformation. The experiment was done at two temperatures, 16·0° C and 18·7° C (see Table 1).

It is evident that in the later loadings, Plasticine work-hardens considerably in the sense that $n > 2{\cdot}0$. There is no evidence of regular variations of n with time of loading unless possibly there be a slight increase towards the end of the loading.

The second type of experiment presents a novel experimental feature which was discussed in a short preliminary note.[9] In the earlier experiments and, as far as the authors are aware, in all experiments described by others using sphere indenters, the use of a constant load during the indentation has meant that the stress falls as the strain increases. We have shown[9] that with some materials it is not difficult to increase the load in such a way as to maintain an approximately constant stress by moving the weight along the arm of the instrument at a suitably selected constant speed. But this depends on the nature of the material and is not possible with Plasticine. In order to study the work-hardening of Plasticine at constant stress the following experiment was done. The

Table 1. *Increase in yield-value with increasing depth of indentation*

Temp. 16·0° C	Δ (cm)	—	0·317	0·336	0·350	0·367	0·381	0·396
	S_0 (kg.cm^{-2})	—	4·780	5·052	5·275	5·443	5·625	5·834
Temp. 18·7° C	Δ (cm)	0·350	0·377	0·397	0·415	0·435	0·452	0·469
	S_0 (kg.cm^{-2})	4·025	4·170	4·425	4·575	4·730	4·920	5·025

It will be seen that these curves are approximately linear. It is not felt, however, that the data are sufficient to justify any conclusions beyond the fact that S_0 certainly increases with increasing values of Δ and, for this reason no functional relation between the variables has been assumed.

Unfortunately it is not possible to get more readings, since the total depth of indentation available for all the loadings taken together is limited; but there can be no possible doubt that, if work-hardening is defined as increase in yield-value on repeated loading, Plasticine work-hardens.

In order to determine the Meyer hardness coefficients, values of $\log W$ were plotted against $\log d$ for a series of times during the loading process. A random selection of these curves is shown in Fig. 1. It will be seen that the curves are only linear for the greater loads. At small loads the Meyer exponent is increasing with load probably starting at a value less than 2·0 (softening) and, by the time the third load (11·7 kg) is reached, stabilizing at an average value of 3·44 at 16·0° C and 3·57 at 18·7° C. The straight lines shown were fitted through the five top points by the method of least squares.

With regard to possible softening at small loads, Plasticine is known to show some false-body or thixotropy and the sample was left for at least 24 h between tests. It should also be pointed out that there is certainly no state of equilibrium in the system, and it is therefore not remarkable that the log-log curves are not in all cases linear. The slopes (n) for both the experiments are given in Table 2.

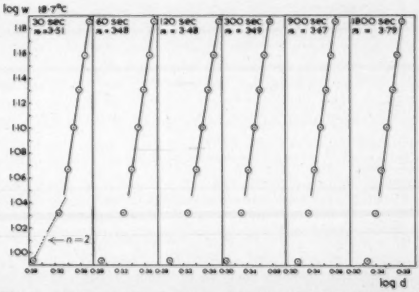

Fig. 1. Selected Meyer curves showing effect of duration of loading

sample was compressed with a small pre-straining load until the dial reading indicated that a convenient previously selected stress (load/projected area of indent) was reached. A table had been previously prepared giving the position of the weight on the lever arm corresponding to a series of dial readings. A series of readings was taken and immediately after each reading the weight was moved along the lever arm so that the stress was

Table 2. *Values of n obtained at two different temperatures*

Temperature (°C)	Time (sec) 30	60	120	300	900	1 800
16·0	3·35	3·29	3·39	3·33	3·57	3·71
18·7	3·51	3·48	3·48	3·49	3·67	3·79

adjusted to the selected constant value (6·21 kg/cm²). It is appreciated that in the earlier stages of the experiment, when readings were taken every 15 sec, this compensation was not fully satisfactory but, especially during the later part of the experiment, this procedure does maintain a comparatively constant stress. In this experiment, work-hardening as defined in terms of either the rate of increase of projected area of indent with time or the Meyer coefficient, n, has been investigated. It should be noted that the first definition involves area of indent or d^2 rather than depth of indentation Δ. If n, which is given by the slope of the log W/log d curve, is more than 2·0 the sample is said to work-harden. Thus the former definition refers to an increasing resistance to indentation with time and the second to the effect of changes in load. These two effects cannot be altogether separated, since increments of load take place during the course of the experiment, but load does not increase linearly with time. A loading curve plotting d^2 against time for constant stress is shown in Fig. 2.

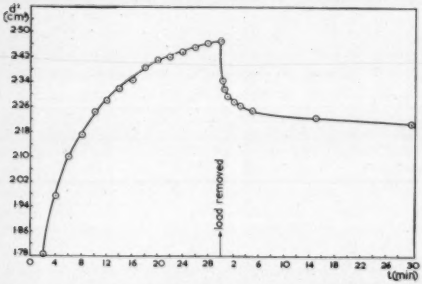

Fig. 2. Increase in projected area at constant stress and also elastic recovery curve

After the experiment was completed, the load was removed and the curve for elastic recovery obtained. This is also included in the figure. Further recovery (unrecorded) was observed even after 48 h. It is clear that the rate of increase of d^2 at approximately constant stress, progressively decreases with time, and in this sense Plasticine may again be said to work-harden.

Finally, the Meyer exponent was calculated from the data shown in Fig. 2 and also from those of an exact repeat of this experiment. The values of n, calculated by the method of least squares, were 1·85 and 1·87 respectively. It should be noted that the loads in this experiment range from 11·2 kg to 16·0 kg. There is no sign of curvature in the log W/log d curves and the low value of n does not correspond to that found in the earlier stages of the variable stress experiment. If a value of $n > 2$ is taken as a criterion of hardening, then Plasticine does not work-harden at the constant stress chosen for this experiment.

The elastic recovery curve is approximately linear plotting Δ against log t. The main interest lies, however, in the fact that the experiment shows that Plasticine recovers elastically to a much greater extent and much more slowly than was previously generally realized. In planning any rheological experiments on this material this fact must be taken into account.

CONCLUSIONS

Does Plasticine work-harden? The answer depends on how work-hardening is defined. By the usual definitions—increase in yield-value on repeated loading or a Meyer exponent greater than 2·0—it certainly does, except under the latter definition for small loads. But, using the sphere hardness tester as it has always previously been used, a very complex situation obtains, since stress, strains and times are all varying. When the experiments are carried out at a constant stress, the load varies and it is thus possible to calculate a Meyer exponent. This exponent, in the case of the stress chosen, is certainly less than 2·0. Yet it might be misleading to say that Plasticine does not harden under constant stress conditions, since the rate of increase of area of indentation certainly slows down as time goes on. It is clear that more precise definitions of work-hardening are needed if we are to discuss the work-hardening of complex non-metallic systems. A comparison with the work-hardening of metals would be interesting. Tabor[10] in a recent paper has proposed a number of relationships between elastic moduli, hardness and hardening coefficients for metals. It had been hoped to compare the behaviour of Plasticine with his results, but it does not seem possible to obtain accurate enough data for this purpose; partly, no doubt, on account of difficulties in getting adequate homogeneity of sample and partly because of high temperature coefficients and the impossibility of preventing significant temperature changes in the samples during the experiments.

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The measurement of dielectric constants of conducting liquids

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The measured force exerted between two plates immersed in a liquid when maintained at a known alternating potential is proportional to the dielectric constant of the liquid, and is independent of any conductance or phase change; this has been used to obtain values of the constant for sodium chloride and tetraethylammonium iodide solutions up to 0·01 N and the apparatus is described. It may be used to determine the dielectric constants of conducting liquids, such as electrolytic solutions, up to a specific conductance of $1 \cdot 2 \times 10^{-3}$ mho-cm, with a reproducibility of $\pm 1\%$.

The determination of the true dielectric constants of conducting liquids at low frequencies can only be carried out by a "force" method. The best-known form of this method is due to Fürth[1]; by means of ballistic techniques[2,3] some workers[4,5] have obtained results repeatable to about $\pm 1\%$ at concentrations of strong electrolytes up to 0·02 N; the agreement of results by different workers on the same electrolytes is, however, poor. The force between two charged plates is independent of conductance or phase change in the medium, or of the direction of the field, and is given by $V^2A\epsilon/8\pi d^2$, where V is the potential, A the area of the plates, ϵ the dielectric constant of the medium, and d the distance apart of the plates. To reduce the heating effect due to the conductance of the medium, d should be small, thus giving maximum sensitivity and so permitting measurement at low potentials and in the shortest time.

CONSTRUCTION OF FORCE APPARATUS[6]

One of the plates immersed in the medium was rigidly fixed to the wall of the non-conducting cell while the other was free to move; the plates were made of stout platinum sheet and were prevented from touching by small non-conducting separators which regulated the value of d. The motion of the moving plate was confined to the axis normal to the surface of the plates by fixing it rigidly to the underside of a stiff light brass beam by means of stiff braced platinum-gold alloy wires, and this beam was suspended by four long thin wires in the same manner as a ballistic pendulum. A small magnet was attached below the other end of the beam so that its axis lay along the line normal to the surfaces of the plates passing through their centre of pressure [Fig. 1(*a*)]. The magnet moved in the uniform field of a Helmholtz coil. An iron framework [Fig. 1(*b*)] 1·2 m high, was fixed to a base equipped with levelling screws. Means were provided at the top of the framework for adjusting the length of the suspension wires; the cell containing the fixed plate, and the Helmholtz coil system, were attached to the base. The suspension wires were made of 30 s.w.g. tinned copper wire, which gave a negligible restoring force. The beam was levelled approximately by adjusting the length of the suspension wires, and the position of the cell was then fixed accurately so that the plates were plane parallel and did not overlap. The position of the moving plate could then be adjusted by tilting the whole

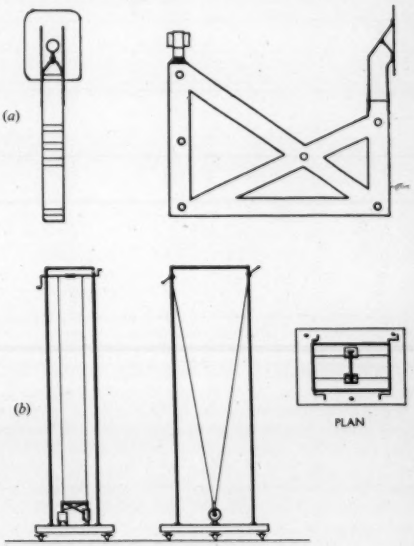

Fig. 1. Plates and framework of apparatus

apparatus. Vibration was eliminated by the use of a Julius type of mounting.[7] The cell was constructed from "Polythene" sheet, $\frac{1}{16}$ in thick. The rectangular inner cell, $3 \cdot 5 \times 2 \cdot 5 \times 5$ cm deep, was surrounded by a jacket, also made of "Polythene" sheet. The plate was rigidly fixed to an inner end wall of the cell by

fusion of the ends of the "Polythene" separators to the wall.

The force between the plates was measured by balancing it against the magnetic force produced by passing an increasing current through the Helmholtz coils until a further slight increase was sufficient to cause the plates to separate. The longitudinal movements of the beam were detected by a combined mechanical and optical lever system (Fig. 2). A galvanometer mirror *M* was attached to a light balanced frame *F*, 5 cm long, which was soldered at each end to phosphor-bronze suspension strips *S*, 0·85 mm wide by 0·035 mm thick, and each 12 cm long. This suspension was placed inside a vertical ½ in bore copper tube, fitted at the top with a torsion head and at the bottom with a flat heavy brass plate with levelling screws. The middle portion of the copper tube was partly cut away to expose the mirror. The lower end of the suspension was soldered to a spring held on the base plate, and the upper end was attached to the torsion head. The wire frame also served to support a taut single nylon fibre *N*, 0·05 mm diameter. This fibre was attached so that it was parallel to the suspension strips, but was displaced about 1 mm laterally from their axis; horizontal tension of the fibre thus rotated the mirror. A light wire connecting link *L*, 1 cm long, was attached to the mid-point of the fibre for this purpose, its other end being soldered to the vertical tip of a sewing needle *P*. A small glass cup *B* was attached to the side of the instrument beam by a light lateral aluminium strip at such a height that when the suspension assembly was placed on the instrument base, the needle point could rest in the glass cup, thus connecting the beam to the mirror frame. Rotation of the mirror was detected by means of a lamp and scale placed 1 m away, so that a direct magnification of 1 000 was obtained. Direct deflexion measurements by microscope and comparison with coil current values showed that no parasitic errors or friction had been introduced. When any shock was accidentally transmitted to the instrument the needle tip was lifted out of the bearing cup, and the beam automatically disconnected from the delicate suspension system. The apparatus, including the anti-vibration mounting, was enclosed in a stout draught-proof box, suspended independently and fitted with doors and windows where necessary.

ELECTRICAL APPARATUS

The measuring circuits and Helmholtz coil system were operated by accumulators standardized by a Weston cell. For the plates of the force apparatus an alternating potential of 2 500 c/s was used. A stable source of this potential was provided by an 80 V, 500 W tandem generator, with output potential regulated to ± 1%. A Variac transformer was used for adjusting the potential below 20 V. The waveform of the output of this source was sinusoidal and free from harmonics.

The alternating potential was measured by the resistance change on applying it to the 100 Ω heating coil of an indirectly heated thermistor (Stantel type L) of about 10 000 Ω bead resistance, and standardizing against a known d.c. potential. Readings of alternating potentials could thus be taken with a reproducibility of ± 0·2% within about 20 sec. The impedance of the heater winding at 2 500 c/s was negligible.

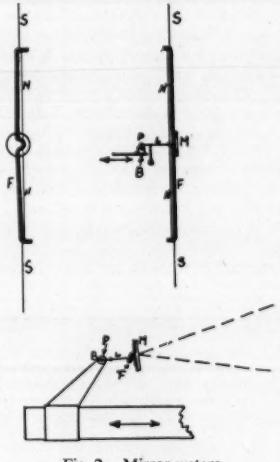

Fig. 2. Mirror system

DIELECTRIC CONSTANT MEASUREMENTS

The cell was filled with the liquid and the levelling screws of the force apparatus adjusted so that the moving plate just rested against the separators. The coil current necessary to move the plate was determined. Coil currents were then determined for various alternating potentials applied to the plates, and plotted against the squares of the plate potentials. A straight line was obtained, the slope of which was theoretically proportional to the dielectric constant of the liquid. The proportionality was experimentally verified to within ± 1% by measurements of ethanol-water and dioxane-water mixtures.[8,9,10,11] As an example the values for the dielectric constants of aqueous solutions of sodium chloride and tetraethylammonium iodide at four concentrations up to 0·01 N are shown in the table. The experimental reproducibility for these electrolytes was found to be ± 1%.

Dielectric constants of NaCl and Et_4NI solutions

Electrolyte	Concn. N.	Temp. °C	ε soln.	ε' water(3)	$\frac{\varepsilon - \varepsilon'}{\varepsilon'} \times 100$
NaCl	0·0025	20	80·2	80·4	−0·25
	0·0050	20	80·2	80·4	−0·25
	0·0075	18	82·0	81·0	+1·23
	0·0100	16	83·0	81·7	+1·59
Et_4NI	0·0025	20	80·7	80·4	+0·37
	0·0050	20	80·4	80·4	0
	0·0075	20	80·3	80·4	−0·12
	0·0100	20	80·3	80·4	−0·12

ACKNOWLEDGMENTS

The authors wish to thank Dr. J. A. Kitchener for helpful discussions during the development of this apparatus. Thanks are also due to the Ministry of Education for a grant to one of us (J. B-R.).

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Absorption edges on Debye-Scherrer X-ray photographs

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[*Paper first received* 14 *September*, 1950, *and in final form* 6 *December*, 1950]

On powder photographs taken with Kα radiation obtained by filtering X-rays from a pure metal target through the appropriate "β-filter," sharp-edged bands which are not normal powder lines, may occur in the low-angle region. These are shown to be due to residual X-rays of short wavelength which give rise to dark bands where fluorescence of silver and bromine occurs in the photographic emulsion. It is concluded that in the analysis of mixtures it should be borne in mind that bands of this sort accompany all strong Kα reflexions from the specimen, and they should be carefully distinguished from weak Kα reflexions.

In the identification of the crystalline constituents of mixtures by X-ray powder methods in this laboratory it is often necessary to give rather heavy exposures in order to obtain as much information as possible about the minor constituents. We find the best compromise between the conditions required for speed and the conditions required for obtaining photographs with low background intensity is achieved by working the X-ray tube at a voltage above that which gives the highest proportion of characteristic radiation, and to pass the X-rays through one of the standard "β-filters." Commonly we use 60 kV, 20 mA, with an effective focal area about 6 mm by 0·3 mm. Such conditions are probably often used in other industrial laboratories. Weak spurious reflexions occur under these conditions which could cause difficulty or mistakes in interpretation. These spurious reflexions, which are distinct from the very weak lines due to Kβ radiation, accompany every strong line in the powder pattern. In favourable conditions they may be seen to be bands which are fairly sharp-edged on one side, and they are not comparable with normal powder lines, but this fact is not obvious when the bands are partially obscured in a complex pattern.

ORIGIN OF SHARP-EDGED BANDS ON POWDER PHOTOGRAPHS

The sharp-edged bands are due to the presence of X-rays of short wavelength in the filtered beam. This may be demonstrated by taking a photograph of sodium chloride with radiation from a copper target filtered through the usual 0·024 mm of nickel and through 0·03 mm of iron in addition. The iron filter allows much of the short-wave radiation to pass, but it greatly reduces the intensity of CuKα. Under these conditions the sodium chloride pattern is practically invisible, but the bands are still very easily seen. The origin of the bands is as follows. Each set of "reflecting" planes in the powder specimen spreads a spectrum of X-rays of all wavelengths present in the incident radiation, though the powder line formed by the predominant wavelength is normally the only indication of the existence of the spectrum which is noticed. Fluorescence of silver or

bromine and high absorption in the emulsion occurs where the wavelength of the X-rays reaching the film is suitably low. The effect of this will be equivalent to an unusually high degree of X-ray scattering within the emulsion, which will lead to an increase in the proportion of silver halide grains that become sensitive to development. In this way there arises a series of weak bands with fairly sharp edges which in a sense repeat the powder pattern given by the predominant wavelength, but at spacings on the film corresponding to the wavelengths of the silver and bromine absorption edges.

CONFIRMATORY EXPERIMENTS

Measurements supporting this view were obtained from films exposed in a 9 cm Bradley-Jay type camera, using normal powder specimens. The high-angle edges of the dark bands were measured. The wavelengths of the radiation at these points were determined by attributing the "white" radiation spectra to the same sets of reflecting planes as gave the powder lines, associating the most prominent edge with the strongest line and so on. By this means most of the bands were found to be due to silver absorption, but in two cases (C3 and C4 below) bands due to bromine absorption were also observed. The results are given in the table.

be seen from observations A, C and D, E that a change in the nature of the X-ray target has no influence on the wavelength attributed to the edge of a band. This is consistent with the explanation which has been advanced.

It is also to be noted that no bands attributable to bromine fluorescence were observed when nickel-filtered copper radiation was used. This fits in with the fact that only a small proportion of such radiation has a wavelength near that of the absorption edge of bromine, the residual "white hump" being of shorter wavelength, near that of the absorption edge of silver. In cobalt radiation filtered through iron, however, a greater proportion of the background has a wavelength near 0·92A, and bands due to both silver and bromine absorption were observed [see table (C)]. This also is consistent with the account given above.

In a complicated pattern the edges of fluorescence bands might be confused with weak Kα reflexions, and in seeking a complete and accurate interpretation of complex patterns this should be borne in mind. For example, from the measurements given in the table it can be deduced that if sodium chloride is present in a mixture absorption edges would occur at the positions of normal Kα reflexions from spacings of 9·22 and 6·38A, or 10·70, 7·41, 5·53 and 3·89A according to

Measurements of absorption edges, using Co or Cu radiation filtered through 0·03 mm of iron or 0·024 mm of nickel, using various powder specimens

Observation letter and number		A	B	C	D	E	F	G	H	I
Substance			NaCl			αFe		CaO	MgO	SiO_2(Quartz)	ZnO
X-ray target			Cu		Co	Cu	Co	Cu	Cu	Cu	Cu
Filter		Ni	Ni + Fe	Fe	Ni	Fe	Ni	Ni	Cu	Ni	Ni
1	Measurement between corresponding pairs of edges, in cm	1·52	1·53	1·52	2·16	2·17	1·85	2·05	1·29	1·75
2		2·19	2·18	2·19		2·15	2·60			
3			2·66	2·94						
4			3·45	4·20						
1	Wavelength of X-rays which reached the film at the observed edges, in Angstrom units	0·47	0·47	0·47	0·48	0·48	0·48	0·47	0·47	0·48
2		0·47	0·47	0·47			0·48			
3			0·48	0·90						
4			0·47	0·95						

DISCUSSION

The absorption edges of silver and of bromine are at 0·484A and 0·918A respectively, which agrees closely enough with the observed figures to establish qualitatively that low-angle bands due to fluorescence accompany strong reflexions in powder photographs. It will be the radiation used. Reflexions in these regions would therefore need to be examined with special care.

ACKNOWLEDGMENT

The X-ray photographs used in this work were taken by T. C. Alcock and J. L. Matthews.

Erratum.—In the article *An apparatus for studying the laws of the magnetic field due to an electric current in a long straight conductor* by D. Owen, p. 6, January 1951 issue, the two lines immediately above Fig. 3 should read "to the axis until the needle sets parallel to that axis. The centre of the needle is then opposite to a neutral point."

A stereoscopic technique for use with an electron microscope*

By J. W. SHARPE, M.A., F.Inst.P.,† General Chemicals Division, Imperial Chemical Industries Ltd.

[*Paper received* 21 *July*, 1950]

Details are given of the method of operating a stereo-holder for use with the Metropolitan-Vickers type EM2 electron microscope. The construction of the holder is described.

With the normal transmission type electron microscope, the image formed is a two-dimensional projection or "silhouette" of the specimen. In some applications, however, such as the examination of surface structure and porosity, it is an advantage to have a three-dimensional picture of the specimen. Owing to the great depth of focus of the electron microscope it is possible to obtain this picture by taking stereo-micrographs which are in fairly clear focus for a considerable depth of the specimen. The technique used in electron microscopy, however, differs from that of normal stereophotography, where two pictures are taken of the object from slightly different positions, the object remaining fixed. In the case of the electron microscope two successive electron micrographs are taken of the same area of the specimen, the camera remaining fixed and the specimen being tilted between exposures about a line perpendicular to the axis of the electron beam. The two prints are then mounted as a stereo-pair along a base-line which, in the specimen, is at right angles to the axis of tilt.

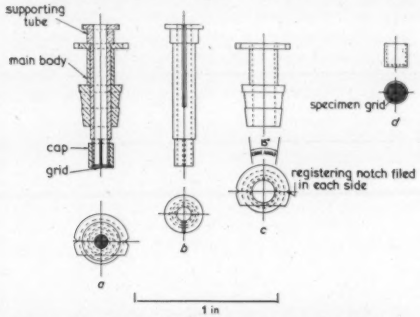

Fig. 1. Assembly and details of normal specimen holder

(*a*) Specimen holder assembly. (*b*) Details of specimen supporting tube. (*c*) Details of main body of holder. (*d*) Specimen cap.

In the most recent designs of electron microscopes[1] provision is made for introducing the required tilt by means of external controls, and the specimen is not removed from the instrument between the two exposures. No such provision is, however, made in the case of the Metropolitan-Vickers Type EM2 microscope and major alterations would have to be made to the specimen stage of this instrument before such techniques could be used. A simply constructed type of holder has, however, been designed by the author for use with the EM2, and a description of this is given in the present paper.

CONSTRUCTION OF THE STEREO-HOLDER

The normal specimen holder supplied with the EM2 is shown in Fig. 1. The main body of the holder [Fig. 1(*c*)] is of brass and has a $\frac{5}{32}$ in diameter central hole. The lower part is conical in shape and is very accurately ground so as to fit rigidly into the mechanical stage carrier plate of the microscope, whilst at the upper end it has a flange with a flat on one side and two notches at 180° for locating the specimen holder in the "stirrup" of the specimen insertion block. Into the main body fits a specimen supporting tube, which carries the cap containing the specimen grid at its lower end.

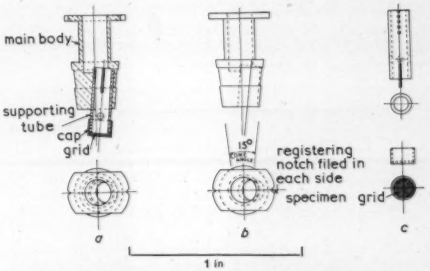

Fig. 2. Assembly and details of stereo specimen holder

(*a*) Specimen holder assembly. (*b*) Main body of holder. (*c*) Specimen supporting tube and specimen cap.

The stereo specimen holder is shown in Fig. 2. The main body [Fig. 2(*b*)] is again of brass, and has exactly the same external dimensions as the one supplied with the instrument, with the exception of the flange at the upper end which now has two flats parallel to and on opposite sides of the line joining the notches. When placed in the "stirrup," these flats are parallel to the long edge of the photographic plate. The $\frac{5}{32}$ in diameter central hole, in this case, runs from the top of the main body to a point about $\frac{1}{16}$ in below the top of the conical portion, where it joins the hole into which the new specimen supporting tube fits. This hole, which is $\frac{1}{8}$ in. in diameter, is inclined at an angle of 5° to the axis of the holder and the

* Paper read at the International Convention on Modern Microscopy held at Newcastle on 18 April, 1950, under the auspices of the Royal Microscopical Society and the Electron Microscopy Group of The Institute of Physics.

† Now at the Royal Technical College, Glasgow.

projection of its axis on to the top flange is accurately parallel to the flats.

The specimen supporting tube is of brass, and is $\frac{47}{64}$ in long, $\frac{1}{8}$ in. in diameter, with a wall thickness of $\frac{1}{64}$ in. A normal specimen cap cut down to a height of $\frac{3}{32}$ in is used with the supporting tube, which is split at both ends to make a spring fit with both the main body of the holder and the specimen cap. Three holes are bored in the tube at 120° intervals just above the level of the top of the cap. These allow a needle to be used as a tommy-bar to rotate the tube inside the main body. Fine markings on the tube and on the under surface of the conical portion of the holder enable a 180° rotation to be made accurately.

METHOD OF USING THE STEREO-HOLDER

Taking stereo-micrographs. The complete operation of taking a stereo-pair consists of four stages as follows:—(1) The first exposure is taken of the selected part of the specimen, and the magnification, exposure and focusing conditions are carefully noted. (2) The specimen holder is removed from the microscope and the tube carrying the cap and specimen grid is rotated through 180° relative to the main body of the holder. (3) The holder is rotated as a complete unit through 180° and replaced in the "stirrup" of the specimen insertion block. The specimen has now been tilted through 10° and, since a total rotation of 360° has been applied, it has the same general orientation as before. (4) The specimen is re-inserted in the microscope and a second exposure taken of the same area with the same conditions of magnification, exposure and focusing as before.

Manipulation of the holder is relatively simple and the problem of preventing accidental movement of the specimen grid whilst introducing the tilt has been overcome. The complete operation can be carried out in under ten minutes.

Relocating the specimen. The difficulty of relocating the desired area of the specimen after removing it from the microscope in order to introduce the tilt has been overcome by the use of a new type of supporting grid made by Messrs. Kodak Ltd. These new grids have a centrally marked "origin" and a rectangular "co-ordinate" system made up of bars of two different thicknesses. With the help of the intermediate image, there is no difficulty in returning rapidly to any selected square on these new grids, and, once the correct square has been found, it is an easy matter to relocate the exact area of the specimen by observing the final image at low magnification.

The portion of the grid which can be "seen" in both positions is limited to an area near the centre [see Fig. 2(*a*)]. It is, however, necessary in any case to work near the centre of the grid, in order to avoid a change in the level of the selected area of the specimen between the two exposures, which would result in a difference in the magnification.

CALIBRATION OF THE STEREO-HOLDER

Magnification. Calibration of the magnification obtainable with the stereo-holder can be carried out by standard methods.[2]

Rotation. Owing to the helical nature of the path described by an electron as it passes through the magnetic fields of the lenses of the microscope, a rotation of the image relative to the object is produced. The angle of rotation depends on the lens excitation currents and is given by the equation

$$\phi = K\int_{-\infty}^{+\infty} H_z dz$$

where K is constant for a given accelerating voltage, H_z is the axial magnetic field, and z is measured along the electron optical axis. (The rotation is clockwise if H_z acts downwards, anti-clockwise if H_z acts upwards.) In general, therefore, it will be necessary to rotate the stereo-prints back to correct for this effect before examining them in the stereo-viewer.

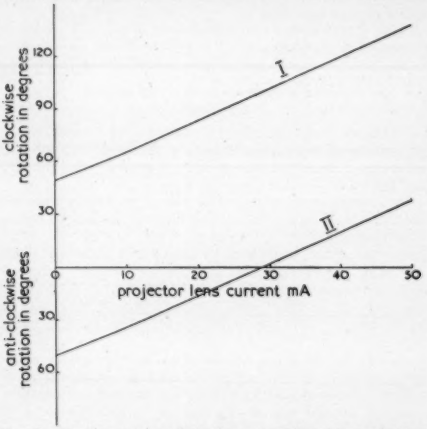

Fig. 3. Total rotation of the image produced by objective and projector lenses. (Objective lens current setting for stereo-holder—29 mA)

I. Lens currents in same sense. II. Lens currents in opposite senses.

In order to determine the amount of rotation produced by the objective and projector lenses, a platinum wire of diameter 7 μ was mounted on a specimen grid and placed in the stereo-holder in such a way that on insertion in the electron microscope, its horizontal projection was parallel to the long edge of the photographic plate. With the lens currents in the same direction, the total rotation introduced by the two lenses was measured for various projector current settings. The objective lens current was set at 29 mA (the setting required for

focusing) throughout. This was then repeated with the lens currents in opposite senses. The resulting graphs of rotation against projector current are shown in Fig. 3. From these it can be seen that there is a very convenient curfent setting for zero rotation with the two currents in opposite senses and equal to 29 mA. With these particular settings no back rotation of the prints is necessary and the long edge of the photographic plate is the common base-line.

ACKNOWLEDGMENT

The author gratefully acknowledges the assistance of Mr. J. E. Taylor, B.Sc., with the experimental work described in this paper.

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Correspondence

The detection of moisture in electrical insulation by a single test

The entry of moisture into insulation is a frequent cause of breakdown and a satisfactory test for moisture would be valuable. Such a test should be both sensitive and positive and should be easily and rapidly applied.

Two simple tests for moisture are at present in common use, namely (i) the test for d.c. conductivity, usually by means of a "megger" type of insulation tester, and (ii) the 50 c/s power-factor test, sometimes by means of a Schering bridge but more usually by means of a special form of wattmeter (see, for example, Burton, Forrest, Warren, *J. Inst. Elect. Eng.*, **89**, II, p. 288, 1942). The d.c. conductivity test is highly sensitive to moisture which is continuously distributed between the test electrodes but fails on complex insulation when one constituent resists the entry of moisture, although other constituents may allow it. The 50 c/s power-factor test will detect localized moisture but is much less sensitive than the d.c. test because the loss due to moisture must be detected in the presence of other losses.

There is thus a case for either test, but both have the serious practical difficulty that results, although readily obtainable in the field, are difficult to interpret because the tests are essentially relative. Except for gross deviation from normality, the values measured are of no individual significance, since the d.c. conductivity and the power factor of insulating structures in sound condition may vary widely. Detection of moisture must therefore depend on changes in the measured quantities. Since there are substantial variations due to ageing, temperature and surface films of moisture, changes must be relatively large to be significant, so that only relatively large quantities of moisture can be ascertained. The need to keep detailed records over long periods adds to the inconvenience of testing the large number of individual units of insulation present in modern transmission systems.

By making measurements over a suitable range of frequencies (e.g. from 0·05 c/s to 50 c/s) more information is available than from any test at a single frequency or with d.c. It is well known that tests of this kind can take a number of experimental forms which are superficially different but which yield the same fundamental information. Using sinusoidal voltages, series or shunt capacitance or conductance, power factor or loss factor may be plotted against frequency or against each other. Alternatively, square waves may be used and instantaneous voltage, current or charge may be plotted against time. There is thus a wide choice of experimental methods each of which puts all the available information in a different form. It appeared to be possible that, among the available information, there might exist a contribution which could form a direct measure of moisture content which would be relatively unaffected by other conditions. If this were so, a suitable choice of experimental method might serve to separate it from the remainder.

An examination of a new method of test (fuller details will be made available in E.R.A. reports), consisting of the measurement of shunt capacitance and shunt conductance over the frequency range 0·05–50 c/s, has revealed the possibility of developing a direct test for moisture as suggested above. It has been found, for paper insulation (which is the material of greatest practical interest), that dry specimens show no change in shunt capacitance with frequency. Moist specimens have a change of shunt capacitance with frequency which increases progressively with the moisture content. Although the shunt capacitance itself varies with temperature, the absence of change of shunt capacitance with frequency may be used as a direct criterion of dryness because it is unaffected by either temperature or surface moisture.

Tests have so far been made only on small laboratory specimens. Interest lies mainly in practical insulation such as transformer and switchgear bushings. Since it is difficult to control the moisture content of such large specimens, field-testing experience will be required to assess the value of the new method in relation to that of existing methods.

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Lissajous or Lissajou

On p. 334 of your issue for December 1950, in a review of "Text Book of Physics, Part IV, Sound" by J. Duncan and S. G. Starling, the spelling of the name *Lissajous* is queried. The name is in fact correctly rendered in the text-book, as can be verified by reference to Lissajous' original papers in *Comptes Rendus* and elsewhere.

JULES ANTOINE LISSAJOUS (1822–1880) was born in Versailles, studied at the *École Normale* and taught in Paris at the *Lycée St. Louis*.

A. J. WOODALL

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Shrivenham, Swindon, Wilts.

I am grateful to Dr. Woodall for his authoritative statement, and also to Dr. J. M. A. Lenihan of the Western Infirmary, Glasgow, who previously sent me the same information. So far the source of my misapprehension has not been traced.

E. J. IRONS

Queen Mary College,
University of London.

New Books

The Art of Scientific Investigation. By W. I. B. BEVERIDGE. (London: William Heinemann Ltd.) Pp. xii + 171. Price 10*s*. 6*d*. net.

There are two characteristics of this book which should be mentioned at the outset. Firstly, it does not provide, and does not profess to do so, a profound or systematic analysis of the methods by which scientific discoveries are made. It rather offers a survey of the conditions, both external and internal, to the worker which make for success in research. Secondly, it is mainly concerned with biological or medical research. That is not to say that the physicist will find it lacking in interest or value, but merely that he will wish that a corresponding book with a physical bias were available. The author admits his inability to arrange his matter in a logical sequence, but there is no need for him to take this too much to heart. It is not profitable to attempt to analyse too closely the mental processes of the successful researcher, but it is well worth while to observe him at work and consider to what special characteristics his success may be attributed. This is in effect what the author has done, and his conclusions are supported by a wealth of example and anecdote. Although they are drawn mainly from the records of biological research, which does differ in some fundamental respects from physical research, there is much for the physicist to learn from his brother scientist. Even if he is not in a position to put this into practice he will find Professor Beveridge a very agreeable guide and companion in the exploration of the avenues of scientific investigation.

W. E. CURTIS

A Text-Book of General Physics. By G. R. NOAKES, M.A., F.Inst.P. (London: Macmillan and Co., Ltd.) Pp. viii + 414. Price 10*s*. 6*d*. net.

This is a welcome addition to the author's textbooks for students at university scholarship level and deals with the principles of Newtonian mechanics and their application to the study of solids, liquids and gases. The emphasis is experimental rather than mathematical, for, as the author points out, it is not really the mathematics that often makes mechanics a difficult subject, but the physical concepts themselves. The author is therefore at pains to clarify the essential physics of each topic as he develops it.

Chapter 1 concerns fundamental definitions and concludes with a section on graphical representation. Most teachers know how clumsy even mathematically able pupils can be in representing data graphically to the best advantage. The summary dealing with this is admirable. The laws of motion and the principle of conservation of energy are treated in an interesting way and there is ample discussion of difficulties. There follow chapters on circular motion, pressure in fluids, stresses and strains, viscosity, surface tension, simple harmonic motion, wave motion and gravitation. The chapter on stress and strain is rounded off by a description of the actual behaviour of metal wires, and of rubber, with an up-to-date physical picture of the structural changes in these materials when stressed. The student's difficulties in comprehending the phenomena of surface tension are squarely faced and a very careful exposition of the behaviour of liquid surfaces is given. The plentiful diagrammatic illustrations in the chapter on wave motion make this chapter a particularly valuable one, which will be appreciated for its thoroughness.

The diagrams are numerous, well chosen and well executed. There is a plentiful selection of questions at the end of each chapter. The book is excellently produced and can be unreservedly recommended as a clear, thorough and interesting treatment of these properties of matter.

G. VAN PRAAGH

Bibliography of Electron Microscopy. Prepared by the Electron Microscopy Group of The Institute of Physics. Edited by V. E. COSSLETT, M.A., Ph.D., F.Inst.P. (London: Edward Arnold and Co.) Pp. 352. Price 40*s*. net.

In order to assist its members, the Electron Microscopy Group of The Institute of Physics decided to compile a centralized bibliography from the card indexes which had already been assembled in several laboratories. The resulting material, further amplified from such bibliographies as had already appeared, has been now made available to all interested by publication in book form. Its 3 000 entries are believed to provide the most complete source book yet available on the electron microscope and its application. The aim has been to include all work published up to the end of 1948, from which date the subject is covered by the quarterly bulletin of abstracts now issued by the Group in co-operation with the Royal Microscopical Society. There are full cross-references by authors' names and short abstracts are given of approximately half the entries, the remainder having the full title of the paper as well as author and journal reference. When the paper is not in English, the original title is given as well as a translation. Arrangement is alphabetically by name of the first author. No attempt has been made at subject classification owing to the wide range of subjects covered.

For the convenience of those who wish to make their own card index from this bibliography, the publishers have kindly agreed to supply copies in folded and collated sheets printed on one side of the paper only at 45*s*. net.

The Identification of Molecular Spectra. By A. G. GAYDON, D.Sc., and R. W. B. PEARSE, D.Sc., F.R.A.S. (London: Chapman and Hall, Ltd., 1950.) 2nd edition. Pp. xi + 276. Price 50*s*. net.

This book differs from all others on band spectra in that it is specially planned to enable the practical investigator to identify completely and accurately the bands in a complicated spectrum and avoid errors of assignment such as have frequently appeared in published papers. It is not possible in this brief note even to outline the procedure suggested by the authors for the best use of the tables of persistent band heads and of individual band systems which form the two main sections of the book; nor, perhaps, is this desirable nine years after the appearance of the first edition. The second edition includes material from papers up to 1947 with additional references and data inserted in the proofs as late as November 1949. New band systems or other new results

are now given for about one-third of the molecules covered by the first edition and for many other molecules. The consequent expansion of the tables and accompanying notes amounts to 55 pages, and four plates (26 well reproduced spectrograms) are added to the former eight. The new edition will, no doubt, be an essential item of the equipment of every spectroscopic laboratory as well as a valuable complement to other books on the subject. W. JEVONS

Der Ultraschall. By Dr. L. BERGMANN. (Zürich: S. Hirzel Verlag.) Pp. xi + 748.

The only edition of this well-known book available for comparison is the English edition published in 1938 and described as an unabridged translation of the book published by the VDI-Verlag in Berlin in July 1937. This old edition, a volume of 264 pages with 483 references, is completely out of date. The new volume with its 748 pages, 2 322 references and 460 illustrations, is neither more nor less than a comprehensive handbook on the subject. As such it has all the advantages and disadvantages of the typical German handbook. Although almost every piece of research has been mentioned and in many cases described, there is no attempt at a critical discussion of the various results. Indeed such a task would be almost impossible for one man. Still more so as Professor Bergmann has worked under great difficulties. Owing to the war conditions he has been able to get the title only of a number of papers and has been unable to consult the original works. There is no indication of which papers have been read in title only, but post-war papers were particularly inaccessible to him. In the preface he makes a special appeal to all workers in the subject to send him reprints so that they can be dealt with in the next edition.

The original division into five sections is preserved, but the treatment is much fuller as can be judged from the number of pages now needed: Chapter I, Generation of ultrasonics (105 pp.); Chapter II, Detection and measurement of ultrasonic waves (124 pp.); Chapter III, Determination of velocity and absorption in liquids and gases (171 pp.); Chapter IV, Determination of velocity in solids and of their elastic and photoelastic constants; and Chapter V, Further applications (191 pp.). The variety of these applications is too great to merely list. The ultrasonic stroboscope and fluorometer, television, communication, testing of materials, chemical, physical and biological applications are treated in nearly 200 pages where less than 40 pages sufficed in the first edition.

None of the arbitrarily selected topics used to test the index and references was found missing. For example, during the summer of 1950 the reviewer was able to see one of the newer applications in medicine in Austria where Dr. K. Th. Dussik is using ultrasonics for diagnosis, as distinct from their extensive and controversial therapeutic use. It is found that whilst brain tumours differ little from the brain in their absorption of X-rays, sufficient difference with ultrasonic waves occur to enable early diagnosis and treatment before serious symptoms arise. The searcher would have been put on the track of this work, representing the close collaboration of a surgeon and a physicist, by references on the last page of the book.

In a fascinating short appendix on ultrasonics in nature, the physicist finds himself reading of researches on bats, moths, grasshoppers, crickets and crabs.

The book is well produced and bound but there is an ominous omission of the price from the dust-cover. Inquiry in Zürich revealed that 50 Swiss francs or $13.00 are needed by the British buyer. Nevertheless the book is quite indispensable to any science library. W. H. GEORGE

Acoustical Designing in Architecture. By VERN O. KNUDSEN and CYRIL M. HARRIS. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd.) Pp. x + 457. Price 60*s.* net.

It is appropriate that a journal concerned with applied physics should contain a notice of this book, for it is addressed to the practitioner rather than the theoretician, who will look in vain for a derivation of Eyring's reverberation formula or an account of Bolt's work on wave acoustics. The authors strike the key-note of their composition on page 2 by stating that "Architectural acoustics is an exact science and a practical art," and their subsequent concern is to justify this assertion. This they do by considering the relevant physical and physiological (or decibel, phon and sone) aspects of the subject, sound absorbing materials and constructions, and the control of noise, before examining in detail design desiderata for various types of building. They also include an interesting discussion of open-air theatres and deal in a comparatively brief manner with sound amplification systems.

Perhaps the most striking feature of the book is the wealth of data displayed in curves, nomographs and tables. Of particular interest to designers are two figures summarizing the authors' recommendations on the desirable reverberation time *v.* frequency for rooms, and appendices containing tables of sound insulation data and details of (predominantly American) sound absorbents.

As already hinted the book is not one for physics students, who might readily become too myopic examining the details of the trees to appreciate the form and origins of the wood. However, those concerned with architectural acoustics in this country will doubtless be glad to have in such a convenient form so much information on what, from the authors' standing, may be regarded as current American practice. It suffices to add that the production is also in accordance with the best of this practice. E. J. IRONS

An Introduction to Experimental Stress Analysis. By G. H. LEE. (London: Chapman and Hall Ltd.) Pp. xiv + 319. Price 44*s.* net.

The extensive developments which have taken place in recent years in various methods of experimental stress analysis have made this an important subject of study for all engineers, and a survey of available methods and their applications has been badly needed. Professor Lee's book is designed to meet this need. Quite rightly he emphasizes the necessity for a sound knowledge of theory both in the designing of experiments and in the interpretation of experimental results, and he begins with a well presented outline of basic elastic theory. The survey of methods and instruments which follows is comprehensive, including all kinds of mechanical, optical and electrical strain gauges, photoelasticity, and brittle lacquers. There are useful chapters also on the solution of strain rosette equations, structural similitude, and the use of various analogies. The book is amply illustrated with photographs and clear diagrams, and very well printed and produced. Bibliographies are appended at the end of each chapter, but these are not always well selected. In that on elasticity, for example, Filon's fundamental work on Generalized Plane Stress was omitted, while some highly specialized and relatively unimportant papers were included. Although in scope and content the book is in many ways

admirable, there is a certain lack of realism in the accounts given of some of the methods. In practice the most important application of stress analysis is to the determination of the stresses at points of high stress concentration. It is at these points that difficulties occur with all methods. Many of the methods as described and illustrated in this book would be ineffective at such points, while others would need special techniques. In view of the amount of relatively unimportant detail given in some places, these difficult and important cases might have received some attention. One would have thought, too, that three-dimensional photoelasticity merited a much fuller and better informed account than is given in the one page the author devotes to it.

The value of the book is marred also by several quite astonishing mis-statements and errors of which two may be quoted. One is the statement that "*the distribution of stress in an elastically deformed body is independent of the elastic constants of the material.*" The second occurs in the illustrative example on cumulative errors where the total error in a continued product is obtained by adding the *actual* instead of the *percentage* errors.

H. T. JESSOP

Physical Chemistry. By G. VAN PRAAGH, Ph.D., A.Inst.P. (London: Cambridge University Press.) Pp. xiv + 295. Price 15*s*. net.

In this atomic age it is difficult for a science student in an ordinary undergraduate course of normal duration to cover adequately the extensive groundwork now considered necessary prior to specialization in physical science. Physical chemistry, for example, as part of this fundamental structure, is now receiving earlier and more specialized attention than formerly in the curriculum, often at the secondary school stage. Van Praagh's *Physical Chemistry* meets this new situation. He provides an admirable collection of topics, and, of its type, it is the most satisfactory book the reviewer has met with in his experience of such texts. Even for the biologist, or for students who do not intend to specialize, the book has much to recommend it. The material presented is sufficient to give a real understanding of this border-line subject, and to provide an introduction to its practical problems. In a space of less than 300 pages a wide field is surveyed, with a treatment based on the atomic theory and having emphasis on kinetic molecular concepts rather than on the laws of thermodynamics. To cover this curriculum it has been necessary to restrict the mathematical treatment, and some fundamental items, such as radioactivity and spectroscopy, are omitted on the presumption that they are dealt with in the physics syllabus. The material is well selected. Each topic is introduced by a few appropriate experiments, mostly for class demonstration. These are followed by a theoretical section, with a historical background. Finally, a number of practical exercises are outlined, some of which are intended to be carried out by the student himself. In all, over 150 exercises are described, and with the necessary practical details for their execution. To take one example, in the section on Electrode Potentials, there are experiments on (i) the measurement of relative electrode potentials, (ii) preparation of saturated calomel electrodes, (iii) use of the hydrogen electrode, (iv) measurement of e.m.f., (v) measurement of the pH of a solution, (vi) electrometric titrations, (vii) use of galvanic couples, etc.

The set-up, including the type, plates, diagrams, paper and binding are of the usual excellent standard of the Cambridge University Press, and the price is reasonable. The book can be highly recommended.

JOSEPH REILLY

Analytical Absorption Spectroscopy. Edited by M. G. MELLON. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall Ltd.) 1950. Pp. vii + 618. Price 72*s*. net.

In this book, which is intended for analytical chemists, nine authorities on analytical absorption spectroscopy, eight of them American and one British, have contributed theses dealing with different branches of the subject within one cover under the editorship of Dr. Mellon. As little attempt has been made to restrict the field covered by each author, theoretical foundations, such as the Laws of Absorption, are repeated several times. The reader has the impression that each author was unaware of the coverage of the other contributors.

The first chapter introduces the whole subject by discussion of general chemical techniques, including the preparation of the sample and the choice of suitable reagents. This information is amply supported by tables summarizing the methods recommended for determining various constituents. The bibliography of this chapter alone has 574 references, of which all but about 20 are American. A second introductory chapter deals with the principles of measurement, the process of absorption and the Laws of Beer and Bouguer (usually known as Lambert's Law). The treatment is brief, but other authors in recapitulating it have amplified it where necessary. The following six chapters are devoted to the principles underlying the design and use of instruments, only those originating in the United States being dealt with, except in Chapter 6. Although only Chapter 4 claims to deal with filter photometers, the filters themselves are left for Chapter 5; this involves the reader in a good deal of cross reference which might have been avoided. Chapter 6 deals with photographic methods, the instruments, sources of error and means of reducing them. The relative merits of photoelectric and photographic methods are also discussed to the ultimate advantage of the former except in certain special cases. A section of great assistance to analytical chemists is in Chapter 7, which deals very fully with applications of ultra-violet and visual spectrophotometric data. Apart from a further restatement of the symbols used, the description of specific applications in this chapter alone would make this book invaluable to an analytical chemist. Sections dealing with infra-red analysis and true colour measurement conclude the book. These sections, one feels, deal too deeply with the theory to be appropriate for a book addressed to analytical chemists. The theory as propounded requires at least an elementary knowledge of geometrical optics and electronics.

The general presentation of the book is good, and is well provided with illustrations. The bibliography is adequate as far as American publications are concerned. Two mistakes may be cited; an obvious error in Table 6.1, page 276, while the sensitivity of the barrier layer cells mentioned on page 167 refers to an obsolete type of photocell which has been superseded since 1937 by several makes giving about 4 times this sensitivity. The only other criticism is that identically the same numerical code is used for chapters, sections, tables, graphs, photographs and equations. With several such parallel systems running concurrently, the location of some of the illustrations becomes rather confusing.

R. A. C. ISBELL

A Modern Electricity. By M. NELKON, ESQ. (London: William Heinemann Ltd.) Pp. viii + 237. Price 7*s*. 6*d*. net.

All interested in the teaching of elementary physics are acquainted with Mr. Nelkon's text books. This, his latest publication, covers the electricity section of the physics for the ordinary standard of the General Certificate of Education. It is clearly written and fulfils its purpose admirably. The first thing which attracts the reader is the excellent arrangement and setting of the subject matter. This has characterized most of the author's publications and is an important point in the production of school text books. Electro-statics occupies a prominent position and the matter covered is much more than most examining boards require. Much of the content of the last two chapters on the "Principles of a.c. circuits" and "Electron devices—atomic theory," is outside the present syllabus. These chapters are good and students will read them with interest. The conception of an electric current as a flow of electrons in a conductor is used throughout. Although this is a very simple idea to teach, it seems likely that it will require modification and, already, many object to its use. Notwithstanding the clarity of the book in general, the reviewer could not understand the working of the Van de Graaff machine from the description given and the diagram is definitely bad. Each chapter is followed by a generous number of examination questions and in many instances numerical examples are worked out in the text.

W. ASHHURST

Notes and Comments

Elections to The Institute of Physics

The following elections have been made by the Board of The Institute of Physics:

Fellows: E. Jones, N. F. Roberts, E. D. Trout.

Associates: G. D. Archard, R. W. Bain, A. G. Bastin, F. Bryant, A. G. Duerdoth, O. C. Gay, S. Geddes, J. B. Hodgson, P. A. Lee, D. H. Lucas, J. M. Menzies, R. Parker, A. Pennington, M. M. Qurashi, E. A. G. Shaw, D. W. Slimming, F. Sterry, T. B. Thomas.

Twenty-five Graduates, five Subscribers and twenty-six Students were also elected.

Summer School on the theory of dielectrics. The Department of Theoretical Physics and the Department of Extra-Mural Studies of the University of Liverpool, in co-operation with The Institute of Physics and with the support of the British Electrical and Allied Industries Research Association, will hold a Summer School on the theory of dielectrics from 19–21 July. Lectures will be given by Professor H. Fröhlich, Dr. B. Szigeti and others.

The Summer School will be followed by a conference on dielectrics from 23–25 July. Papers presented will cover both experimental and theoretical aspects of the following subjects: dielectric loss; relation between dielectric properties and structure, including phase transitions; ferroelectrics; dielectric breakdown in solids and liquids; and related matters. Surveys introducing the various subjects will be given by Dr. L. Hartshorn, Professor J. G. Kirkwood, Dr. M. Magat, Professor J. C. Slater, Professor C. P. Smyth, Dr. S. Whitehead and Professor H. Fröhlich.

Further particulars can be obtained from Dr. B. Szigeti, Department of Theoretical Physics, University of Liverpool, 6 Abercromby Square, Liverpool, 7.

Journal of Applied Chemistry. We welcome the appearance of the first issue of our sister *Journal* published by the Society of Chemical Industry and wish this new/old contemporary continued success under its new title and dressed in the height of fashion. For, indeed, it is none other than the continuation of that Society's *Journal* which commenced publication as long ago as 1881. The applied chemists choose single-column printing on a page slightly bigger than ours and it is interesting to compare this with the double-column used in these pages. Should any of our readers happen to have the two Journals side by side and make the comparison, we should be glad to receive their observations in what is usually referred to in these columns as *private communication*.

Journal of the Franklin Institute—125th Anniversary. We have received a copy of the 125th anniversary issue of the Journal of the Franklin Institute, Philadelphia. This beautifully produced *Journal*—one of the oldest in the United States—contains a symposium of fifteen papers on recent and expected scientific developments written by leading authorities in several fields of science and technology. These include Dr. O. E. Buckley on communications, Dr. I. I. Rabi on nuclear physics, Dr. H. C. Urey on chemistry. Dr. F. Seitz on solid state physics, Dr. V. K. Zworykin on electronics and Dr. W. F. G. Swann on cosmic rays.

We extend cordial felicitations to the Editors of our contemporary on this occasion and wish their *Journal* continued prosperity.

Journal of Scientific Instruments

Contents of the March issue

ORIGINAL CONTRIBUTIONS

A low-power micromanipulator and microdissector. By R. Barer and H. E. Saunders-Singer.

The preparation of pinhole free silver mirrors. By E. Bovey and P. Unger.

A directional scintillation counter for clinical measurements. By E. H. Belcher and E. H. B. Evans.

An integrating flame photometer for small quantities. By J. A. Ramsey, S. W. H. W. Falloon and K. E. Machin.

A hardness tester employing micro-indentations for attachment to the Vicker's projection microscope. By S. J. Lloyd and D. J. Norris.

A new recording raingauge. By A. Hauer.

A sensitive vapour pressure recorder. By W. C. Swinbank.

A simple temperature controlled laboratory furnace. By R. S. Barnes.

New Instruments

Spectrographic exposure controls—Substage illuminator—Warm chamber—Wagner type shaker—Polishing aluminas—Automatic aperiodic balance—Tank level gauge—Singer micromanipulator.

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The electrical resistance method of measuring soil moisture

By D. Croney, B.Sc., J. D. Coleman, B.Sc., A.Inst.P., and E. W. H. Currer, Road Research Laboratory, Harmondsworth, Middlesex

[*Paper first received* 11 *December*, 1950, *and in final form* 29 *December*, 1950]

The continuous measurement of the moisture content of soil, and the suction or tension at which soil moisture is held, is of importance in both agriculture and civil engineering. In this paper an analysis is made of the electrical resistance method of measuring soil moisture. It is shown that whilst the method can be used to determine soil suction with a reasonable accuracy, its application to the measurement of moisture content is limited. A new type of concentric electrode moisture gauge of very small size and rapid response is described.

The moisture content of soil and the suction (pressure below atmospheric pressure) at which soil moisture is held, are of importance both in agriculture and in civil engineering. Research in both subjects calls for continuous methods for studying these factors, involving the minimum disturbance to the soil.

Continuous measurements of soil moisture have been made by numerous observers[1] using (*a*) field tensiometers, in which the suction of the soil water is measured directly; (*b*) electrical resistance methods, in which the resistance of the soil, or of a moisture absorbent material buried in the soil, is related to the soil moisture content; (*c*) electrical capacitance methods, in which variations of the dielectric constant of the soil are related to the moisture content; and (*d*) thermal conductivity methods, in which the relationship between the thermal conductivity of the soil and its moisture content is used.

In the study of moisture conditions in soils under structures, the point at which measurements are required will not normally be accessible. This accentuates the need for a reliable instrument which will function without attention over a period of several years. Further, in the case of a road or airfield, the nearest place at which the measuring apparatus can be set up may be a considerable distance from the point at which measurements are required. These considerations will in general preclude the use of tensiometers, which require frequent attention during service, and electrical capacitance methods in which the use of short leads is very desirable. Of the two remaining alternatives, the resistance method has much to recommend it from the point of view of simplicity. For this reason it was decided, when work on moisture meters was started at the Road Research Laboratory several years ago,[2] to concentrate on the resistance method.

PRINCIPLE OF THE RESISTANCE METHOD

In the most elementary form of resistance type moisture meter, the resistance between two electrodes buried in the soil is measured under different soil moisture conditions, and the calibration obtained is used to interpret subsequent changes which occur in the moisture content of the soil. This has the advantage of being a direct method in the sense that any change of soil moisture content is the direct cause of the measured change in resistance. In practice, however, the contact resistance between the electrodes and the soil, in addition to being a very variable factor, may considerably exceed the actual resistance of the moist soil. It was largely to overcome this difficulty that the absorbent type gauge was introduced.[3] The electrodes, instead of being placed directly in the soil, are enclosed in an absorbent material which is itself buried in the soil. When an absorbent moisture gauge is used, the relationship between gauge resistance and soil moisture content is no longer a direct one. When moisture transfer between the gauge and soil has ceased, both are in a state of suction equilibrium, but they will not normally have the same moisture content. Any change in the moisture content of the soil is in general accompanied by a change in moisture suction. A suction gradient between the gauge and the soil is thereby created, which in turn causes a change of moisture content within the absorbent and a corresponding change in electrical resistance. Unless the electric field associated with the electrode system of an absorbent gauge is contained wholly within the absorbent, the contact resistance between the absorbent and the soil enters into the gauge resistance as a secondary factor. To obviate this, Slater[4] first proposed the use of a concentric electrode system.

DESCRIPTION OF THE ABSORBENT METER

The apparatus used in this investigation comprised (*a*) absorbent gauges of various types, and (*b*) the bridge and associated equipment used for measuring the resistance of the gauges.

Gauges.—The first gauge used consisted of a gauze cylinder closed at one end, provided with a central straight wire electrode. Several absorbent materials in powder form such as sand and silt were packed between the electrodes. In theory this gave a gauge the characteristics of which could be varied to suit a wide range of soil types. The arrangement proved unsatisfactory, however, due to variations in the contact resistance between the electrodes and the absorbent, and it was decided to modify the electrode design and to use

absorbent materials with a rigid structure. Plaster of paris (free from setting and retarding agents) and high alumina cement (Ciment Fondu) have been used as absorbent materials in the work described in this note. Both plaster of paris and gypsum have been used in previous investigations principally by American workers.[3, 5]

The electrode system, and the method used for casting the plaster and cement gauges are shown in detail in Fig. 1. Both electrodes are of tinned copper wire, the straight wire being No. 18 gauge and the spiral electrode No. 22 gauge. The spiral is machine wound and is accurately gauged for length and concentricity prior to

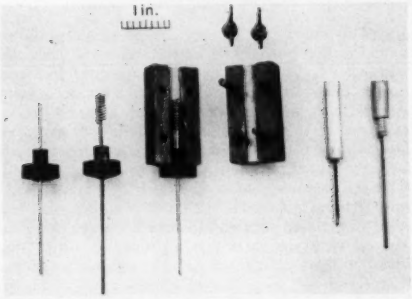

Fig. 1. Design and construction of plaster and alumina cement moisture gauges. (In the two gauges shown on the left the connexion to the spiral electrode is behind the central rod)

casting. A wet mix is essential to prevent air being entrapped during casting. A gauge of such small dimensions ($\frac{5}{16}$ in diameter by $1\frac{1}{2}$ in long) has two advantages in that the time required to reach equilibrium with soil is less than 24 h (an important consideration during the calibration) and that the gauge can be inserted with comparatively little disturbance to the soil. The use of the spiral electrode in preference to a gauze screen also simplifies the construction of the gauges. (The resistance of a saturated gauge was found to change by less than 0·5% on immersion in water. It appears, therefore, that with this electrode system only a negligible part of the current circulates outside the gauge.) In some additional experiments, nylon fabric was used for the absorbent. Several layers of the fabric were wound round the central electrode which consisted of a sheet of stainless steel gauze about 1 in square. This assembly was tightly enclosed in a flat container of the same gauze which formed the outer electrode.

Resistance measuring equipment.—To prevent electrolysis within the absorbent during the measurement of the resistance of the gauge it is essential to use an alternating current method. Even so, the transient evolution of gas at the electrodes causes the gauge to feed back an alternating current into the bridge circuit, the effect being that the gauge behaves as a resistance shunted by a frequency-dependent capacity. This apparent capacity decreases as the gauge becomes drier. The bridge used at the Laboratory is of the parallel resistance type,[6] working at a fixed frequency of 1 000 c/s. To balance the capacity component of the gauge impedance, a variable capacity 0–1 μF is used across the arm opposite to that in which the moisture gauge is included. Three ratio arms (1 : 1, 1 : 10, and 1 : 100) enable resistances up to 10 MΩ to be measured. Headphones are normally used to balance the bridge, but a null-indicator of a type recently described[7] is used for measurements in the field which may have to be made under noisy conditions. The oscillator is transformer coupled to the bridge to prevent the passage of any direct current through the moisture gauge.

THEORY OF THE ABSORBENT RESISTANCE GAUGE

When an absorbent gauge is in moisture equilibrium with soil, the moisture suctions in the gauge and the soil have the same value, although the moisture contents of the materials will in general be different. The variation of gauge resistance with soil moisture content involves three relationships which can be determined experimentally, viz. those between (1) the suction of the water in the absorbent and the moisture content of the absorbent; (2) the moisture content of the absorbent and the resistance of the gauge; (3) the suction of the water in the soil and the moisture content of the soil. Combination of (1) and (2) gives the variation of gauge resistance with the suction in the gauge and the soil, which together with (3) gives the relationship between gauge resistance and soil moisture content. Both these further relationships can be determined experimentally. Since, however, the relationships (1), (2) and (3) will not be in general unique, any analytical approach must be based on a separate consideration of all three relationships. These relationships and the methods by which they are determined are discussed in detail below.

RELATIONSHIP BETWEEN SUCTION AND MOISTURE CONTENT FOR POROUS MATERIALS

The wide range of suctions encountered in porous materials calls for the use of a logarithmic scale. On the pF scale introduced by Schofield,[8] the common logarithm of the suction expressed in centimetres of water is equivalent to the pF value of the suction. Thus, a suction of 1 atm corresponds to pF 3·014 (1 032 cm of water).

Relationships between suction and moisture content which have been determined for samples of plaster of paris and alumina cement are shown in Fig. 2. The mix proportions used and the method of preparation were similar to those employed in the manufacture of the gauges made from these materials. The moisture content is expressed as a percentage of the dry weight. There

is an appreciable hysteresis between the curves representing the wetting and drying conditions for both materials. The drying curve was obtained by increasing the suction applied to an initially saturated sample and the wetting curve by decreasing the suction applied to an initially dry sample. (With materials such as these, which may lose moisture by dehydration at temperatures in the region of 100° C, oven-drying is not possible. It was known that the plaster of paris retained little hygroscopic moisture at humidities less than 50%; this material was therefore air-dried, and the moisture contents are referred to the air-dry weight. For the alumina cement, drying to a lower humidity was essential, and the samples used were dried over concentrated sulphuric acid in vacuum.) The wetting and drying curves shown for each material represent the maximum hysteresis; the loop would in both cases be smaller if a limited suction range had been considered.

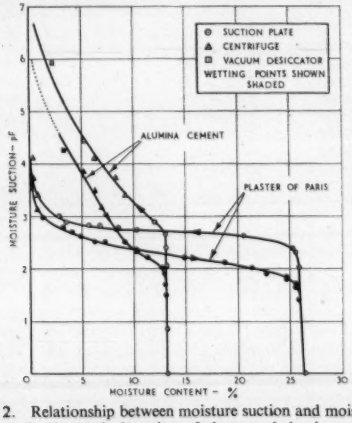

Fig. 2. Relationship between moisture suction and moisture content for the particular mixes of plaster and alumina cement used for experimental moisture gauges

The curves for the nylon fabric used in the construction of the nylon gauge are not shown. On drying, this material was found to lose the greater part of its moisture at suctions below pF 1 and the remainder above pF 4. There was in addition a large hysteresis between the wetting and drying conditions below pF 1. It was clear that the shape of the curves was a function of the particular weave used.

The relationship between suction and moisture content can be explored over the entire moisture range using the suction plate, the centrifuge and vacuum desiccator methods. The first of these is applicable to the range pF 0 to pF 3. A thin flat porous plate of maximum pore size about 2 μ is arranged to seal a water-filled reservoir the pressure within which can be reduced from atmospheric pressure to a value close to zero. Since the pores of the plate will remain full at all suctions within this range, continuous moisture films are established when a moist porous material is held in close contact with the plate. Moisture transfer takes place across these films until a condition of suction equilibrium between plate and sample is reached. If the moisture content of the sample is then determined, one point on the relationship is obtained. If the dry weight is known, the suction can be changed by increments and the corresponding moisture contents calculated from the equilibrium wet weights. At the Road Research Laboratory sintered glass plates (No. 5 porosity) have been found very satisfactory for use in this apparatus.

The centrifuge method can be used over the range pF 3 to pF 4·5. A cylinder of porous material with a flat upper surface and its lower end in contact with free water is fitted in the centrifuge cup. With this arrangement the suction exerted on the sample under test, which is placed on the top face of the cylinder, is equal to that exerted by a column of water equal in height to the distance of the sample above the water table, when the cylinder is in a field of 1 g. The suction is increased to n times this value in a mean field of n g.

The vacuum desiccator method covers the range pF 4·5 to pF 7, and makes use of the relationship between the suction of the moisture in a porous material and the humidity of the enclosure with which the material is in equilibrium.[9] Samples of the material under test are enclosed in a filter flask over sulphuric acid, the concentration of which is adjusted to give the humidity corresponding to the required suction. The flask is evacuated to accelerate moisture transfer and to reduce the effect of temperature fluctuations.

THE RELATIONSHIP BETWEEN THE MOISTURE CONTENT OF THE ABSORBENT AND THE GAUGE RESISTANCE

The relationship between moisture content and resistance for the plaster and alumina cement gauges is shown in Fig. 3. In each case the relationship was studied over the same range of moisture suctions as that used in the investigation of the moisture suction/moisture content characteristics of the materials. Again there is a considerable hysteresis between the wetting and drying conditions for both gauges. This hysteresis is smaller if a restricted moisture range is used. Broadly similar curves were obtained for the nylon gauge, the hysteresis, however, being greater than for the other absorbents. To obtain the curves shown in Fig. 3, the dry weight of the gauge (the "dry" condition of the absorbent being defined as in the suction/moisture content investigation) and the weight when saturated under vacuum were both determined. Since the saturation moisture contents of the materials were already known, it was possible from these weights to deduce the moisture content of the absorbent corresponding to any weight of gauge between saturation and dryness. The resistance in the saturated

condition was measured, after which each gauge was allowed to dry in air for a short period. It was then sealed in a small air-tight container until an equilibrium value of resistance had been reached. The gauge was then weighed and its moisture content deduced. This process was repeated until the dry condition was reached. In the cement gauge this entailed final measurements over sulphuric acid solutions. The wetting curves were obtained by a similar procedure in which the gauges were wetted by increments.

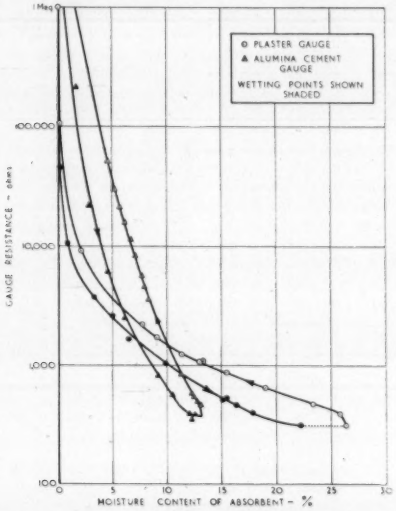

Fig. 3. Relationship between resistance of gauge and moisture content of absorbent for plaster and alumina cement gauges at 20° C

RELATIONSHIP BETWEEN MOISTURE SUCTION AND GAUGE RESISTANCE

The variation of the resistance of the gauge with the suction of the moisture in the absorbent (i.e. with the suction of the soil moisture when the gauge is in equilibrium with soil) is obtained by combining the two relationships previously discussed. The curves shown in Fig. 4 are obtained in this manner from Figs. 2 and 3. Considering first the curves for the plaster of paris gauge, the loop between the wetting and drying conditions in the range of moisture suction pF 0 to pF 2·2 has no practical significance. It arises from the small reduction of moisture content which was detected when the suction applied to the saturated material was increased over this range. This loss of moisture (which was not found in the cement gauge) was probably due to surface drying rather than the entry of air into the gauge structure. The gauge operates between pF 2 and pF 4. It is between these suctions that the greater part of the moisture required to saturate the absorbent enters or leaves the gauge. At suctions above pF 4 the plaster is dry and the resistance rises to a value too great to measure. Between suctions of pF 2 and pF 3 there is an appreciable hysteresis between the wetting and drying conditions with the result that there is a range of possible suctions corresponding to any value of resistance. The true suction, which must lie within this range, will depend on the previous moisture history of the gauge. The magnitude of the suction range corresponding to different values of resistance is shown in Table 1.

Table 1. *Accuracy of plaster of paris gauge when used as a suction meter.* (*Calibration over the range pF 0 to pF 4*)

Resistance of gauge (ohms)	*Possible range of suction (pF scale)*	*Mean suction (cm of water)*	*Possible error in suction*
500	2·23–2·55	265	± 35%
1 000	2·36–2·72	375	± 39%
1 500	2·44–2·74	410	± 33%
2 000	2·55–2·78	480	± 26%
4 000	2·80–2·90	710	± 11%
6 000	2·94–2·97	900	± 3%
10 000	3·15–3·18	1 450	± 3%
50 000	3·76–3·81	6 100	± 6%
100 000	3·96–4·02	9 800	± 7%

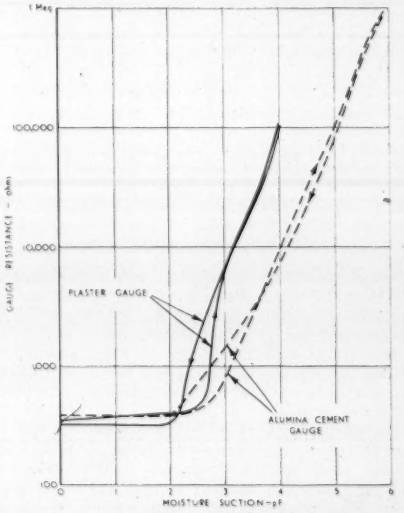

Fig. 4. Relationship between soil moisture suction and gauge resistance for plaster and alumina cement gauges at 20° C

At suctions below pF 2·8 the possible error is of the order of ± 35%, but above pF 2·8 the error is considerably smaller. If the gauge were required for use only over a limited range of suctions, it would be possible, by carrying out suction/moisture content and gauge resistance/moisture content determinations in the wetting and drying conditions within this limited range, to deduce the correspondingly smaller hysteresis in the suction/gauge resistance relationship. Resistance measurements at low suctions could in this way be interpreted in terms of suction with considerably greater accuracy. To investigate this, the hysteresis in both the suction/moisture content and the gauge resistance/moisture content relationships for the plaster gauge were studied experimentally over the range pF 2·1 to pF 2·8, but to avoid confusion the curves have not been included on Figs. 2 and 3. On the basis of the deduced hysteresis in the suction/gauge resistance relationship, the figures shown in Table 2 have been obtained.

Table 2. *Accuracy of plaster of paris gauge when used as a suction meter.* (*Calibration over the range pF 2·1 to pF 2·8*)

Resistance of gauge (ohms)	*Possible range of suction (pF scale)*	*Mean suction (cm of water)*	*Possible error in suction*
500	2·23–2·24	172	± 1%
1 000	2·49–2·60	355	± 13%
1 500	2·59–2·75	475	± 18%
2 000	2·77–2·78	600	± 1%

The possible error over this limited range is thus reduced to about half the error obtained when the larger range of suction is considered. An error of 15%, though large, might be acceptable for some purposes. The same general features are apparent in the suction/gauge resistance relationship for the cement gauge. The intersection of the wetting and drying curves at pF 3·65 may be due to small experimental errors in the determination of the suction/moisture content relationship for the cement. Table 3 gives the possible error in suction, based on the main hysteresis loop. Below suctions of pF 3·3 the errors are of the order of 40%. Between pF 3·3 and pF 5 the mean error falls to about 15%.

Table 3. *Accuracy of alumina cement gauge when used as a suction meter.* (*Calibration over the range pF 0 to pF 7*)

Resistance of gauge (ohms)	*Possible range of suction (pF scale)*	*Mean suction (cm of water)*	*Possible error in suction*
500	2·24–2·69	330	± 48%
1 000	2·73–3·11	910	± 41%
1 500	3·04–3·28	1 500	± 27%
2 000	3·24–3·39	2 100	± 17%
4 000	3·65	4 800	0
6 000	3·80–3·89	7 000	± 10%
10 000	4·00–4·15	12 100	± 17%
50 000	4·70–4·84	60 000	± 16%
100 000	4·98–5·07	107 000	± 11%

No tests over a limited range of suction were made on this gauge.

RELATIONSHIP BETWEEN SUCTION AND MOISTURE CONTENT FOR SOILS

The relationship between suction and moisture content for soils is more complicated than that for porous materials with a rigid structure, owing to the effect which changes in dry density (weight of dry material in unit volume) have on the relationship. In a granular soil which does not swell or shrink with changing moisture content, the relationship at any given dry density resembles that for a rigid structure, showing a similar hysteresis effect. There is, however, a family of such relationships each corresponding to a different density condition of the soil. It follows that any disturbance to such a soil will affect the suction/moisture content relationship, particularly if the density suffers a material alteration. In clay soils the problem is further complicated by the effect which moisture content has on the dry density of the soil. Undisturbed heavy clay soils are normally saturated under field conditions, in temperate and wet climates, so that any change in moisture content is accompanied by swelling or shrinkage. If such a soil is slurried to break up the natural structure, and the change of moisture content is studied as increasing suction is applied to the soil, what may be regarded as one limiting suction/moisture content curve for the soil is obtained. If the drying process is continued to oven-dryness, the equivalent of a very large consolidation pressure is applied to the soil, with the result that the particle arrangement is modified and a new structure created. If the suction/moisture content relationship is then studied as the soil is again wetted, what may be regarded as a second limiting curve is obtained. (Experimental evidence suggests that more than one wetting and drying cycle may be required to give a true limiting curve.) If the initial drying of the slurried soil is discontinued before the oven-dry condition is reached, e.g. at pF 5, and the soil is wetted again from this suction, the wetting curve will fall between the two limiting curves already discussed. Further wetting and drying over the range of suctions pF 0–pF 5 would have resulted in no further modification to the soil structure, although there would have been a small hysteresis between the wetting and drying curves, probably associated with the resistance to deformation offered by the shear strength of the soil. In practice the suction/moisture content relationship for an undisturbed clay would lie between the limiting curves, and the relationship would follow a closed hysteresis loop, provided the corresponding range of suctions did not result in any modification to the structure of the soil as determined by the pre-consolidation pressure to which the soil had been subjected during its geological history. It follows, as in the granular soil, that any disturbance to a clay will result in a modification of the suction/moisture content relationship. To illustrate the magnitude of the effect which disturbance

can have, the two limiting curves for a heavy clay soil may give moisture contents as different as 60% and 30% at pF 2.

Fig. 5 shows the relationship between moisture suction and moisture content for a heavy clay soil in the undisturbed state. At the commencement of the test, the moisture content was approximately 24% and the relationship was studied both in the wetting and drying conditions from this moisture content. Some points on the subsequent wetting cycle from pF 0·85 are also shown. From these experimental data it is possible to sketch the remaining portions of the wetting and drying curves covering the approximate suction range pF 0 to pF 4·6. These portions are shown dotted on the figure. The point of maximum curvature at about pF 4·2 is associated with the preconsolidation pressure already referred to. Any attempt to dry this soil to a moisture content below about 17% would result in a more marked hysteresis on subsequent wetting. This is apparently due to a reorientation of the clay particles when the preconsolidation pressure is exceeded.

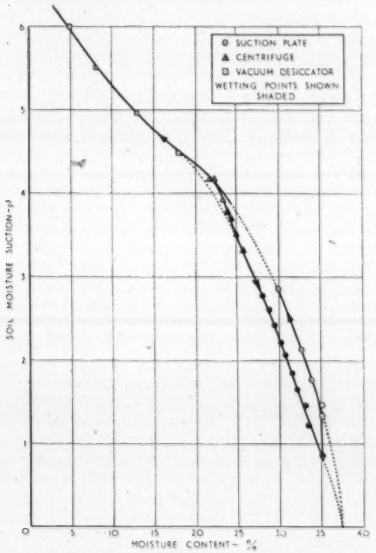

Fig. 5. Relationship between soil moisture suction and moisture content for an undisturbed sample of heavy clay

THE RELATIONSHIP BETWEEN SOIL MOISTURE CONTENT AND GAUGE RESISTANCE

The variation of gauge resistance with soil moisture content is obtained by combining the relationships between soil moisture content and suction and between suction and gauge resistance. For the clay referred to in Fig. 5, the relationship between soil moisture content and gauge resistance over the suction range pF 0 to pF 4·6 is shown in Fig. 6. In the alumina cement gauge

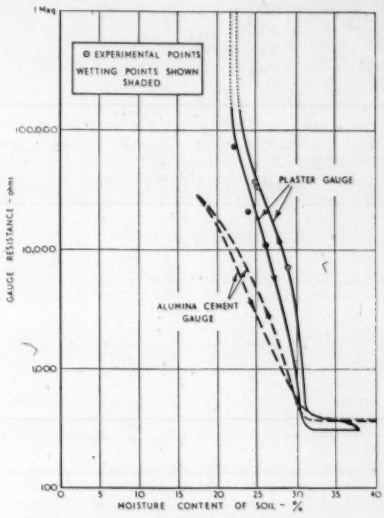

Fig. 6. Relationship between resistance of gauge and moisture content of soil for plaster and alumina cement gauges at 20° C

the relationship above pF 3·65 (resistance 3 900 Ω) is based on a mean curve between the wetting and drying curves relating gauge resistance and suction shown in Fig. 4. (It has already been pointed out that the hysteresis loop in this relationship above pF 3·65 may be due to small experimental inaccuracies in the determination of the suction/moisture content relationship for the cement.)

For moisture contents of the clay in excess of about 32%, both gauges are relatively insensitive. As the soil is dried below this value, the resistance in each case rises steadily. At a moisture content of 22% the resistance of the plaster gauge becomes too large to measure. Thus, 22 to 32% is the useful moisture range of this gauge for the particular clay and the suction range under consideration. The cement gauge, however, continues to respond satisfactorily down to a moisture content of 17%, corresponding to pF 4·65, the upper limit of the suction range considered. There is a hysteresis between the wetting and drying conditions of the soil for both gauges. As a result, a given value of resistance can only

be interpreted in terms of percentage moisture content with an accuracy of about $\pm$ 1 on a dry weight basis.

For the plaster gauge several points on the relationship between gauge resistance and soil moisture content were obtained by direct calibration of the gauge in the soil. These points are shown on Fig. 6. Both gauge and soil were wetted and dried together and care was taken to cause a minimum of disturbance to the soil on insertion of the gauge.

EFFECT OF TEMPERATURE ON THE CALIBRATION OF RESISTANCE GAUGES

It has been shown[9] that the suction/moisture content relationship for soils and other porous materials is not materially affected by changes of temperature. Temperature does, however, affect the resistance of an absorbent gauge at constant moisture content due to the influence which it has on the mobility of the ions in the gauge. It might, therefore, be expected that the gauge resistance would vary with temperature in the same manner as the magnitude of the viscosity of water. Tests carried out in the laboratory in the temperature range 0–30° C have shown that this is so to a very close degree of accuracy. Gauge resistances measured at any atmospheric temperature can therefore be readily converted, using viscosity data, to equivalent readings at 20° C, on which the calibration curves can be conveniently based.

DISCUSSION

It has been shown in this note that electrical resistance gauges provide a method of determining the suction of soil. Their reliability as soil moisture content meters is doubtful owing to the effect which disturbance has on the suction/moisture content relationship for most soils. With the shallow water-table conditions normally prevailing in this country the civil engineer will usually be concerned with suctions (negative pore water pressures) within the range pF 0–pF 3, although in the study of the influence which drought or fast-growing vegetation has on the stability of structures, suctions outside this range may be encountered. Neither the plaster nor the cement gauges described in this note are sensitive in the range pF 0 to pF 2·5. There is therefore a need to develop gauges made from absorbent materials which will give up moisture continuously from a suction less than pF 2. The agriculturalist is particularly interested in suctions in the neighbourhood of the wilting coefficient pF 4·2–4·3. Both the plaster and alumina cement gauges are suitable for measuring tensions of this order. Civil engineering measurements are largely confined to undisturbed soils or soils which have been compacted to known density conditions. In such soils electrical gauges can be used to determine moisture contents with a greater degree of accuracy than is possible in agricultural soils which are constantly being disturbed by cultivation. The difficulties involved in the calibration of electrical moisture gauges emphasize the importance of producing gauges of standard characteristics. Work at the Laboratory has shown that small differences in the geometry of the electrode system are of secondary importance in the suction/resistance relationship of an absorbent gauge. The factor of principal importance is the suction/moisture content relationship for the absorbent used. Very small differences in mixing and curing have an important influence on this relationship.

ACKNOWLEDGMENT

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Gloss measurement of papers—the effect of luminance factor

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For a range of papers of luminance factor varying from 20% upwards and of gloss varying from almost complete matt to gloss high enough to give fair mirror images the equation $G = S(a + 10/L)$ is proposed, where G is a measure of visual gloss, S is the relative intensity of reflected light under 45° — 45° conditions of illuminating and viewing, L is the luminance factor expressed as a percentage and a is a constant which, under the conditions used by the authors, is approximately 0·20. The visual gradings of eight papers out of ten are not markedly affected by the conditions of observation, but for the remaining two there is a highly significant shift in grading according to whether these conditions permit formation of mirror images or not. Gloss is considered to be a *gestalt*, corresponding to no single physical property of a surface, being judged by appraisal of a physical situation taken as a whole. The validity of the method used in establishing the psychological gloss scale is checked by experiments in weight grading.

A previous investigation* has shown that instrumental gloss grading of a certain range of papers is best made by measurement of the intensity of reflected light when the angles of incidence and viewing are 45°. The papers then examined, however, did not vary substantially in colour; and when, later, other papers of widely different colours were tested, it was found that this conclusion was no longer valid. In consequence, the present paper records an attempt to extend the earlier work to cover papers varying in colour from near-white to dark brown. The papers used in the work to be described were similar to those commonly used in the manufacture of paper bags, namely, rough on one side and glazed on the other. Such papers are known in the trade as *machine glazed* (M/G) and are prepared by allowing the damp paper to dry in contact with a heated polished steel roller, in much the same way as a photographer glazes his prints. The visual gradings of the selected papers were established by a kind of postal ballot, similar to those reported previously, and the information here recorded is based on the 164 valid sets of replies received. The observers were required to rank the papers by eye in order of gloss (glossy sides only). Conditions of viewing were not requested, since experience has shown that the description of experimental conditions given by observers in a ballot of this nature is rarely sufficiently precise to be of value and only an average grading can be expected.

Having ranked the papers, the observers were asked to indicate by means of a code whether the gloss differences between the consecutive papers of the series were large, moderate, small or negligible. This information was analysed but not used; for it was found to add little to that which could be obtained from consideration of the ranking data alone and its inclusion merely complicated the treatment.

THE VOTING DISTRIBUTIONS

The actual voting at the close of ballot is reproduced in Table 1. From these figures histograms may be drawn having number of votes as ordinates and ranking as abscissae. These are only pseudo-frequency-diagrams, for the rank numbers are not measured quantities but mere ordinals, and there is no guarantee that the gap between, say, 2 and 3 is equal to the gap between 3 and 4. Nevertheless, the histograms do in fact approximate closely to one or other of the standard Pearson types, and in most cases do not differ significantly from a Gaussian distribution; they may therefore be treated as such. The main reason for believing that this procedure is valid is an empirical one which is discussed in the Appendix. Table 2 shows some of the important constants for the data in Table 1 treated as ordinary frequency distributions. The figures after ± are not probable errors, but standard deviations or standard errors, either calculated directly or estimated from Pearson's *Tables for Statisticians and Biometricians*. The first two columns give the respective means and standard deviations for the distributions. β_1 and β_2 are calculated from the second, third and fourth moments of the distributions about the mean. For a Gaussian distribution $\beta_1 = 0{\cdot}0$ and $\beta_2 = 3{\cdot}0$. For a total of 164 votes only, the standard errors of β_1 and β_2 are large and it cannot be said that the distributions (with the exception of those for *I* and *C*) differ significantly from the normal; although for the most part they appear to be of Pearson's Type IV, that is, leptokurtic and slightly skew. The Pearson Skewness Coefficient is shown in the fifth column. The distributions for papers *I* and *C* are J-shaped, as would be expected, and the skewness tends to rise as the mean of the distribution approaches the ends of the scale, clearly on account of the curtailment of the scale at the

Table 1. *Voting distribution* (164 *observers*)

	Least glossy			*Ranking*					*Most glossy*	
	1	2	3	4	5	6	7	8	9	10
I	97	35	25	7						
D	21	59	57	16	8	3				
H	6	32	73	33	14	5		1		
E	1	5	28	70	39	18	1	2		
G		3		5	40	60	32	22	2	
L		1	2	5	24	46	33	45	5	3
F		1		5	29	35	48	42	3	1
K			1	1	6	21	39	80	11	5
B							2	6	107	49
C									11	153

* HARRISON, V. G. W. *J. Sci. Instrum.*, **26**, p. 84 (1949).

Table 2. *Means, modes and frequency constants of voting distributions in Table* 1

	Mean	σ	β_1	β_2	*Sk*	*Mode*
I	1·65 ± 0·069	0·89 ± 0·071	1·22 ± 0·34	3·08 ± 0·44	—	1·00
D	2·63 ± 0·085	1·09 ± 0·063	0·55 ± 0·40	3·85 ± 1·28	+ 0·37 ± 0·13	2·23 ± 0·15
H	3·23 ± 0·098	1·26 ± 0·090	0·62 ± 0·66	4·55 ± 3·20	+ 0·30 ± 0·17	2·85 ± 0·23
E	4·27 ± 0·086	1·10 ± 0·074	0·14 ± 0·27	3·97 ± 2·15	+ 0·13 ± 0·10	4·13 ± 0·12
G	6·12 ± 0·095	1·22 ± 0·083	0·11 ± 0·26	4·04 ± 2·42	+ 0·11 ± 0·11	5·98 ± 0·14
L	6·65 ± 0·112	1·39 ± 0·080	0·071 ± 0·094	3·17 ± 0·57	− 0·13 ± 0·10	(6·47 ± 0·10)*
F	6·62 ± 0·098	1·26 ± 0·071	0·15 ± 0·14	3·08 ± 0·52	− 0·22 ± 0·10	6·90 ± 0·14
K	7·47 ± 0·087	1·11 ± 0·082	0·50 ± 0·64	4·54 ± 3·68	− 0·25 ± 0·17	7·75 ± 0·19
B	9·24 ± 0·045	0·57 ± 0·042	0·19 ± 0·50	4·63 ± 11·0	− 0·13 ± 0·21	9·31 ± 0·12
C	9·93 ± 0·020	0·25 ± 0·034	*c.* 12	*c.* 13	—	10·00

* Distribution probably bimodal.

Table 3. *Approximate analysis of paper L histogram*

Position	*Recorded votes*	L_1	L_2	$L_1 + L_2$
1	0	0	0	0
2	1	1	0	1
3	2	2	0	2
4	5	5	1	6
5	24	22	2	24
6	46	37	8	45
7	33	20	14	34
8	45	14	30	44
9	5	1	5	6
10	3	0	2	2

Constant	L_1	L_2
n	102	62
Mean	6·09	7·50
σ	1·26	1·25
β_1	0·10	0·35
β_2	3·41	3·87
Sk	−0·14	−0·25
Mode	6·26	7·81

Table 4. *Approximate analysis of paper F histogram*

Position	*Recorded votes*	F_1	F_2	$F_1 + F_2$
1	0	0	0	0
2	1	0	0	0
3	0	1	1	2
4	5	2	3	5
5	29	7	22	29
6	35	20	16	36
7	48	39	9	48
8	42	37	3	40
9	3	2	1	3
10	1	1	0	1

Constant	F_1	F_2
n	109	55
Mean	7·00	5·76
σ	1·12	1·13
β_1	0·56	0·22
β_2	4·13	3·38
Sk	−0·32	+0·24
Mode	7·36	5·50

two ends. From the skewness coefficient the position of the mode can be estimated, and the results of this calculation are displayed in column six. The location of each distribution can be expressed in terms of either the mean or the mode, and in the present investigation it makes little difference which is used. A fair case can be made for the use of the mode, which here corresponds to a simple majority vote; but in this paper the mean has been preferred, since it is calculated directly from the observations instead of indirectly, and it is also more stable with regard to sampling variations—that is, its standard error is smaller. The means shown in column one of Table 2 have been adopted as an index of the relative gloss of the papers as judged by a panel of 164 observers and expressed on an arbitrary psychological scale ranging from 1 to 10. These means are not strictly a measure of the gloss sensation; taken together with the standard deviations they express rather the probabilities that two papers will be adjudged the same or different in respect of gloss by a group of observers. From this point of view they are equivalent to a measure. In any case, there can be no reasonable doubt about the *ranking* of the papers in the series. Some further observations on this subject will be made in the Appendix.

ABNORMAL DISTRIBUTIONS

A study of the figures in Table 1 shows that the distribution for paper *L* is abnormal, being apparently bimodal. It is true that occasional samples from a normal universe would give an apparently bimodal distribution of this type; but sufficient experience has been gained in this work to show that this is very unusual and that the abnormality deserves further investigation. Such abnormality could arise from sampling variations; but precautions were taken when the papers were selected to see that sampling variations were as small as possible. A more likely reason is that the observers did not all use the same conditions of observation. Table 3 shows the result of an attempt to analyse the distribution for paper *L* into two simpler distributions L_1 and L_2. In this analysis it is stipulated that: (1) the sum of L_1 and L_2 should approximate closely to the observed bimodal distribution; (2) the values of β_1 and β_2 for the two sub-distributions L_1 and L_2 should be in reasonable agreement with those shown in Table 2. In fact, the values of β_1 and β_2 are extremely sensitive to the transfer of a vote from one position to the next, and there are fewer solutions to this problem than at first sight might be thought. Table 3 shows the best solution obtained after a lengthy process of trial-and-error. From this analysis it appears that the observed voting distribution for paper *L* can be explained on the assumption that the observers can be divided roughly into two groups, of 102 and 62 members respectively, using different conditions of observation and grading paper *L* at 6·09 and 7·50 respectively.

A further examination of the data in Table 1 suggested that the distribution for paper *F* was also slightly

abnormal, though less so than that for paper *L*. A similar analysis was therefore attempted, with the results shown in Table 4. Here there are two groups of 109 and 55 members, grading paper *F* at 7·00 and 5·76 respectively. This grouping of approximately 2 : 1 is in good agreement with that shown in Table 3, if the rough nature of the analysis is taken into consideration. Examination of paper *F* showed that it was capable of forming fairly good mirror images, whereas paper *L* was not. If, therefore, some of the observers examined the papers under conditions allowing mirror image formation, they could be expected to grade paper *F* higher in the scale than those examining the papers under conditions not allowing mirror image formation. The forms of the histograms for papers *L* and *F* could thus be explained. Further information on the effect of viewing conditions on grading was therefore sought.

GRADING UNDER CONTROLLED CONDITIONS OF OBSERVATION

Opaque strips of black paper 1 cm wide were stuck on to a circular piece of glass 12 cm in diameter, leaving 1 cm of clear glass between strips. A second series of strips was mounted at right angles to the first so that the unmasked part of the glass formed a set of squares of 1 cm side. A disk of thin tissue paper was then attached to the masked glass on the side intended to face the source of light, and the assembly was fixed with wax to the open end of a metal reading lamp containing a 60 W electric light bulb. The reading lamp was arranged so that light passing along the axis of the masked disk of glass met the sample of paper to be examined at an angle of 45° at a distance of 17 cm from the disk and the observer was seated so that his eyes were about

Table 5. *Voting distribution* (42 *observers*)

Mirror image conditions of observation

	Least glossy			*Ranking*					*Most glossy*	
	1	2	3	4	5	6	7	8	9	10
I	28	12	2							
D	8	17	15	2						
H	3	12	13	9	4	1				
L	1	1	6	10	12	7	4	1		
E			6	13	15	5	3			
G	1			3	5	19	11	2	1	
K				4	6	9	12	7		4
F						1	5	19	14	3
B				1			2	12	20	7
C						1	3	3	5	30

50 cm from this point, the mean line of vision being at 45° to the surface of the paper. Under these conditions the papers of highest gloss gave fairly clear mirror images, whereas the papers at the lower end of the scale did not. Forty-two observers were consulted. Table 5 shows the voting under these conditions, and Table 6 the means and standard deviations. It will be seen at once that the anomaly noted with paper *L* has now disappeared and the paper is graded much lower in the series. Examination of the data in Table 6 and comparison with the corresponding values in Table 2 shows that there is no significant shift in grading with papers *H* and *G*. With papers *I*, *D*, *E*, *K*, *B* and *C* the observed differences between the respective means taken from Tables 2 and 6 vary from 2·4 to 3·4 times the estimated standard error and they may be considered significant; but as the shift is in most cases downwards by roughly half a unit, the *relative* grading of these papers is not markedly affected. With papers *L* and *F*, however, there is a highly significant shift in grading, since the observed differences between the means are respectively 7·7 and 10·2 times the estimated standard error of the difference between means. Imposition of mirror image conditions of observation raises the grading of paper *F* and lowers that of *L*, exactly as was suspected after examination of the histograms obtained in the main test. The bimodal distribution shown for paper *L* in Table 1 is hereby explained and the significance of the analyses given in Tables 3 and 4 is confirmed.

Table 6. *Means and standard deviations of voting distributions in Table 5*

	I	*D*	*H*	*E*	*L*	*G*	*K*	*F*	*B*	*C*
Mean	1·38	2·26	3·05	4·66	4·74	6·05	6·67	8·31	8·67	9·43
σ	0·58	0·82	1·17	1·08	1·45	1·29	1·60	0·86	1·06	1·05

It is concluded that for eight of the ten papers used in this investigation, the relative grading is not markedly dependent on the conditions of viewing and the establishment of a scale of gloss for general purposes is for them a possibility. The grading of the remaining two papers is so sensitive to changes in the conditions of illuminating and viewing that, whatever scale of gloss be adopted, it will give unsatisfactory grading of these two unless viewing conditions be suitably chosen. In the present work the values shown in Table 2 have been adopted as giving the best available gradings of papers *L* and *F* for general purposes, but it is realized that these values may be misleading when the papers are examined under special conditions.

PHYSICAL MEASUREMENTS ON THE PAPERS

Goniophotometric measurements were made in the manner explained in earlier papers. Table 7 shows the intensity of light received for equal and opposite angles of incidence and viewing ranging from 10° to 75°. The figures are calculated in terms of an incident beam giving

Table 7. *Intensity of scattered light: angles of incidence equal and opposite*

Angle	*I*	*D*	*H*	*E*	*G*	*L*	*F*	*K*	*B*	*C*
10°	81	63½	75½	105	118	132	104	91½	86½	121½
15°	80½	63½	76	106½	120½	134	105½	103½	87	123
20°	81½	64½	78	111	124	137	109½	106½	86½	125½
25°	84	67	80½	112½	132½	140	111½	107½	87	136
30°	86½	70½	84	119½	143½	147	115½	113	97½	143
35°	90½	74½	91	127	150	163	121½	122	105	157
40°	97	82	100	137½	175½	174	132½	135	118	168
45°	106½	90½	111	151	199½	195	152	154	131	187
50°	118½	106	130	177	235	226	176½	186	160	223½
55°	141½	131½	161	212	335	310	215	234	208	280
60°	176	168½	211	293	424	419	294	337	281	384
65°	235	228	312½	395	675	769	414	469	401	532
70°	350	345	477	518	884	960	638	687	628	832
75°	550	526	743	856	1 350	1 515	988	1 085	938	1 305

a photometric reading of 100 when incident on a magnesium oxide surface at 45° and viewed normally. The results shown are for single specimens only, but exactly the same part of these specimens was illuminated at all the angles shown. In the work previously reported it was shown that for papers not differing appreciably in colour, the use of measurements at 45° incidence and viewing gave satisfactory correlation with visual assessments; now, however, all angles of incidence and viewing give unsatisfactory grading.

Table 8. *Intensity of scattered light: angles of incidence and viewing 45°: means of 20 specimens*

	I	*D*	*H*	*E*	*G*
S	$107\frac{1}{2} \pm 1$	$96 \pm 1\frac{1}{2}$	$103\frac{1}{2} \pm 2$	$132\frac{1}{2} \pm 3$	$183\frac{1}{2} \pm 3\frac{1}{2}$.
	L	*F*	*K*	*B*	*C*
S	173 ± 4	$156\frac{1}{2} \pm 4\frac{1}{2}$	$128\frac{1}{2} \pm 4\frac{1}{2}$	155 ± 2	222 ± 5.

Table 8 shows the intensities of light for 45° incidence and viewing expressed as the means of determinations on twenty specimens: as before, the figures after ± are standard errors. These values, plotted against the visual assessments as given by the means in column one of Table 2, are shown in Fig. 1. The correlation coefficient is +0·81, which, judged by Fisher's z-test and "Student's" t-test, can be regarded as significant at a 1 : 500 level; but the grading is nevertheless highly unsatisfactory. Examination of the positions of these points showed, however, that the papers for which the instrumental readings appeared too low were for the most part comparatively dark in colour, whereas for the lighter papers the instrumental readings mostly appeared too high. Attempts were therefore made to find a suitable correction factor for the luminance factor of the paper.

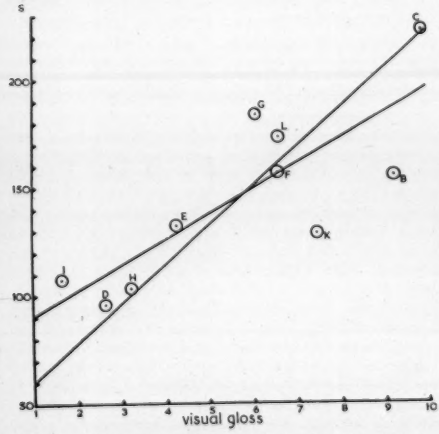

Fig. 1. 45° specular reflexion plotted against visual gloss

The spectrophotometric curves of the papers were measured over the range 390 to 780 μμ at 10 μμ intervals, the illumination being normal and "viewing" at 45° by a zone mirror. From these curves the C.I.E. trichromatic coefficients and luminance factor were computed, illuminant *A* being selected since the gonio-photometric measurements—and certainly some of the visual assessments—were made with tungsten light. The results are displayed in Table 9. It was found later that the results were relatively much the same (though absolute values were different) if the luminance factors were computed for illuminants *B* or *C*. Various types of function were examined, the object being to derive an expression which should show high correlation with the gloss grading of the papers as decided by the visual tests. Finally an expression of the following form was found to give the best results:

$$G = S(a + 10/L)$$

where *G* is the measure of visual gloss, *S* is the relative intensity under 45° — 45° conditions of illuminating and viewing (intensity for magnesium oxide at 45° — 0° being 100), *L* is the luminance factor for 45° — 0°

Table 9. *Colours of papers examined: C.I.E. illuminant A*

	x	*y*	*z*	*L*
I	0·487	0·418	0·095	52·3%
D	0·501	0·418	0·081	36·0
H	0·501	0·418	0·081	37·0
E	0·496	0·415	0·089	39·3
G	0·494	0·414	0·092	41·7
L	0·464	0·412	0·124	49·5
F	0·508	0·416	0·076	37·9
K	0·511	0·413	0·076	24·4
B	0·508	0·419	0·073	21·3
C	0·500	0·419	0·081	33·2

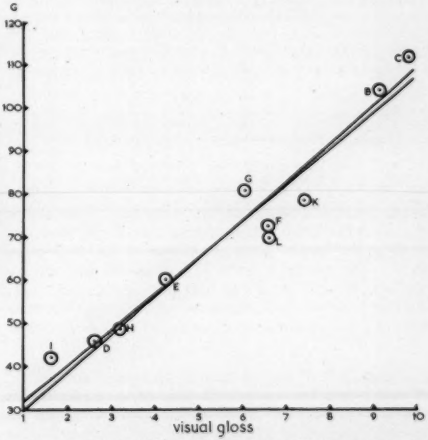

Fig. 2. $G = S(0{\cdot}2 + 10/L)$, plotted against visual gloss

conditions of illuminating and viewing expressed as a percentage, and a is a constant. Under these conditions the best results are given when $a = 0{\cdot}2$. The factor $(a + 10/L)$ can be evaluated mentally with the aid of a table of reciprocals, and the multiplication with S can be performed with a slide rule, so that G is readily calculated. The values of S and L are given in Tables 6 and 9; when $a = 0{\cdot}2$, Fig. 2 results, where G is plotted as ordinate and visual gloss as abscissa. The correlation coefficient is now $+ 0{\cdot}97$ and improvement in the grading is evident. As a further check on the validity of this type of correction for luminance factor, measurements were made with the Sheen Glossmeter, a commercial instrument operating at $45° - 45°$ angles of incidence and viewing. In this instrument the intensity of the incident beam is adjusted so as to give a reading of 500 with a piece of polished black glass. Table 10

Table 10. *Sheen glossmeter data*

	S_{45}	S_0	G
I	$58{\cdot}6 \pm 0{\cdot}10$	$26{\cdot}8 \pm 0{\cdot}14$	43·5
D	$49{\cdot}9 \pm 0{\cdot}16$	$18{\cdot}1 \pm 0{\cdot}05$	46·0
H	$58{\cdot}6 \pm 0{\cdot}25$	$20{\cdot}0 \pm 0{\cdot}11$	50·9
E	$63{\cdot}2 \pm 0{\cdot}32$	$20{\cdot}5 \pm 0{\cdot}06$	54·1
G	$73{\cdot}3 + 0{\cdot}23$	$22{\cdot}2 \pm 0{\cdot}10$	60·1
L	$77{\cdot}8 \pm 0{\cdot}28$	$30{\cdot}8 \pm 0{\cdot}11$	54·0
F	$68{\cdot}4 \pm 0{\cdot}23$	$21{\cdot}1 \pm 0{\cdot}03$	57·7
K	$56{\cdot}9 \pm 0{\cdot}23$	$14{\cdot}3 \pm 0{\cdot}03$	60·8
B	$62{\cdot}5 \pm 0{\cdot}15$	$13{\cdot}0 \pm 0{\cdot}03$	71·1
C	$86{\cdot}8 \pm 0{\cdot}25$	$20{\cdot}7 \pm 0{\cdot}06$	74·0

shows the results for the papers under test. Column one (S_{45}) gives the instrumental readings for 45° incidence and viewing, the figures being the means of 200 determinations. The second column (S_0) gives the instrumental readings for 45° incidence and normal viewing, being the means of 10 determinations, and the third column (G) gives the gloss values, calculated from the equation

$$G = S_{45}(0{\cdot}37 + 10/S_0)$$

a is taken as $0{\cdot}37$ instead of $0{\cdot}20$ because of the method of calibrating the instrument with black glass; this is merely an adaptation of the earlier equation to suit the changed intensity of incident beam and does not involve a new fitting of the visual gloss results. If S_{45} is plotted against the visual grading it is found that the correlation coefficient is only $+ 0{\cdot}64$ which, under these conditions, is barely significant even at a 1 : 20 level. If, however, the new gloss number G is used instead, the correlation coefficient is $+ 0{\cdot}96$ and the points lie evenly distributed about a straight line, with the exception of L which, as we have already seen, is abnormal.

Finally, the new gloss equation was applied to the series of low-gloss papers examined previously,* with the results shown in Fig. 3. These first papers did not vary widely in luminance factor, so the effect of the correction is comparatively small; nevertheless, some improvement is evident. The correlation coefficient is $+ 0{\cdot}99$ and now only the point for paper *A* appears to be in error.

* *J. Sci. Instrum.*, **26**, p. 84 (1949).

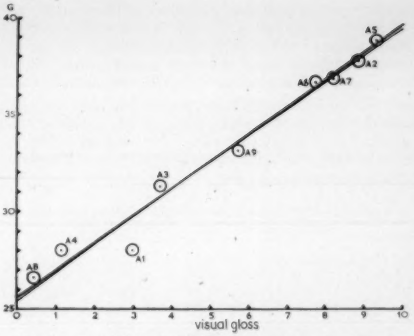

Fig. 3. $G = S(0{\cdot}2 + 10/L)$, plotted against visual gloss: original "A" series

DISCUSSION

It is concluded that the dimensionless number G given by the equation $G = S(a + 10/L)$ is highly correlated with the gloss of papers as judged by a panel of 164 observers. The equation is valid over a range of luminance factor from 20% upwards and for gloss ranging from almost perfect matt to gloss high enough to give fair mirror images under the conditions of viewing described in detail in this paper. Warning must be given against the use of the equation outside these limits, at any rate for the present. In point of fact, several different empirical equations can be found, quite unlike in form but nevertheless capable of giving satisfactory grading of the papers over the limited range examined. Among those examined were

$$G = S/L^{\frac{1}{2}}$$

and

$$G = S/\log_{10}(1 + L/10)$$

both of which give correlation coefficients of $+ 0{\cdot}97$, though numerical values are, of course, different. All three equations grade the papers relatively in a very similar manner over the range examined, but they diverge considerably in the three cases for low values of L. At present there is no experimental evidence to show which, if any, of these equations is true in extreme cases. The range already examined, however, covers the bulk of white and brown uncoated glossy papers on sale in this country. The equation $G = S(a + 10/L)$ has been adopted because it is found empirically so far to give the best results. It suffers from the defect that for a perfect Lambert diffusing surface the value of G is not zero but a positive number, namely $21{\cdot}2$ in our present notation; moreover the values of G for perfect diffusers of different luminance factors would be different. It is easy to modify the equation to make $G_0 = 0$ for all perfect diffusing surfaces, but it becomes more unwieldy and the grading of the papers is not thereby improved;

therefore it is thought better for the present to leave the equation as it is until more information is available.

It is clear from Fig. 2 that the grading is not perfect even with the correction for luminance factor. Papers *G*, *F* and *L* proved particularly troublesome and these anomalies will be discussed in a further paper now in preparation.

Finally, it should be noted that the equation for *G*, if written in the form

$$G = aS + 10S/L$$

can be regarded as made up of two parts. The first is a measure of the intensity of light reflected at the specular angle when the angle of incidence is 45°. The second involves a term S/L, which is a measure of contrast when the angle of viewing is changed from 45° to 0°, the angle of incidence remaining unchanged at 45°. *G* is simply a weighted average of the two. In other words, the average observer judges gloss neither by pure specular reflexion nor by pure contrast, but by a combination of the two. Gloss is judged by an appraisal of a physical situation *as a whole*. That is, gloss is a *gestalt*. It is likely that specular reflexion and contrast are not the only components of this *gestalt* and the subject is being investigated further. It is not certain that the weighting factor *a* is truly constant, though it is approximately so over a wide range. Later work may, however, show that it is a function of *S*, *L* or both. The effect of the mechanical smoothness of the surface examined on the visual assessment of gloss also requires investigation.

APPENDIX

We have been desirous of getting an independent check on the validity of our methods of establishing the relative "visual gloss" of specimens and to illustrate the procedure used we have made supplementary experiments on the ranking of weights. Ten similar cylindrical tins 11 cm high and 7·5 cm in diameter were loaded with copper strips attached to the bottoms internally with cement. The observers were requested to arrange the closed tins in increasing order of weight, no time limit for the operation being set. They were allowed to lift the tins, but not to shake or rotate them, or to remove the lids. The resulting voting distribution of 42 observers is set out in Table 11. By treating these distributions in exactly the same way as before, estimated weights in arbitrary psychological units are arrived at which, plotted against physical weights as determined by measurement on a balance, give linear regression with a correlation coefficient of + 0·992 (Fig. 4). It was interesting to find that no observer succeeded in getting the ranking quite correct, though several made nearly correct attempts: the means of the distributions, however, give correct ranking and nearly correct relative grading, as judged by physical methods. The similarity of Figs. 4, 3 and 2 confirms our belief that the methods described in this and previous papers do provide a reasonably correct visual grading of the relative gloss of papers and that the high correlation of this visual grading with a combination of physical properties is genuine. In the weight-grading experiments the subjective sensation of lightness or heaviness is found to be highly correlated with the single physical property of mass, a conclusion which probably will not be contested. The subjective sensation of relative gloss can be estimated equally readily, but these estimates are highly correlated with, not a single physical property, but a combination of (at least) two physical properties, the intensity of the reflected light and its distribution.

Table 11. *Voting distribution* (42 *observers*): *estimation of relative weights*

		Lightest			*Ranking*					*Heaviest*
	Wt(*g*)	1	2	3	4	5	6	7	8	9	10
X	76·4	30	9	1	2						
R	80·6	9	19	7	6	1					
U	82·6	1	10	18	6	5	1	1			
W	83·6	2	1	9	9	14	5		1	1	
T	86·6		2	4	14	13	2	5	2		
Z	90·7			1	2	6	20	7	4	2	
Y	93·7		1	1	3		11	13	6	5	2
V	96·8						3	9	17	5	8
Q	97·6				1		1	4	9	18	9
S	100·5			1		2		2	3	11	23

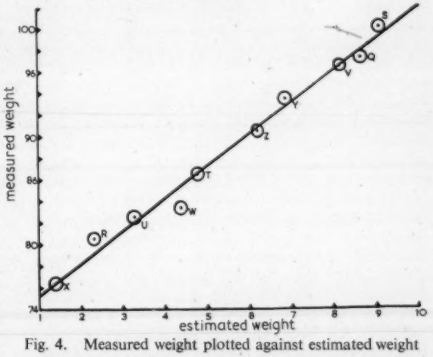

Fig. 4. Measured weight plotted against estimated weight

Gloss thus emerges as an entity neither purely physical nor purely mental. It is not purely physical because it corresponds closely to no single physical property of a surface that we have been able to discover, and there is no way of evaluating the constant *a* of our equation apart from appeal to subjective judgments. On the other hand, it is not purely mental, since relative gloss is determined approximately for most observers by a combination of physical properties. It can be regarded as a mental construct based upon appraisal of a physical situation as a whole. Whether it differs in kind or only in degree from physical properties such as mass or extension in space, is a nice philosophical question into which it would be out of place here to enquire; but we do not regard the *gestalt* nature of the gloss of surfaces as rendering it on that account any the less real or less worthy of scientific study.

"Heat developed" and "powder" Lichtenberg figures and the ionization of dielectric surfaces produced by electrical impulses*

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A brief outline of the characteristics of Lichtenberg figures is given. Some experiments on "heat developed" and "powder" figures are reported. With regard to the former it is shown that the effect is restricted to certain kinds of solid dielectrics and associated with the state of the surface; an explanation of the mode of their formation is suggested. The effect of repetition of impulses of alternating polarity is investigated by means of "powder" figures. It is shown that the effect of a discharge of given polarity is not neutralized or cancelled by a succeeding discharge of the opposite polarity. This leads to the conclusion that local electrical or mechanical failure in the surface may occur owing to the chance that positively and negatively charged particles will in the course of time be brought very close together. A difference between the Lichtenberg figures produced by negative impulses of short and long wave front and tail is demonstrated; an explanation based on the theory of spark discharge is given. The electrostatic charges produced on the dielectric surface by single and multiple impulses are measured. The results indicate that in order to produce the charge, electron emission from the dielectric surface, when this is the cathode, is necessary; this is evidence that formation of a cathode spot on the dielectric surface is possible, and may be capable of triggering the process which results in failure of the dielectric.

1. Introduction

The surface of a solid electrical insulating material is affected in a characteristic way when it is adjacent to or in contact with a conductor charged to a potential sufficiently high to produce an electrical discharge or ionization in the ambient gas. The effect is not immediately visible but its existence is demonstrable by dusting the surface with a dielectric powder. When the surplus powder is gently blown away, the powder which remains adhering to the surface forms a pattern which is star-like, diffuse or confused, according to the polarity and time variation of the applied potential. These patterns are called "Lichtenberg figures" after their discoverer.[1] If the dielectric surface is unexposed photographic emulsion and if this is developed and fixed by ordinary photographic procedure after exposure to the electrical discharge, then images in the film identical in size and shape with the "powder" figures are obtained. With some materials, e.g. glass, latent Lichtenberg figures may be produced owing to local physical changes of the dielectric surface and be rendered visible merely by breathing on the surface, owing to selective condensation of moisture, or by the reaction of the surface with a silvering solution, the silver being selectively deposited;[2] in such cases a number of repetitions of the discharge is necessary before the effect is appreciable.

The present author observed during the course of an investigation on a silicone varnish that latent Lichtenberg figures on the surface of certain kinds of solids could be developed by the application of heat.[3] It has, however, since been ascertained that a similar observation was reported in 1897 by Swan[4] working with clear "Bordeaux" resin containing some resin oil, but this effect does not appear to have received attention from any other investigators.

Having regard to the investigations of the mechanism of the electrical breakdown of solid insulating materials which are now in progress, any information appertaining to the interaction of such materials with a gaseous discharge is of value; it was thought worth while therefore to undertake a short experimental investigation of these "heat developed" figures. This comprised tests on a limited range of materials in order to obtain an indication of the type of material which possessed the requisite property, and further tests to enable this to be identified. During the progress of this work, however, other effects were observed incidentally which prompted some further research with respect to Lichtenberg figure formation in general, apart from the "heat developed" phenomena. For example, certain features of interest associated with the figures obtained in uniform or very slightly divergent fields are reported. These results appear to break new ground since hitherto the study of Lichtenberg figures has been more or less restricted to the patterns obtained in strongly divergent fields, presumably because under these conditions their size is directly related to the amplitude of the applied impulse. In addition the effect of discharge repetitions is studied and measurements of the electrostatic charge which is the primary cause both of the "powder" and "heat developed" Lichtenberg figures are made.

2. Characteristics and Applications of Lichtenberg Figures

The first comprehensive investigation of Lichtenberg figures was undertaken by Pedersen.[5] The photographic figures have been extensively used in "Klydonographs" for investigations of surge voltages on transmission lines.[6] An interpretation of the mechanism of their formation, based on modern theory of discharges in gases, has been given by Merrill and Von Hippel;[7] these authors indicate how ionization by electron impact,

* Based on Report Ref. L/T232 of the British Electrical and Allied Industries Research Association.

space charge and plasma formation, and charge neutralization are all involved in the process.

It is well established that Lichtenberg figures owe their origin to the movement of electrons in the ambient air or gas when the applied electrostatic potential is of sufficient magnitude to produce ionization. The evidence for this is, first, the curvature of the rays of the patterns which is observed when they are formed in the presence of a magnetic field;[7, 11] secondly, that only electrons possess the mobility needed to give the observed time of formation of the figures which is of the order of 10^{-8} to 10^{-7} sec; and, thirdly, that no figure is produced in the absence of an ionizing potential. Electrons are deposited on or removed from the dielectric surface according to the polarity of the applied surge, and the Lichtenberg figure is a result of the characteristic paths which they traverse in the gas adjacent to the surface.

"Powder" figures are a consequence of the powder particles being attracted to and retained by the surface charges; each particle adjacent to a localized charge becomes a dipole by induction and so subject to an attractive force.

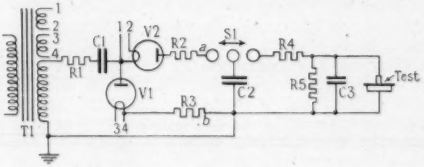
The origin of the photographic figures has been a source of some controversy but, in the main, the experimental evidence indicates that they owe their existence to the emission of actinic radiation from the ionization generated by the electrons in their passage through the air or gas adjacent to the surface of the solid. The particular pattern produced by an applied impulse depends on the polarity and waveform; its overall size in a divergent field is related to the voltage amplitude.

A detailed discussion, including a fairly complete bibliography, of the characteristics of Lichtenberg figures has been given recently by Anstey[8] and need not be repeated here. It may, however, be mentioned that Anstey has carried the subject a stage further by demonstrating that the wave front time of a surge potential may be estimated if a positive and a negative figure are simultaneously obtained.

Two novel developments which are related to the Lichtenberg figure formation are of interest. First, Selenyi[9] has described a method of recording fast electrical phenomena which is based on the modulation of an electron or ion beam falling on a dielectric surface; the record is revealed by dusting; with the dielectric surface moving past the beam at a rate of 10 m/sec an alternating current of 20 kc/s is resolved. Secondly, a dry printing process, to which the name "Xerography" has been given, has been described;[10] in this case a surface composed of photoconducting material is charged by exposure to an electrical discharge, and then exposed to light transmitted through a photographic negative; the photoconducting property of the surface causes a loss of charge at those regions which are illuminated; the surface is then dusted with a dielectric powder, the surplus blown off and the powder figure which is produced is analogous to a positive print and can be fixed by application of heat.

3. THE SURGE GENERATOR

Lichtenberg figures are produced and studied most conveniently by means of unipolar impulse potentials. The circuit of the impulse generator used in the present tests is shown in Fig. 1. The only feature which calls for comment is the special sphere-gap switch. The central sphere is movable and when it is in contact with the left-hand sphere, capacitor C_2 is charged to 8·6 kV. It is attached to a spring so that on release of a stop it quickly moves towards the right-hand sphere but comes to rest without touching it; the gap between the spheres breaks down and C_2 is discharged through the resistors R_4 and R_5, the latter being connected in parallel with capacitor C_3 and the test specimen. Short wave front surges are obtained when R_4 and R_5 are 1 000 and 3 000 Ω respectively; for longer wave fronts these values are changed to 10 000 and 30 000 or 100 000 and 300 000 respectively. The waveforms were calculated and the results confirmed by a cathode-ray oscillograph; the values are given in Table 1. The surges are free from



R1, R2, R3, 20 kΩ
R4, 1,10 or 100 kΩ, carbon/w
R5, 3,30 or 300 kΩ, "
T1, 240/3500 V
C1, 0·1 μF
C2, 0·002 μF
C3, 100 pF (including test cap.)
V1, V2, U16
S1, Special sphere-gap switch

Fig. 1. Surge generator

oscillation. In Fig. 1 the generator is connected to give a surge of negative polarity; a positive surge is obtained by interchanging the connexions at *a*, *b*. As a safety precaution capacitor C_1 is shunted by a leakage resistance of 10 MΩ which serves to discharge this capacitor quickly when the supply is switched off.

Table 1. *Waveforms of surges used in the tests*

	Wavefront time μ secs	*Time-to-half crest value* μ secs	*Crest voltage* kV
Short wave front	0·4	6·2	6
Long wave front	4·0	62	6
Very long wave front	40·0	620	6

4. PREPARATION OF "HEAT DEVELOPED" FIGURES

"Heat developed" Lichtenberg figures can be prepared on certain kinds of dielectric materials. Some resinous substances yield particularly well-defined patterns, and in fact after testing a variety of substances the most satisfactory material for the purpose of the present investigation was found to be ester gum containing

about 10% by weight of tri-cresyl-phosphate, the latter being added merely to reduce brittleness and to prevent wrinkling of the surface with age. (Ester gum is the product of the interaction between glycerol and rosin and consists mainly of glyceryl triabietate.) The plasticized ester gum or other resin is heated sufficiently to liquefy it, and a small quantity is poured on to a clean glass plate which is supported horizontally and maintained at a temperature sufficient to prevent solidification. The molten resin is spread uniformly over the plate with a glass rod. The plate is then allowed to cool and a hard and smooth film of solid resin on the glass is obtained. This is placed between a metal electrode, e.g. a needle point, disk, sphere, etc., and an earthed metal plane, and an impulse of magnitude + 5 kV or more is applied. No change of the exposed surface is visible, but by gently heating the resin film, say by holding it over a hot plate, thread-like grooves appear just as the softening temperature is reached. In a few seconds the complete star-like pattern of a positive Lichtenberg figure, which seems to be engraved in the resin surface, is obtained. At this instant the heating must be stopped otherwise the figure rapidly disappears owing to the complete liquefaction of the resin. If the applied surge is of negative polarity, the "heat developed" figure is an annular depression which usually contains several thread-like grooves. The uniformity of the figures is improved by coating the underside of the glass plate with tin foil before exposure to the discharge. The "heat developed" figure may be reproduced in an ordinary photographic enlarger, the test film being used as a negative after removing the tin foil; photographs of positive and negative figures obtained in this way are shown in Figs. 2 (*a*) and (*b*). Sometimes, however, a dark-ground photographic method is better.[3]

5. EXPERIMENTAL INVESTIGATION OF "HEAT DEVELOPED" LICHTENBERG FIGURES

5.1. A range of solid insulating materials was tested for "heat developed" Lichtenberg figure formation in order to obtain an indication of the special properties involved. Carnauba wax was specially included, since this substance is an outstanding example of an "electret" forming solid. (An "electret" is a dielectric material which when allowed to solidify in the presence of a strong electric field shows a strong − charge on the surface in contact with the anode and a + charge on the opposite cathodic surface.) The results are given in Table 2.

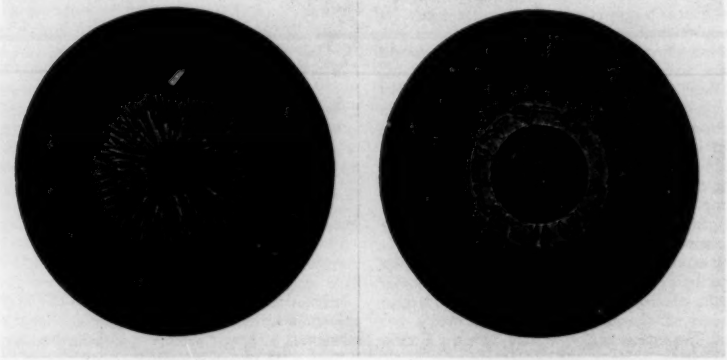
Table 2. *Substances tested for "heat developed" Lichtenberg figure formation*

"Heat developed" Lichtenberg figure formation	
Substances which give figures	*Substances which do not give figures*
Ester gum (with and without plasticizer).	Carnauba wax.
Cumar resin.*	Carnauba wax and ester gum (1 : 1 mixture).
"Bordeaux" resin (French rosin)† (with and without plasticizer).	Beeswax.
Maleic anhydride resin.	Mixture of petroleum waxes (A and B. 1 : 1).
"Novolak" resin.	Paraffin wax.
Silicone varnish.	Shellac.
Petroleum wax (A) m.p. 65·6/71·1° C.	Clear air drying insulating varnish.
Petroleum wax (B) m.p. 82·2/85° C.	
Polythene (moulded disk).	

* Cumar resin is a polymerization product of the coumarone and indene constituents of coal-tar naphtha.

† An extract from the exudation of *Pinus maritima*.

It will be seen that the effect is not related to "electret" formation, nor is it confined to resin or to materials



(*a*) + surge (0·4/6·2 μsec) (full size). (*b*) − surge (0·4/6·2 μsec) (twice full size).

Fig. 2. Photographic reproductions of "heat developed" figures obtained with a ⅜-in diameter electrode

with indefinite melting points. It was confirmed that all the materials tested gave normal "powder" figures.

5.2. It was concluded from the foregoing results that the effect was related to some property possessed by the surface of the substance. This was confirmed by the following experiment. An ester gum test specimen was prepared and a thin layer removed from one half of the surface by abrasion with carborundum paper. An electrode was placed on the line separating the untouched and the abraded areas, and a surge applied. The outcome of the usual heating procedure was that a "heat developed" figure appeared only on the virgin surface; the abraded part remained unaffected. On the other hand a "powder" figure could be produced whether or not the surface layer had been removed.

5.3 The next step was to find what after-treatment was effective in suppressing formation of "heat developed" or "powder" figures. Ester gum containing plasticizer was used as test material for the "heat developed" figures; beeswax and petroleum wax were used in similar tests for "powder" figures. The results are given in Table 3. As a matter of interest it may be noted that if the test surface was previously sluiced with water and the exposure to the discharge made in a damp atmosphere, no figure was obtainable.

Table 3. *Results of experiments on suppression of "heat developed" and "powder" Lichtenberg figure formation by treatment after exposure to a discharge*

Treatment which suppresses figure formation	*Treatment which does not suppress figure formation*
Immersion or sluicing with water.	Exposure to a moist atmosphere (96% R.H.) for 24 hours.
Immersion in mercury.	Tin foil pressed on surface and wrapped round the specimen.
Exposure to a Bunsen flame. (The flame must be passed rapidly over the surface for about 30 secs; a single momentary exposure is not effective.)	Exposure to ultra-violet light.
Heating until the substance is quite soft (the formation of a figure reduces but does not entirely remove the surface charge).	

5.4. An experiment, made in pure nitrogen, demonstrated that the formation of "heat developed" or "powder" figures was independent of the nature of the ambient gas.

5.5. The presence of an electrostatic charge on the surface of the test material after exposure to a discharge was confirmed (see Section 7). The polarity of the charge is always the same as that of the potential applied to the adjacent electrode and neither a "heat developed" nor "powder" Lichtenberg figure can be obtained unless a charge is present.

5.6. "Heat developed" figures were obtained by Swan after a lapse of two months from the time of the discharge. In the present investigation figures with apparently no loss of definition have been obtained a week after exposure to the discharge; there is no doubt that the charge adheres stubbornly to the surface in a way which suggests that it is trapped or strongly bound to the molecules of the solid.

6. EFFECT OF DISCHARGE REPETITION ON LICHTENBERG FIGURES

In the treatment of discharges in voids in dielectrics under a.c. potentials[12] there is an implicit assumption that the charge transferred to the surface of the material by one discharge is completely neutralized by the succeeding discharge of opposite polarity. That this assumption is not strictly valid may be shown by means of Lichtenberg figures. "Heat developed" figures may be used but for the present purpose it was found more convenient to use "powder" figures; "photographic" figures were obviously ruled out because with these the effects produced are essentially irreversible. As regards the substance used as powder, previous workers have generally recommended either a mixture of red lead and sulphur or lycopodium powder, but after trying these and also talc, magnesia, zinc oxide and jewellers' rouge, it was found that the last gave the best pattern definition and enabled good contact prints to be made from the test specimen.

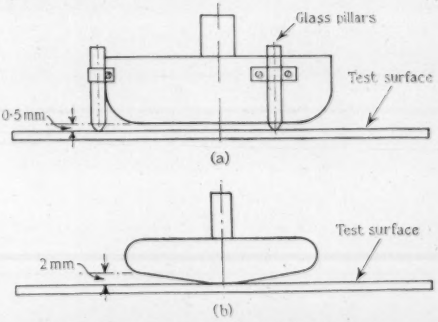
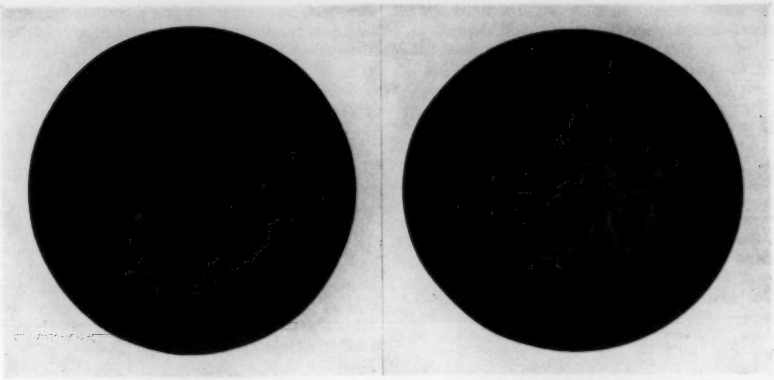


Fig. 3. Electrodes used for Lichtenberg figure tests

Ester gum containing 10% by weight of tri-cresyl-phosphate was used as test material; this gives a transparent coating on a glass plate. The underside of the latter is coated with tin foil stuck on with a thin coating of petroleum jelly. After exposing the test specimen to one or more discharges the surface is dusted with the rouge and the surplus gently blown off. The tin foil is peeled from the glass and the grease washed away with benzene. This leaves the test specimen in a condition in which it can be treated as a photographic negative and a contact print made by the usual procedure. Tests were made with two types of electrodes, one a brass cylinder having a circular plane area of $1\frac{1}{4}$ in diameter with rounded edges of $\frac{1}{4}$ in radius of curvature and the



(*a*) + surge (0·4/6·2 μsec). (*b*) − surge (0·4/6·2 μsec).

(*c*) Sequence of four surges − + − + (0·4/6·2 μsec). (*d*) Sequence of four surges + − + − (0·4/6·2 μsec).

Fig. 4. Lichtenberg "dust" figures obtained with a parallel plane electrode

other a truncated conical electrode (see Figs. 3(*a*) and (*b*)). The plane surface of the former is supported parallel to and at a distance of 0·55 mm from the surface of the test specimen; the central region of the discharge therefore takes place in a uniform field. With the conical-shaped electrode, the discharge occurs in a slightly divergent field, the air space varying from zero to 2 mm in thickness. The crest value of the surges applied in the tests is 6 kV; this is found by experiment to be about 20% higher than the discharge inception voltage of the parallel plane electrode (Fig. 2(*a*)). In the tests with 50 c/s a.c. the peak voltage is 5·2 kV.

The main object of these experiments was to obtain "powder" Lichtenberg figures for single and repeated surges of alternating polarity and thus get some indication of the distribution of the surface charge which was produced; in addition the difference between short and long wave front surges was observed. The contact prints made from the test specimens are reproduced in Figs. 4 to 7, the corresponding details of the tests being given in Table 4. In the photographs the light areas show the presence of adhering powder and therefore the location of the surface charge but not its polarity, whereas the dark background shows absence of charge since the unaffected parts of the test specimens are transparent.

No reproduction of "powder" figures for very long wave front surges (40/620 μsec) are given since these do not differ significantly from the 4/62 μsec surge figures.

Table 4. *Lichtenberg "powder" figures*

Electrode used	*Voltage applied*	*Wave form of surge*		*Polarity of surge*	*"Powder" figure obtained, see Fig. No.*
		Wave front time μsec	*Time to half value μsec*		
Parallel plane	Surge (short wave front)	0·4	6·2	+	4 (*a*)
Do.	Do.	0·4	6·2	−	4 (*b*)
Do.	Sequence of four surges of alternating polarity	0·4	6·2	− + − +	4 (*c*)
Do.	Do.	0·4	6·2	+ − + −	4 (*d*)
Do.	50 c/s sine wave for 30 secs followed by surge	0·4	6·2	+	5 (*a*)
Do.	Do.	0·4	6·2	−	5 (*b*)
Do.	Surge (long wave front)	4	62	+	5 (*c*)
Do.	Do.	4	62	−	5 (*d*)
Conical	Surge (short wave front)	0·4	6·2	+	6 (*a*)
Do.	Do.	0·4	6·2	−	6 (*b*)
Do.	Sequence of four surges of alternating polarity	0·4	6·2	− + − +	6 (*c*)
Do.	Do.	0·4	6·2	+ − + −	6 (*d*)
Do.	Surge (long wave front)	4	62	+	7 (*a*)
Do.	Do.	4	62	−	7 (*b*)

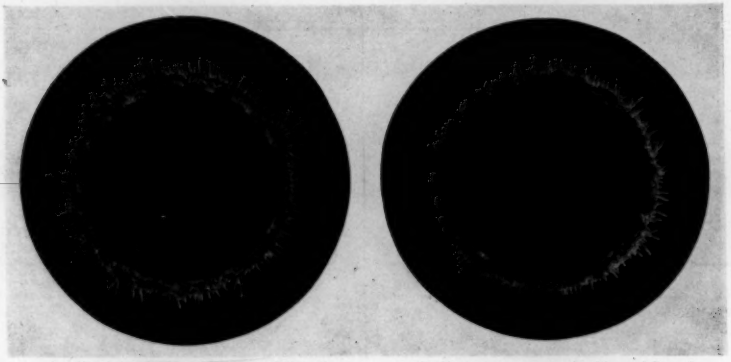
The points of interest to be noted are:—

(i) A surge of given polarity does not cancel the effect of a previous surge of the opposite polarity.

(ii) No "powder" figure under the plane surface of the parallel plane electrode is obtained with 50 c/s a.c. followed by a surge.

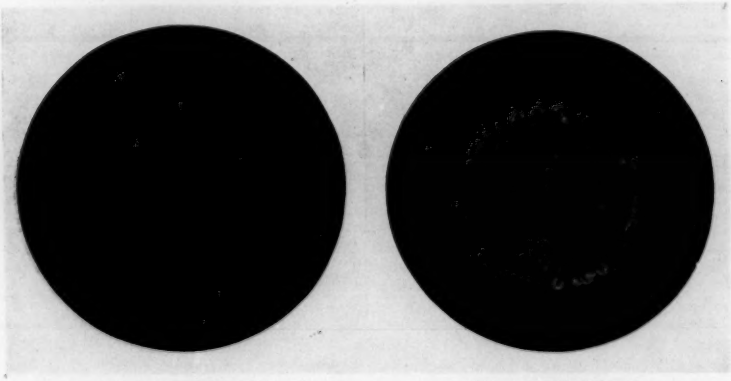
(iii) There are significant differences between short wave front (0·4/6·2 μsec) and long wave front (4/62 μsec) surges.

With regard to the tests with 50 c/s a.c. a mixed figure similar to Fig. 4(*d*) was obtained for times of application of the a.c. up to 2 sec. As the time of application was lengthened the figure in the central area became fainter



(*a*) 50 c/s sine wave a.c. for 30 sec followed by + surge (0·4/6·2 μsec).

(*b*) 50 c/s sine wave a.c. for 30 sec. followed by − surge (0·4/6·2 μsec).



(*c*) + surge (4·0/62 μsec).

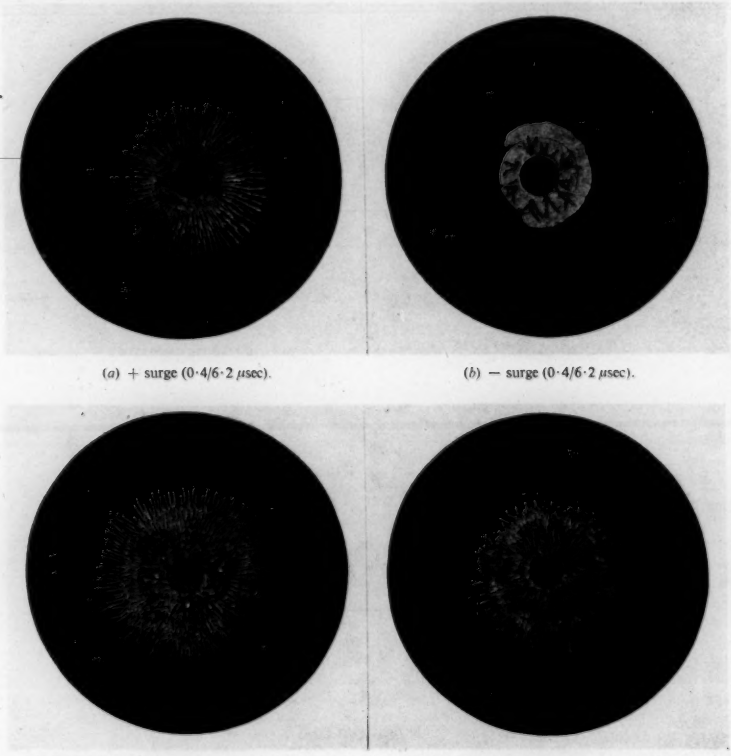
(*d*) − surge (4·0/62 μsec).

Fig. 5. Lichtenberg "dust" figures obtained with a parallel plane electrode

until with 30 sec application practically no powder adhered. This effect can be seen in Figs. 5(*a*) and 4(*b*) in which the application of the 50 c/s a.c. is followed by a + and − surge respectively in order to show that the result is independent of the polarity of the last discharge. The results of these tests and the conclusions to be drawn thereform are discussed in Section 9.

7. MEASUREMENT OF THE SURFACE CHARGE

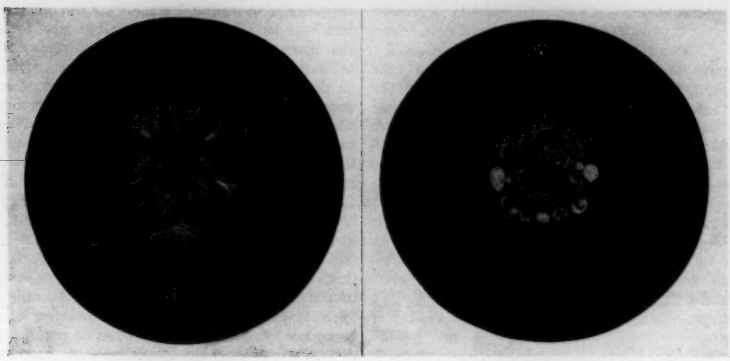
The magnitude and polarity of the charge which is deposited on the surface by a discharge and which is the primary cause of Lichtenberg figure formation is measured by the method illustrated in Fig. 8. This is similar in principle to that which has been used for the investigation of "electrets."[13] After the application of the test potential, the h.v. lead is removed and the electrode is attached to a cord as shown. This electrode is now earthed momentarily; this causes it to acquire a charge, opposite in sign to that on the surface of the dielectric. The charged electrode, after removing the earth connexion, is raised up from the test surface, by lowering the counterweight, until it makes contact at *a* with the conductor of the polythene insulated cable, the other end of which is connected to a capacitor *C* and the needle of a Lindemann electrometer. Thus the charge on the electrode is shared with *C* (which includes the capacitance of the cable and electrometer). The gain of potential of the electrometer needle is measured by its deflexion or preferably by a "null" method.



(*a*) + surge (0·4/6·2 μsec). (*b*) − surge (0·4/6·2 μsec).

(*c*) Sequence of four surges − + − + (0·4/6·2 μsec). (*d*) Sequence of four surges + − + − (0·4/6·2 μsec.).

Fig. 6. Lichtenberg "dust" figures obtained with conical electrode



(*a*) + surge (4/62 μsec). (*b*) − surge (4/62 μsec).

Fig. 7. Lichtenberg "dust" figures obtained with conical electrode

Let Q = charge on the dielectric surface.
Let Q_1 = charge induced on the upper earthed electrode.
Let C_1 = capacitance formed by the air gap and upper electrode (see Fig. 7).
Let C_2 = capacitance formed by the resin and glass plate (see Fig. 7).

Then
$$Q_1 = -QC_1/(C_1 + C_2) \qquad (1)$$

Let C = capacitance of the capacitor C plus that of the cable and the electrometer.
Let V = potential acquired by the electrometer needle.

Then $Q_1 = CV$, since, when the electrode is raised to make contact at a, the capacitance C_1 becomes negligibly small.

From equation (1) $Q = -CV(C_1 + C_2)/C_1$

or
$$Q = -CV(1 + C_2/C_1) \qquad (2)$$

The value of the ratio C_2/C_1 may be estimated from the static dielectric constants of the resin and glass plate and their known thicknesses. It is found to be nearly unity, so that approximately

$$Q = -2CV \qquad (3)$$

The batteries and potentiometers used in association with the Lindemann electrometer for the determination of V are not given as the method is well known.

The material employed for the dielectric surface in these tests is petroluem wax, m.p. 65·6/71·1° C. The upper electrode is the parallel plane shown in Fig. 3(*a*) which is spaced 0·055 cm from the surface of the wax. The glass plate is 0·14 cm and the wax coating 0·08 cm thick. The capacitance of the electrode when in position on the glass plate is 13·5 μμF. The results of the measurements are given in Table 5. It will be seen that the values obtained are consistent and repeatable. These tests are, however, to be regarded merely as indicative; it would have been of interest to estimate separately the

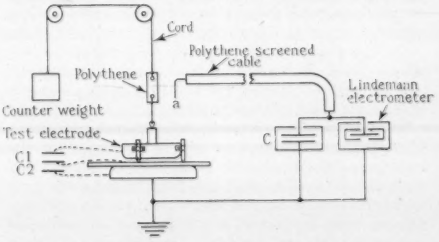


Fig. 8. Arrangement for measurement of surface charge

surface charge located under the plane surface of the electrode from that round the edges, but the time available did not permit of further work on these lines. It is of significance that although no "powder" figure is obtainable in the central area after exposure to 50 c/s a.c. for 30 sec or more, yet the surface charges deposited are of the same order of magnitude as those in the single surge tests. Another point is that the magnitude of the surface charge is unaffected by the wave front time although there are differences in the corresponding Lichtenberg figures. These results are discussed in Section 9.

Table 5. *Values of surface charges obtained with the parallel plane electrodes*

Test No.	Applied voltage	Surface charge μ coulombs	
1	− surge, short wave front (0·4/6·2 μsec)	− 0·0088	Mean for short wave front
2	Do.	− 0·0092	
3	Do.	− 0·0092	− 0·0091
4	− surge long wave front (40/620 μsec)	− 0·0090	Mean for long wave front
5	Do.	− 0·0078	
6	Do.	− 0·0090	− 0·0086
7	+ surge, short wave front (0·4/6·2 μsec)	+ 0·0064	Mean for short wave front
8	Do.	+ 0·0060	
9	Do.	+ 0·0066	+ 0·0063
10	+ surge, long wave front (40/620 μsec)	+ 0·0062	Mean for long wave front
11	Do.	+ 0·0060	+ 0·0061
12	50 c/s sine wave a.c. for 2 min	+ 0·0066	Mean for 50 c/s sine wave
13	− surge to test No 12.	− 0·0098	+ 0·0064
14	50 c/s sine wave a.c. for 2 min	+ 0·0070	
15	− surge to test No. 14	− 0·0088	
16	50 c/s sine wave a.c. for 5 min	+ 0·0050	
17	− surge to test No. 16	− 0·0068	
18	50 c/s sine wave a.c. for 5 min	+ 0·0068	
19	+ surge to test No. 18	+ 0·0068	

8. MECHANISM OF FORMATION OF "HEAT DEVELOPED" FIGURES

The results of the experiments described in Section 5 may be summarized as follows:—

(i) "Heat developed" figures are capable of production only on certain dielectric materials. The materials are not characterized by a particular chemical or structural composition, but in general they are resins or of a resinous nature.

(ii) The particular characteristic of a material which gives a "heat developed" figure is associated exclusively with the surface which is produced by cooling from the liquid state or by pressure moulding.

There are two ways whereby the surface characteristics of a solid may differ from the substratum, first, the surface molecules may be in a state of strain, and secondly, the surface molecules may be oriented. In a material such as ester gum which gives particularly well-defined "heat developed" figures, both of these characteristics are probably present. The presence of residual strains is indicated by the fact that immediately after solidification, the surface of a pure film of ester gum is perfectly smooth, but on being left for a few days it develops wrinkles and becomes corrugated; these strains are a consequence of the contraction which occurs during the change of state.

As regards the existence of surface orientation, it is well established that molecules in the surface layer of a polar substance in the liquid state are oriented[16] and this state will be "frozen in" on solidification. Again it is highly probable that there are residual strains present in pressure-moulded polythene as well as a certain degree of molecular orientation.

To explain the formation of "heat developed" Lichtenberg figures several mechanisms have been considered but have been rejected as incapable of accounting for the known facts, except the one which follows. The explanation suggested, which is at present hypothetical, in that there is no experimental confirmation as yet, depends on two assumptions; first, that it is a necessary condition that residual strains exist in the surface of the material; secondly, that the charged particles, i.e. electrons or positive ions, in the surface layer cause a lowering of the melting point of the material in their immediate neighbourhood by decreasing the energy of the molecular bonds. Thus, as the temperature of the surface increases, a release of the residual strains in the surface occurs at the positions occupied by the charge; this results in the distortion of the surface which leads to the formation of the grooves which constitute the "heat developed" figure. Molecular surface orientation is apparently not an essential condition but may enhance the effect owing to the likelihood that the presence of some order in the molecular arrangement will enable the charged particles to exert a stronger influence.

It follows that materials which do not give "heat developed" figures are those in which there are no residual strains in the surface owing to absence of contraction on solidification or to their plasticity or softness which enables such strains to be relieved. These materials, however, cannot be made to give a "heat developed" figure by maintaining them in a stressed condition by artificial means.

The removal of the surface charge and consequently the suppression of the Lichtenberg figure by treatment with water or mercury as observed in the tests reported in Section 5 would be simply explained as being due to dissipation of the free surface charge by conduction, were it not for the fact that the charge is not removed by tin foil or by exposure to a high relative humidity. It appears that only a liquid can provide the intimate contact with the surface which is necessary and which cannot be obtained with tin foil.

The fact that the charge is not dissipated by exposure to a high relative humidity, which presumably leads to the formation of a film of moisture on the surface of the material, is difficult to understand. Possibly no adsorption can occur in the vicinity of the charge; some such effect is needed to account for the existence of breath figures mentioned in the Introduction (Section 1).

9. THE EFFECT ON THE DIELECTRIC SURFACE OF A DISCHARGE

It has been shown that the charge on the surface of the dielectric produced by the discharge adheres very stubbornly to the solid. Furthermore, Lichtenberg figures corresponding to a sequence of surges of alternating polarity (see Figs. 4(*c*) and (*d*) and 6(*c*) and (*d*)) demonstrate that the effect of a discharge of given polarity is not neutralized or cancelled by a subsequent discharge of opposite polarity. It may therefore be concluded that the charges are not free surface charges.

The charge produced by a negative discharge is due to electrons. These electrons must enter the surface and be trapped in positions where they cannot easily be reached by positive ions. Again, according to the theory given in Section 2 the charge produced by a positive discharge is due to removal of electrons from surface molecules.

If a positive discharge follows a negative, or vice versa, a state is produced in which there are trapped electrons and positively charged molecules of the dielectric in close proximity. It is reasonable to conclude that after many repetitions of such a process there is a high degree of probability that some negative and positive charges will be so close together that the force between them could cause a local electrical and possibly mechanical failure in the surface layer. This might explain the erosion or boring action of electric discharges impinging on a dielectric surface. This effect would be enhanced by a local weakening of the surface structure in the neighbourhood of charged particles which is equivalent to a lowering of the melting point, and which is suggested in Section 8 to account for "heat developed" Lichtenberg figure formation.

Furthermore, ultimate electric breakdown of the dielectric may be a consequence of local chemical activity in the surface, since if this gave rise to a decomposition product which served as a source of electrons, e. g. carbon, the formation of a cathode spot would be possible and further decomposition would be propagated rapidly through the material. The ionization of the surface molecules caused by a positive discharge is favourable to chemical activity. It is in fact difficult to understand how discharges on the surface of a dielectric can lead to breakdown at relatively low voltages in the manner in which it is known to occur unless the impinging electrons penetrate into the material. The evidence afforded by the Lichtenberg figures leads to the conclusion that this penetration does occur.

Repetitions of impulse voltages of amplitude sufficient to produce a discharge are known to cause a reduction of the impulse breakdown strength of pressboard, and the effect is greatest with negative chopped oscillatory impulses,[18] i.e., when the conditions are such as to produce the most rapid changes in electron movement and therefore the highest probability of close juxtaposition of positive and negative charges on the surface of the material.

There is one final point which requires explanation. It will be seen by reference to Figs. 5(*a*) and (*b*) that after a repetition of a large number of discharges, no "powder" Lichtenberg figure can be developed in the central region under the plane electrode, although measurements show the presence of surface charge similar in magnitude and sign to that produced by a positive discharge. Although the Lichtenberg figures of Figs. 5(*a*) and (*b*) show the presence of charge round the circumference of the central clear region, its magnitude cannot be appreciable having regard to the similarity of the absolute magnitude of the charges observed in Tests 17 and 18 of Table 5.

The surface charge on the central portion may therefore be taken as that given in Test 12 of Table 5, viz. $0{\cdot}0064\ \mu$ coulombs, and the area is $11{\cdot}4\ \text{cm}^2$. If it is assumed that the charge is distributed uniformly over the clear circular area the charge density is $+\ 0{\cdot}00056$ μ coulombs/sq cm which with the sign changed is equivalent to 35×10^8 electrons/sq cm. It is easy to show that if these electrons are distributed uniformly over a surface the distance between any two adjacent electrons will be $\sim 0{\cdot}18\mu$. This distance is of the same order as the linear dimensions of the particles of powder. It follows that relative to the size of the powder particles, the electric field normal to the surface will be practically uniform. The same reasoning applies to a distribution of surface molecules possessing a positive charge owing to the loss of an electron. Owing to induction, a dielectric powder particle placed in an electric field behaves as if it were a dipole. It is subject to a force of attraction only if the field is divergent, in which case it tends to be drawn to the region of higher intensity; no attractive force is exerted if the field is uniform. Thus the absence of a Lichtenberg figure in the central regions of Figs. 5(*a*) and 4(*b*) is probably attributable to the uniform distribution of charge which results when a large number of discharges of alternating polarity are applied.

10. MECHANISM OF THE DISCHARGE AND SURFACE CHARGE FORMATION

It is of interest to interpret the charge formation on a dielectric surface in accordance with the modern theory of spark discharge. Considering first the case of a negative discharge (i.e. that case in which a negative potential is applied to the conducting electrode which is therefore the cathode, the dielectric surface being the anode), an electron adjacent to the cathode surface is impelled towards the anode and generates further electrons by collision processes. This leads to an electron avalanche. The positive ions, being relatively immobile, remain practically in the positions where they are formed. In addition to ionization by collision, the fast moving electrons also produce excited atoms and molecules which emit photons of short length radiation travelling with the speed of light, and these produce more electrons by ionization. The most intense action occurs near the space-charge channel of positive ions; the photo-electrons feed into this channel and it becomes a conducting plasma, whilst the positive space-charge develops towards the cathode (positive space-charge streamer development).

The foregoing represents in outline the initial phase of spark discharge according to accepted theory. To explain the subsequent behaviour, which is conditioned by the fact that the anode surface is a dielectric, the following mechanism is suggested. At the anode, negative charge accumulates as the electrons impinge on the

dielectric surface, with the result that the anode-cathode potential fall decreases, ionization dies away and the positive space-charge is neutralized by recombination or ultimately drifts to the cathode. With an impulse of short wave front and tail the potential of the cathode falls so rapidly after passing its peak that an abundant supply of electrons remains in the conducting plasma and serves to neutralize the positive space charge; the negative charge on the dielectric surface is undisturbed (see Fig. 9(*a*)). On the other hand, with an impulse which has a long-wave tail, the cathode potential is of appreciable magnitude for a period sufficient to enable most of the electrons to be swept outwards from the conducting plasma under the combined effects of the anode-cathode fall and the negative charge on the dielectric surface. This leaves a substantial excess of positive charge in the positive space-charge column, so that when the cathode potential ultimately decreases, there is a potential drop between this column and the negative surface charge on the dielectric which is high enough to cause a discharge in the reverse direction (see Fig. 10(*a*)). This back discharge is suggested to account for the positive discharge patterns which are apparently superimposed on the negative patterns when a surge with a long-wave tail is used (see Figs. 5(*d*) and 6(*b*)).

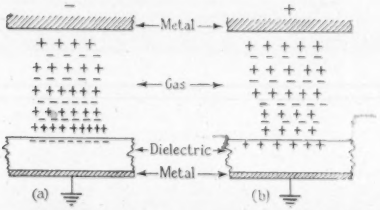


Fig. 9. Discharge with a short wave front and tail: conducting plasma remains when applied potential falls

Secondly, when the dielectric surface is the cathode and the conducting electrode the anode, i.e. when a positive surge is applied, the electron avalanche and conducting plasma formation is initially similar, but in this case the electrons disappear into the conducting anode. At the tip of the conducting plasma which extends nearly to the cathode or dielectric surface, there is a very steep gradient of potential. This produces intense ionization and is capable of removing electrons from the surface molecules of the dielectric, and may indeed lead to the formation of a cathode spot. The accumulation of positive surface charge, however, rapidly reduces the anode-cathode fall and ionization ceases. Since the electrons are drawn to the anode or recombine with the positive space-charge, the condition which leads to a back discharge does not arise (see Figs. 9(*b*) and 10(*b*)) and there is, therefore, little difference between the charge patterns produced by short and long wave tail impulses (see Figs. 4(*a*) and 5(*c*)). The irregularity of the pattern produced in a divergent field by a long wave front impulse as observed in Fig. 7(*a*) has also been noted by Anstey[8] and is considered to be due to conductivity of the dielectric surface.

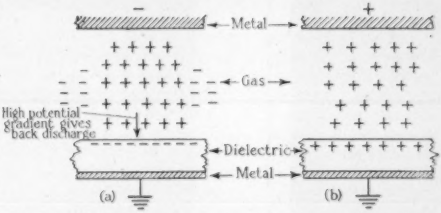


Fig. 10. Discharge with long wave front and tail: positive space charge column remains when applied potential falls

According to Loeb and Meek[14] the breakdown potential of a uniform field may be calculated from the following equation:

$$\alpha\delta + \log\frac{\alpha}{p} = 14{\cdot}46 + \log\frac{X}{p} + \tfrac{1}{2}\log\frac{\delta}{p} \qquad (4)$$

where α = Townsend's first coefficient (number of new electrons created in the gas by a single electron in its advance of 1 cm along the field axis).

δ = gap length, cm.

p = ambient pressure, mm mercury.

X = field strength, volts/cm.

In the present tests δ and p are substantially constant ($\delta = 0{\cdot}055$ cm, $p = 760$ mm mercury) so that equation (4) is, in effect, a relationship between α and X. The solution may be obtained by reference to curves α/p as a function of X/p[15] since any value of X uniquely determines one particular value of α. The procedure is by means of trial and error. In this way it is found that the breakdown potential of the air gap used in the tests with the parallel plane electrode described in Section 6 is 3 460 V and the value of α is 281.

The number, n, of electrons generated by the passage of one electron starting from the cathode and ending at the anode surface is given by

$$n = e^{\alpha\delta}$$

which with $\alpha = 281$ and $\delta = 0{\cdot}055$ cm gives

$$n = 6 \times 10^6 \text{ electrons.}$$

Now in Fig. 4(*a*) there are approximately 300 small star-like patterns in the central area, and if it is assumed that each small star corresponds to one electron avalanche, then the total number of electrons, N, generated in this area by avalanches is

$$N = 18 \times 10^8 \text{ electrons}$$

The total negative charge on the surface produced by a short-wave front negative surge is given in Table 5 as $-0{\cdot}0091\mu$ coulombs. Some of this charge is external to the central area; the actual proportion is not known, but as a rough estimate it may be supposed that edge discharge accounts for 20% of the total so that the charge in the central area is approximately $-0{\cdot}007$ μ coulombs. This is equivalent to 440×10^8 electrons. Thus the number of electrons which are bound to the dielectric surface is approximately 24 times the number produced in the air by collision processes. The bulk of the surface charge is therefore supplied by emission from the cathode or by photo-ionization. Similar considerations apply to the case of a positive impulse. Here in order to produce the surface charge, electrons must be removed from the dielectric, leaving it charged positively. Ionized surface molecules would be chemically reactive and the formation of a cathode spot would appear to be probable if the discharge is many times repeated; such an effect would supply the action which triggers the process which leads to ultimate failure of the dielectric. It is of interest to note that, according to the data given above, the charge associated with a single electron avalanche is 20 $\mu\mu$ coulombs, and this is roughly of the same order as the value estimated from discharge intensity measurements[17] which involve an entirely different principle.

11. CONCLUSIONS

(*a*) It is demonstrated that "heat developed" Lichtenberg figures can be produced only on a restricted range of solid dielectrics, e.g. resin, petroleum waxes, moulded polythene, and that they are associated with certain surface characteristics. It is suggested that the latter are residual strains which are caused by contraction on solidification from the melt, or, in other cases, by pressure moulding. To explain the surface distortion which constitutes the "heat developed" figures, it is suggested that the localized electrostatic charges on the surface of the dielectric which result from the discharge have the effect of lowering the melting point of the solid in their immediate neighbourhood so that on heating a local relief of the residual strains occurs.

(*b*) It is demonstrated by means of "powder" Lichtenberg figures that the effect of an electric discharge of a given polarity on the surface of a dielectric is not neutralized or cancelled by a subsequent discharge of opposite polarity. This leads to the conclusion that in the negative discharge, electrons penetrate into the surface of the material and become strongly bound thereto and that they cannot easily be reached by positive ions. Local electrical and mechanical failure in the surface may thus be caused by the close proximity of electrons and positive ions which probably occurs after a large number of discharge repetitions of alternating polarity.

(*c*) It is concluded from the results of measurements of the surface charge and consideration of the theory of spark discharge that the bulk of the charge is supplied by emission of electrons from the cathode. From this it follows that the formation of a cathode spot on the surface of the dielectric during a positive discharge is probable; this may provide the triggering action which leads to ultimate failure of the dielectric.

(*d*) It is demonstrated that the Lichtenberg figures produced by a negative discharge show characteristic differences according to whether the applied impulse is of long or short wave front and tail. The difference is caused by reverse or "back" discharges which only occur with the latter.

The effect is explained as being due to a discharge between the negative charge on the dielectric surface and the positive space charge column which remains after the applied potential has fallen to a low value.

12. ACKNOWLEDGMENTS

The author desires to acknowledge his indebtedness to Dr. V. Daniel, Mr. A. Morris Cassie and several other of his colleagues for helpful discussions, and to the Director of the British Electrical and Allied Industries Research Association for permission to publish this paper.

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The conductivity of hydrocarbon transformer oil containing water and solid conducting particles

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The d.c. resistivity test is used to determine the quality of transformer oil in service. It is found that water droplets in oil oscillate between the measuring electrodes in a d.c. field, transferring charge and giving rise to a high conductivity. As the test temperature is raised, oil dissolves more water and the quantity held in suspension decreases with consequent decrease of conductivity. This results in an inverted V resistivity-temperature characteristic. Solid conducting particles such as carbon give rise to high conductivity and a resistivity-temperature characteristic parallel to, but of lower level than, new oil. The conductivity is proportional to the square of the electric stress, the quantity of conducting matter and inversely proportional to the viscosity.

Robust equipment has been designed for the measurement of dielectric resistance of transformer oil which has enabled the d.c. resistivity test to be used for checking the quality of transformer oil in service. Forrest[1] described this equipment and quoted typical results, amongst which were curves of resistivity against temperature having an inverted V-shape. The interpretation of these curves is clearly important if the maximum information is to be obtained from the test. Whilst it was believed that water was responsible for the inverted V-shape, a mechanism of conduction was not advanced. This point has been investigated, and it is found that suspended water droplets in oil give rise to a high conductivity. The resistivity of oil containing water is low at room temperatures and rises to a maximum as the temperature is increased, finally decreasing with further increase of temperature. Solid conducting particles also cause high conductivity, but the resistivity-temperature curve is parallel to that of clean new oil, but at a lower level.

WATER IN OIL

Water may enter in to transformer oil in several ways; by absorption from moist air particularly when hot; from imperfectly dried fibrous insulation; or even by ingress of rain or humid air into the oil tank. When a small quantity of water is added to a hydrocarbon oil, in course of time, an equilibrium is set up with some of the water dissolved in the oil and some in suspension in the form of small droplets. The amount which will dissolve depends upon the temperature. Clark[2] published curves of the amount of water necessary to saturate oils of various qualities, for various temperatures. Fig. 1 has been taken from his paper, and was obtained for American-grade oil. It shows that the oil of a transformer operating at, say, 50° C, could carry 150 parts per million (p.p.m.) water in solution; on cooling to 20° C, about 40 p.p.m. would remain in solution, the remainder having come out in the form of a suspension.

Oil containing a suspension of water dyed with potassium permanganate was placed in a small dish ground in a microscope slide. The slide was set on a microscope stage with two tinned copper wire electrodes held by rubber blocks so that the ends dipped in the liquid. The

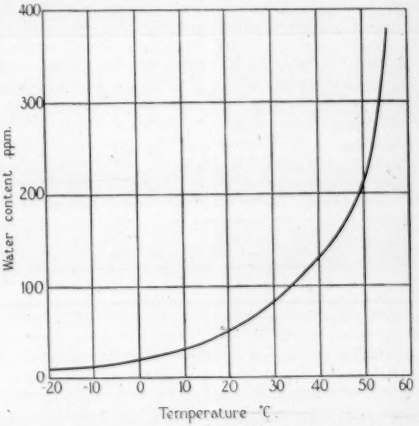

Fig. 1. Solubility of water in transformer oil. (*After Clark*)

electrodes just appeared at the opposite sides of the field of view at a magnification of about 100 times when the spacing was 1 mm.

With 300 V d.c. applied to the electrodes, the droplets were seen to move and the different types of motion noted are described. (*a*) A drop starting midway between the electrodes moved towards one electrode at a constant speed, impinged and immediately rebounded at approximately the same velocity, then on to the other electrode and so on. This was the commonest type of motion and the drops followed paths which mapped out the lines of force. The larger drops which are easily seen with the microscope passed across the gap in roughly $\frac{1}{2}$ sec. There was a small variation in the time taken for different drops to cross the gap. This variation was also noticed with any particular drop at successive crossings. (*b*) Occasionally collisions occurred, and when this happened, both drops were deflected; or the two coalesced; or sometimes the two split up into several

smaller drops. These last two occurrences were rare. (*c*) The whole process slowed down slightly with time of voltage application, the number of moving drops decreasing. On reversal of polarity, the system recommenced movement with its original vigour, the drops in the gap reversing their direction of motion. Gradually the larger drops settled to the bottom of the slide under gravity.

From this it is seen that the conductivity of an oil containing a given amount of water will be high at low temperature because the number of water droplets will be large. On heating, more water will dissolve and the number of droplets in suspension decrease, as will the conductivity. At the temperature at which all the water is in solution the conductivity will be a minimum, and at higher temperatures the conductivity will again increase in the same manner as clean dry oil due to the decreasing viscosity offering lower resistance to the ionic impurities which are always present to a greater or lesser degree. The water in solution in the oil does not ionize and does not, therefore, contribute to the conductivity.

A charged droplet of radius r cms between charged electrodes of large area spaced d cms apart will experience a force F dynes which will cause it to move, with constant velocity given by Stokes' law, as observed under the microscope.

$$F = q \times V_0/d \text{ dynes} \tag{1}$$

$$v = \frac{qV_0}{d6\pi\eta r} \text{ cm/sec} \tag{2}$$

where V_0 = potential difference between the electrodes, e.s.u.
q = charge on the drop, e.s.u.
v = velocity of the drop, cm/sec.
η = viscosity of the oil, poise.

After the droplet has once touched an electrode, the charge q will be constant and for present purposes it is assumed that its charge density will be the same as that of the electrode.

$$q = \frac{r^2KV_0}{d} \text{ e.s.u.} \tag{3}$$

where K = dielectric constant of the sample.

From (2) and (3), the charge transferred per drop per second is given by Q,

$$\text{where} \qquad Q = \frac{r^3K^2V_0^3}{6\pi\eta d^4} \text{ e.s.u./sec} \tag{4}$$

$$\text{or} \qquad \text{current} = \frac{r^3K^2V_0^3}{6\pi\eta d^4 3 \times 10^9} \text{A} \tag{5}$$

Where there is more than one drop, it is unnecessary to differentiate between polarities of charge of the drops because a positively charged drop will transfer the same amount of charge to the cathode as a negatively charged drop to the anode. If the oil contains p_s p.p.m. water in suspension and the volume of sample between the electrodes is $(A \times d)$ c.c.s., the number of drops, assuming uniform size, will be given by N.

$$N = \frac{3p_sAd}{4\pi r^3 10^6} \tag{6}$$

and the total current flowing

$$= \frac{K^2V_0^3p_sA}{24\pi^2\eta d^3} 10^{-15} \text{ A} \tag{7}$$

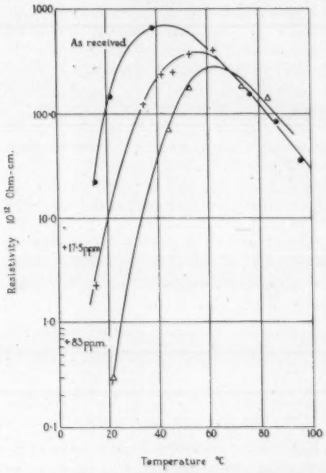

Fig. 2. Resistivity of transformer oil with added water

This is equivalent to a sample resistivity given by ρ.

$$\rho = \frac{72\pi^2\eta d^2}{K^2V_0^2p_s} 10^{17} \text{ ohm-cm} \tag{8}$$

The conductivity is, therefore, proportional to the square of the electric stress in the oil sample, and proportional to the amount of water in suspension, and inversely proportional to the viscosity. The amount of water in suspension and the viscosity both depend upon the temperature.

EXPERIMENTAL RESULTS

The oil test cell[1] used has an electrode spacing of 5 mm and except where otherwise stated, all measurements were made at 5 kV and after 1 min electrification. Samples of B 30 oil were made up with various added water contents. The distilled water added had a conductivity of 10 μ ohms-cm^{-1}, and the wet sample was shaken vigorously and allowed to stand overnight before test. The curves of Fig. 2 show a progressive increase in the temperature at which the maximum resistivity occurs as the water content is increased, until, at the

higher water contents, the curves tend to come together. At temperatures above the resistivity maxima, all tend to come together. A total water content of 76 p.p.m., 26 p.p.m. of which is in suspension, would be sufficient to lower the oil resistivity to 10^{14} ohm-cm at 20° C.

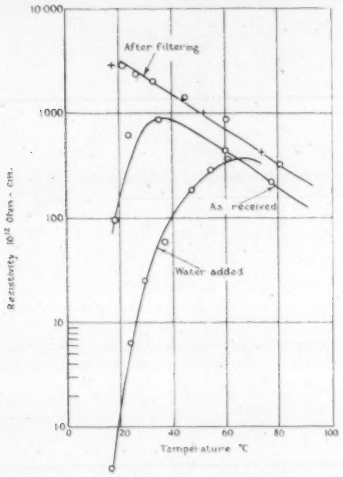

Fig. 3. Use of an edge-type filter to remove water from oil

Fig. 3 shows the resistivity of oil as affected by filtration through a laboratory edge-type paper filter. The sample initially showed the inverted-V characteristic which was removed by filtering. Water was then added, the inverted characteristic reappearing. Finally, the sample was again filtered, and the inverted-V characteristic disappeared again.

A typical example of the effect of voltage on wet oil is illustrated in Fig. 4. The oil contained about 60 p.p.m. water and had an inverted-V characteristic. The curves were obtained at various temperatures, those above that at which maximum resistivity occurs being shown in broken lines. The full line curves are seen to show rising conductivity with increase of voltage. This is owing to the combined effect of conductivity due to the droplets which is proportional to the square of the voltage, and that due to the clean oil which decreases with increase of voltage. It is of interest to compare the curves for 50 and 86° C where the levels of conductivity are of the same order at 5 kV. At the former temperature, droplets are present, whilst they are not at the latter temperature.

SUSPENSION OF SOLID CONDUCTING PARTICLES

Suspensions of aluminium powder in oil were made up to simulate conditions when wet fibres, carbon or other conducting particles are present, and also to afford a check on the theory outlined above. Microscopic examination shows that the phenomenon described for water droplets occurs, the particles crossing and recrossing the gap between the electrodes. The number of

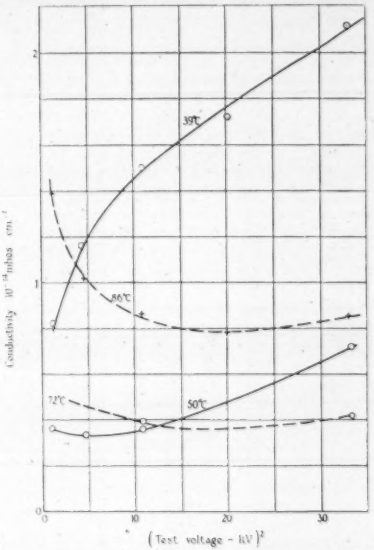

Fig. 4. Effect of voltage on the conductivity of wet oil

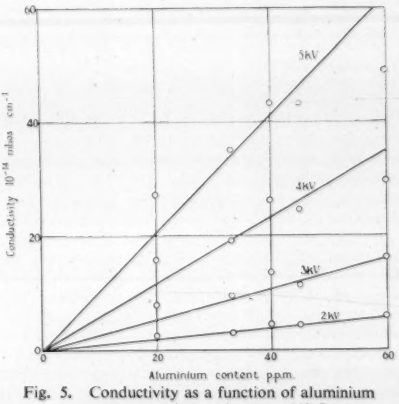

Fig. 5. Conductivity as a function of aluminium content at various voltages at 19° C

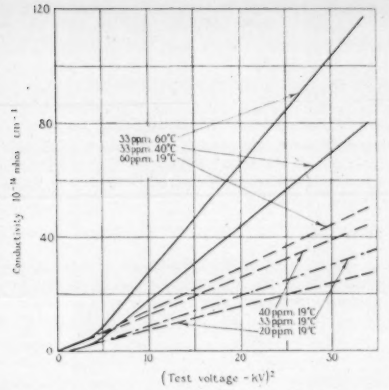

Fig. 6. Conductivity as a function of voltage for various concentrations of aluminium powder and at various temperatures

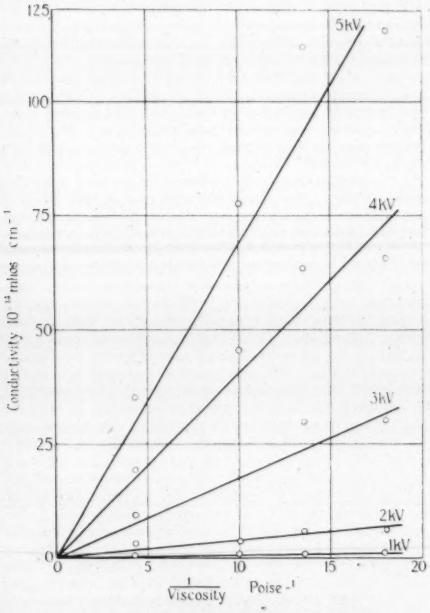

Fig. 7. Conductivity as a function of viscosity. 33 p.p.m. aluminium

particles present is, of course, independent of temperature and the only change of conductivity with temperature is due to change of viscosity. The resistivity-temperature curve will be similar to that of clean dry oil, but displaced to a lower level of resistivity.

The aluminium particles used were, on the average, square plates of 0·06 mm side and 0·004 mm thick. The calculation of conductivity based on spherical water droplets will therefore apply only qualitatively. The oil used was dried by filtering and all samples stored over a desiccant to prevent ingress of water. Fig. 5 shows that the conductivity is directly proportional to the aluminium content, the curves being obtained at 19° C for various test voltages.

Fig. 6 shows that the conductivity is directly proportional to the square of the test voltage. These results were also obtained at 19° C for various concentrations. Fig. 6 also shows the conductivity plotted against the square of the test voltage for various temperatures. At the higher temperature the conductivity is slightly greater than that given by a straight line, due presumably to a slight increase in the number of ions in the oil. Since the amount of aluminium in suspension does not change with temperature, Fig. 7 shows the conductivity inversely proportional to the viscosity. The viscosity range was

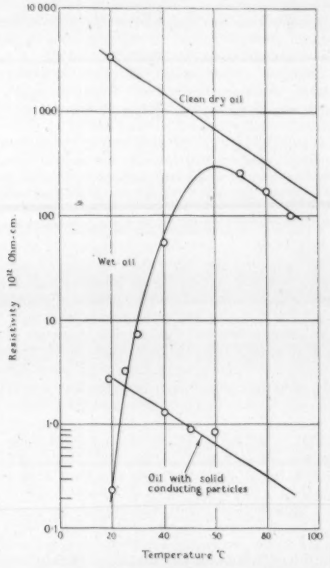

Fig. 8. Resistivity-temperature characteristics of clean and dirty oils

obtained by heating the oil. The same data are presented in Fig. 8 to show the resistivity of the oil varying with temperature. Also on this curve are shown the characteristics of clean and wet oils for comparative purposes.

CONCLUSIONS

It is shown that a suspension of water in oil gives rise to increased d.c. conductivity, owing to the transport of charge by the water droplets oscillating between the electrodes. When the resistance of a wet oil sample is measured, the resulting plot of log resistivity against temperature has the shape of an inverted V. A few parts of million of water in suspension can be detected in this way. Solid conducting particles in the oil give a resistance-temperature curve similar in shape to that of a clean oil, but the resistance is of a lower order of magnitude. In practice, whilst it is reasonable to assume that an inverted-V characteristic is indicative of the presence of water, it is unsafe to assume that a low-level resistivity curve indicates the presence of solid conducting particles. Weak acids due to oxidation would give rise to a similar curve, although these would probably be detected by the chemical acid-value test.

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Physics in Chemical Industry. By R. C. L. Bosworth. (London: Macmillan and Co. Ltd.) Pp. xix + 928. Price £3 10*s.* net.

"This book is written in the hope that it will indicate, in the field of Applied Physics, some of the ways in which the additional knowledge of the 'practical' man can be put in a form amenable to the treatment of a theoretical interpretation." Such is the author's intention given in the introduction. He later amplifies it by expressing the hope that the book will provide a student taking up an industrial career with a background of basic knowledge and will exhibit to those in industrial laboratories developments of a fundamental nature which are directly applicable to their own particular problems. How far are these very wide requirements satisfied?

The book is in four sections, containing thirty-six chapters in all. Chapters I to VII give a mathematical introduction including consideration of vector and tensor quantities, graphical methods, dimensional analysis and statistical methods. The properties of matter are dealt with in Chapters VIII to XVII, comprising atomic theory, thermodynamics, rheology, adsorption, and the general properties of gases, liquids and solids. Part III, Chapters XVIII to XXVII, discusses motion in its various forms, diffusion, fluid flow, heat flow, electrical phenomena, and rate and equilibrium of chemical reactions. The last nine chapters are devoted to instruments both on the works scale and in the laboratory. The author's method in general is to introduce the basic principles of each subject, giving briefly the derivation of fundamental equations and so on, and then to follow this with the facts. The treatment is thus one which will appeal to the physicist who wants to refresh his memory on fundamentals, rather than to the chemical engineer who would normally prefer to be supplied with the facts followed by a theoretical treatment at once less advanced and less condensed than that given here. Of the four sections, the first and third are the most successful; the second will be found generally useful, but suffers to some extent from the failure to give sufficient consideration to the complexities which confront the industrial physicist when he comes to deal with mixtures of material. The fourth section on instruments is, by comparison with the others, superficial and poor.

Bearing in mind the broad intention of the author, there are some serious omissions in subject matter. The reviewer has been unable to find any reference to infra-red and ultra-violet spectroscopy, which is surprising in a book whose express purpose is to interest the physicist in contact with chemical industry. These methods have been very much to the forefront during the last 10 years. One must refer also to the failure to give adequate guidance on the behaviour of gas mixtures at high pressures. By its very nature chemical industry deals with mixtures of substances rather than pure compounds, and the absence of a good discussion of their behaviour is a serious limitation. To close this consideration of the general matter of the book, we may note that there are a few misprints, and in at least one case a figure (that of a rotameter) has been printed on its side. Such an occurrence is out of place in a book which has been on the stocks, or at least in the fitting-out basin, as long as this one; for while it was published in 1950, the foreword and introduction are dated 1945. The book is uneven in treatment—at some times condensed and advanced; at others elementary in the extreme. This unevenness is presumably due to the attempt to cater for an extremely wide range of readers. It is, however, *readable*, which many books in this category are not. Therein lies its merit. The person who consults it as a reference book will probably (but by no means certainly) find the subject of his inquiry is dealt with, and if it is he will find the information presented in such a way that he will continue to read with interest and pleasure.

Considering these remarks, we may say that this book is not likely to appeal to the chemical engineer, except in so far as he may wish to extend his general reading: indeed, it was not the author's intention that it should do so. Neither, in view of its occasional omissions and unequal treatment,

can it be deemed suitable as a textbook for the physicist about to enter chemical industry. On the other hand, the method of treatment makes it valuable to the physicist who wants to refresh his mind on some subject which he may not have had to consider since his university days. Here the theoretical treatment and statement of the general laws and principles of the various subjects, together with brief but well-chosen references, are exactly what is required. Furthermore, it could be read with advantage by those chemists who are responsible for the technical guidance of industry and who are not in contact with chemical engineers.

The binding and reproduction are good, the index adequate, and the treatment of the very extensive list of symbols deserving of praise. P. DOCKSEY

Nuclear Physics. By ENRICO FERMI: JAY OREAR, A. H. ROSENFELD and R. A. SCHLUTER. (Chicago: The University of Chicago Press; London: Cambridge University Press.) Pp. ix + 246. Price 22*s*. 6*d*. net.

It is indeed a notable tribute to a lecturer when his words are taken down by his students and published in the form of a book. This has been done for Professor Fermi by Messrs. Orear, Rosenfeld and Schluter in the volume under review, the subject matter being a course in nuclear physics given at the University of Chicago in the first half of 1949; and the fact that a revised edition has been published within a year may be taken as an indication of the success of the venture.

The book is produced from type-written notes by photographic reproduction, and thus gives the impression of being "hot news." In a second printing, however, the news is obviously not quite so hot as it was and it is a pity that no attempt has been made to alter the hurried, or even careless, style in which much of the book is written. A key to the tone of the book is perhaps given by the statement on page 198 that "these expressions (not given by Fermi) were derived rather hurriedly and may be incorrect." The authors are careful to state in the Preface, and to repeat in the text, as exemplified by the above quotation, that Professor Fermi has not read the material and is not responsible for errors.

The hurry in which the book appears to have been produced is no doubt responsible for such blemishes as the disagreement between the definitions of the roentgen on pages 18 and 241, neither of which conforms with the internationally agreed definition and one of which is actually incorrect. It is presumably also responsible for the rather sketchy treatment given in places; for example, in the section on neutron diffraction. This section incidentally contains the rather striking understatement that "both [neutron and X-ray diffraction] are quite useful in determining crystalline structure." There is a further statement that "the disadvantages of neutron diffraction are mainly monetary and political," which hardly does justice to those aspects of physics which are also involved. The tendency throughout much of the book is to sacrifice coherence for speed, with the result that the editing is somewhat slipshod. Symbols are not always defined, nor are they always adhered to, and the standards of knowledge demanded of the reader are very variable. Familiarity with quantum mechanics is, however, taken for granted throughout.

In the reviewer's opinion the book is not suitable for a student working independently—there are too many gaps and unexplained assumptions—but nevertheless, once its limitations are accepted, the reader will find much useful material at his disposal. The various problems that are scattered throughout its pages form a distinctive feature. Some of these are worked out in considerable detail, and all are instructive. A wide range of topics is covered and there is an adequate index. The chapter headings are: Properties of nuclei; Interaction of radiation with matter; Alpha emission; Beta-decay; Gamma-decay; Nuclear forces; Mesons; Nuclear reactions; Neutrons; Cosmic rays. A Segré chart is appended. J. THEWLIS

Les propriétés optiques des lames minces solides. (Paris: Centre National de la Recherche Scientifique; London: Messrs. H. K. Lewis & Co. Ltd.) Pp. 176. Price 49*s*. net.

The optical properties of thin solid films were the subject of a colloquium, held in Marseilles in April 1949, at which 34 papers were read by guests from seven countries. The collected papers testify to the new vigour of this branch of optics. Twelve of the contributions are in English and the remainder in French.

The papers are arranged in four sections. The first section deals with the theoretical optics of multiple films and thin metallic films. The papers of Section II are concerned with laboratory methods for the deposition, by vacuum evaporation or sputtering, of films of controlled thickness; methods of measuring film thickness and refractive index are also discussed. In Section III experiments on the structure and properties of metal and metal-oxide films (including protective films) are described. The uses of thin films in interferometry, polarimetry, and optical filtering have induced stimulating papers under the heading of "Applications." Dr. A. F. Turner gives detailed descriptions of several types of selectively transmitting and reflecting films, including new interference filters and achromatic anti-reflection films. Professors Jacquinot, Strong, and Tolansky are among other notable contributors. Professor Rouard, the organizer of the colloquium, and P. Cotton have added a valuable bibliographical survey in which nearly 250 references are quoted. This publication must appeal particularly to those who approach the subject from the field of technical optics with a desire to become familiar with the theory of thin films and their practical possibilities as optical components. K. M. GREENLAND

Photometry and the Eye. By W. D. WRIGHT, A.R.C.S., D.Sc. (London: The Hatton Press Ltd.) Pp. ix + 127. Price 7*s*. 6*d*. net.

This is a good book which covers all the aspects of visual photometry in an expert fashion. Dr. Wright reviews past work and points out directions in which future research may progress to achieve fruitful results. A full discussion of photometric terms is followed by a chapter on the luminosity curve, which contains a sound defence of the C.I.E. curve: this is probably addressed to physiologists. The problems of heterochromatic and scotopic photometry are dealt with; measurements involving small light sources are reviewed as a result of the experience gained in recent work, to which Dr. Wright has contributed in no small manner. The chapter on the photometry of flashing light sources contains valuable references to electro-physiological work, which provides the objective counterpart to the subjective results discussed. There are only minor criticisms to be made. Two equations (p. 6) contain printer's errors, and the logic

of statements like "a filter which . . . will change the spectral composition to identity with that of the test light" (p. 48) will probably make Aristotle turn in his grave. The difference between "response" and "sensitivity" (pp. 109–113) deserves perhaps additional emphasis. These and similar minor points can, however, be rectified in future editions of this little book, which can be warmly recommended to all who are concerned with the measurement of light—be they illuminating engineers or astronomers. R. A. WEALE

Piezo-electric Crystals and their application to Ultrasonics. By WARREN P. MASON. (London: D. Van Nostrand Company, Inc.) Pp. xi + 508. Price 56*s*.

This latest volume in the Bell Laboratories Series is a valuable addition to the literature of piezo-electricity and ultrasonics. Instead of duplicating the treatment of the earlier work found in Cady's monumental *Piezo-electricity* (1946) and Bergmann's *Der Ultraschall* (fifth edition, 1949) the author wisely concentrates his attention on some of the latest developments. By this means he avoids the "handbook" mere cataloguing of material and has space to treat in detail the discovery and development of the water-soluble synthetic crystals, in particular di-potassium tartrate (DKT) and ethylene diamine tartrate (EDT), which can often be substituted for natural quartz. At the same time, enough fundamental material on crystallography, stresses, strains, thermal and electrical relations is given to make the book an independent study. It can, in fact, be used as an introduction to piezo-electricity, provided the reader has an adequate mathematical background. An appendix on the use of tensors in calculating the properties of rotated systems helps. The most complete treatment of the author's theoretical work on ferro-electricity in rochelle salt, KDP and barium titanite is included. The last three chapters are of particular interest in molecular physics. It is a saddening thought that the number of crystals used in this field is so much less than the 30 000 000 used in a single year to control the frequency of military communication equipment, after Cady had shown, in 1921, that quartz crystals could be used to control oscillators. The book is very well produced and can be strongly recommended. W. H. GEORGE

Negative Ions. By H. S. W. MASSEY, F.R.S. (London: Cambridge University Press.) Pp. xiv + 136. Price 12*s*. 6*d*. net.

This is the second edition of a monograph originally published in the Physical Tracts Series of the Cambridge University Press. The first four chapters follow the lines of the first edition, though modifications and additions have naturally been made to cover the interval of 12 years between the two publications. As before, Chapter I gives the theoretical basis for the existence of atomic ions and discusses the methods of calculating electron affinities and their experimental determination. Chapter II deals with molecular negative ions and with H_2^-, O_2^- and OH^- in particular. A longer and interesting chapter follows on the modes of formation of negative ions by the capture of an electron under a variety of different conditions and by the dissociation of a molecule through electron impact. Chapter IV is a short one concerning the converse phenomenon of electron detachment. But it is in the application of negative ions to the interpretation of terrestrial, solar and stellar atmospheric phenomena that the most important new results arise. The final chapter which deals with these subjects in a clear and interesting manner has been largely rewritten for the new edition. The author discusses in detail the role of negative ions in the ionized layers of the earth's upper atmosphere and the chapter concludes with an account of the discovery that the negative ion of hydrogen plays a dominating role in determining the spectral distribution of solar radiation over the observable frequency range.

The monograph is well illustrated and it is authoritative and readable. A. M. TYNDALL

Journal of Scientific Instruments

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SPECIAL ARTICLE

Fibre formation in synthetic polymers: methods and features*

By F. HAPPEY, Ph.D., F.Inst.P., Courtaulds Ltd., Coventry†

[*Paper first received* 11 *September*, 1950, *and in final form* 26 *January*, 1951]

In the formation of fibres from polymer molecules three main methods of regeneration have been developed as the dry, the wet, and the melt spinning techniques. These have been demonstrated in the cases of regenerated keratin from waste wool, cellulose acetate, and polythene respectively. A wide variety of straight chain synthetic polymers are now available and have been regenerated into fibres. The physical methods used in the study and development of these fibres are discussed, i.e. mechanical studies, along with the X-ray, optical, electron microscope and infra-red methods of investigation. Certain of these are demonstrated in a comparison of the fibre forming properties of polythene, the Nylons, Terylene, and the synthetic polyesters and polypeptides.

Before the advent of the synthetic polymers, the Rayon industry had two principal methods of regeneration of fibres and filaments from solution. These were the wet and dry methods of spinning and as they are now well known they will not be discussed further at this stage. Probably the first of the synthetic polymers to be spun successfully was Vinyon, the copolymer of vinyl acetate and chloride, and the method of dry spinning this material from acetone solution is now well established. Vinyon spinning is not carried out at room temperature; therefore to demonstrate the principles of wet and dry spinning materials capable of manipulation under very flexible conditions of control must be employed. One such specimen is cellulose acetate dry spun from solution in acetone, and the other a solution of soluble keratin from wool waste spun from alkaline solution and coagulated in a weakly acid bath. In both these cases it is possible to dispense with the accurately controlled spinning conditions which are an essential feature of normal manufacturing practice.

The production of good fibres from synthetic polymers, however, was brought to its present state of efficiency by the introduction of the melt spinning methods, one of which is demonstrated in the case of polythene monofil. This is shown to be readily capable of cold drawing approximately 1 000% (Fig. 1), to produce finally a comparatively strong and transparent filament. This process of cold drawing after spinning is typical of most of the melt spun synthetic polymers which are produced today. A list of the synthetic polymers discussed in this article, together with their constituent monomers, is given below.

THE CHARACTERISTICS OF FIBRE-FORMING SYNTHETIC POLYMERS

At this stage it is convenient to outline what are thought to be the essential fibre-forming properties of the molecules of synthetic polymers. First, of course, it is clear that the polymer must melt and yet in the molten state be capable of maintaining, with little or no degradation, its long chain characteristics. Secondly,

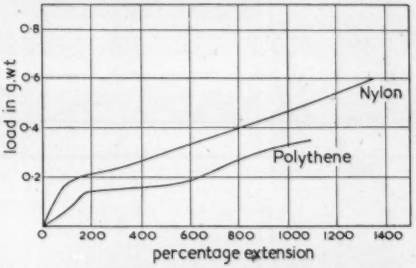


Fig. 1. Load/extension curves of Nylon 6·6 and polythene filaments during cold-drawing. Results expressed for unstretched denier of 1 (constant rate of extension)

it must be capable of orientation, and thirdly, in its orientated state, it must exhibit some evidence of uniaxial crystallinity. The properties of such fibres can be investigated by established physical methods. These include the polarizing microscope, the infra-red and X-ray fibre techniques, and the measurement of mechanical and anisotropic swelling properties of the fibres at various relevant stages during manufacture from the parent polymer substance. During the last few years the polarized infra-red method has been developed by Ambrose and Elliott[1] at the Courtaulds' Laboratories, as an additional tool for the study of the anistropy of polymer molecules. In this method the modes of vibration of the molecules can be studied in relationship to their orientation with respect to the axes of the long molecules. New evidence on the molecular configuration in many of the synthetic polymers has been obtained by the application of this technique, and it has been of vital importance in the investigation of the

* Based on a paper presented to Section B of the British Association for the Advancement of Science, in Birmingham on 1 September, 1950.

† Now at The Technical College, Bradford, Yorkshire.

structure of the α and β forms of the synthetic polypeptides. This method will not be discussed at length, but its importance is likely to be enhanced in the future, and physical methods of investigation of fibre structure could not be discussed without some mention of this comparatively new method of approach. A typical infra-red tracing demonstrating orientated fibre type anisotropy is given for Nylon 6·6[2] in Fig. 2. The numerical definition following the word "Nylon" for polyamides refers to the number of carbon atoms in the parent monomeric constituents of the material. Thus Nylon 6·6 is from the copolymer of hexamethylene diamine $NH_2(CH_2)_6NH_2$ and adipic acid $COOH.(CH_2)_4COOH$. Nylon 6·10 is the copolymer of hexamethylene diamine and sebacic acid $COOH.(CH_2)_8COOH$, and Nylon 6 is the self polymer of ω-amino caproic acid $COOH.(CH_2)_5NH_2$.

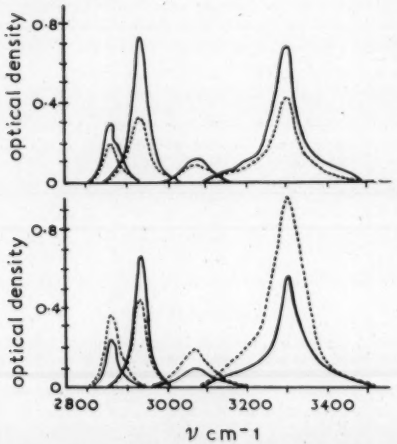


Fig. 2. Polarized infra-red absorption in orientated film of Nylon 6·6 (Ambrose, Elliott, and Temple[2])

(*above*) Film has axis of orientation vertical and plane of film tilted at 36° to direction of horizontal beams of radiation. Continuous line gives tracing with E vector in horizontal plane and perpendicular to axis of orientation. Broken line with E vector vertical and parallel to axis of orientation of film.

(*below*) Film turned through 90° from position in top curves with axis of orientation horizontal and with plane of film at 36° to direction of horizontal beam of radiation. Continuous line gives tracing with E vector horizontal and broken line with electric vector vertical.

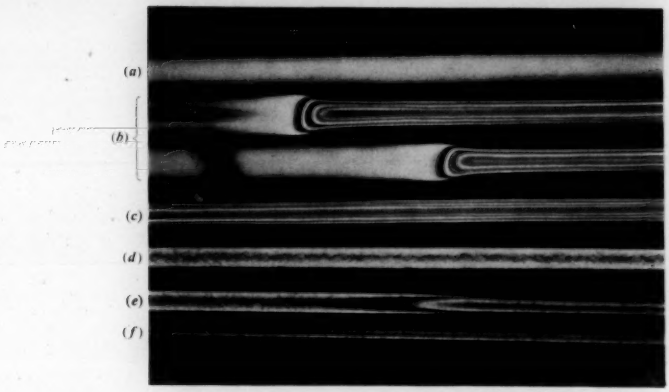
Crystallization does not occur so readily in long chain polymers as in the smaller organic molecules. This will be readily appreciated when the steric difficulties of alining long chain molecules are considered. They fold with varying degrees of facility when not co-ordinated with neighbouring molecules and it is unusual to find polymers in solution as straight chains. If such molecules are deposited singly from dilute solutions, they are usually found to exhibit a coiled structure which is to some extent globular when examined by the electron microscope. An additional factor tending to inhibit crystallization is the variation in chain length of individual molecules. Thus to produce a completely crystalline structure from a polymer, it would be necessary to have all the molecules completely alined and all facing in the correct direction. This orientation would depend upon the necessity of a parallel or anti-parallel arrangement in the crystallographic unit cell. Further, all the chains would require to be correctly co-ordinated relative to one another perpendicular to the fibre axis. Such a state of affairs is not likely to be carried to the limit for obvious reasons; furthermore, if it were, the resulting structure would be useless in a textile fibre because of the rigidity of the continuous crystal structure through the specimen. In fact, the flexibility of a textile fibre depends to a very large extent on the relative amounts of its amorphous and crystalline phases. Considerable variations may occur in the crystallinity of these synthetic materials as will be shown later, and the amorphous/crystalline ratio plays a vital part in determining (*a*) the facility with which the processes of extrusion and drawing can be carried out, and (*b*) the properties of the final yarns.

THE PROTOTYPE MOLECULE OF POLYTHENE

The polythene molecule is chemically probably the simplest of the synthetic polymers. Its structure has been established by Bunn[3] and it can be used as a prototype model of a synthetic fibre-forming polymer. The X-ray diffraction photograph of the extruded fibres [Fig. 3(*a*)] shows little orientation, but some crystallinity. When, however, the filaments have been cold drawn, the work done on the fibre is given by the area under the relevant curve of Fig. 1 and is that required to orientate and crystallize the structure into material giving an X-ray diffraction photograph such as Fig. 3(*b*).

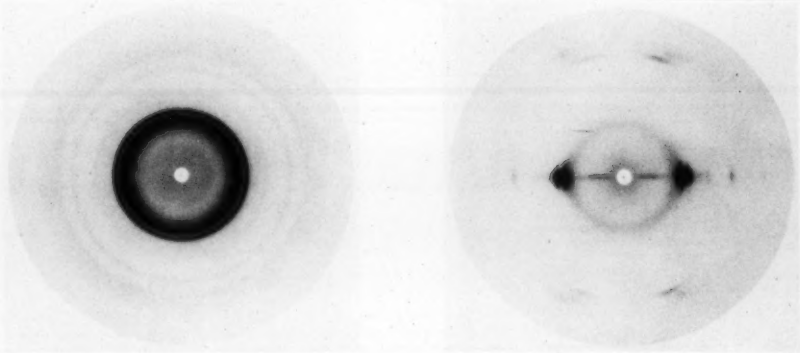
Two processes may be involved in the development of anisotropy in a fibre: (*a*) the fibre may cold draw as in polythene and most other cases of the production of fibres by the melt spinning method. At the yield point there is an almost instantaneous uncoiling and recrystallization from disorientated to orientated molecules, and (*b*) a stretching process in which the orientation and crystallization are achieved gradually. This process is most important in the wet and dry methods of spinning, and to a lesser extent in the case of melt spun polymers.

The spinning and cold drawing of polythene filaments has demonstrated the general principles of the melt spun method, but as its melting-point is in the region of 120° C it is hardly likely that such a fibre would make a good textile yarn capable of manipulation in normal domestic usage. Thus it is clear that to be of universal appeal, polymers with much higher melting-points have



Showing birefringence of polythene and Nylon filaments viewed between crossed Polaroid screens (× 100 approx.)

(*a*) extruded monofil of polythene; (*b*) drawn filaments showing development of birefringence of filaments from yield point during extension; (*c*) fully drawn polythene filament; (*d*) undrawn Nylon filaments showing some slight evidence of birefringence; (*e*) drawn filament of Nylon showing development of birefringence at yield point during extension; (*f*) fully drawn Nylon filament



(*a*) (*b*)

Fig. 3. X-ray diffraction photograph of polythene fibres

(*a*) undrawn, (*b*) drawn

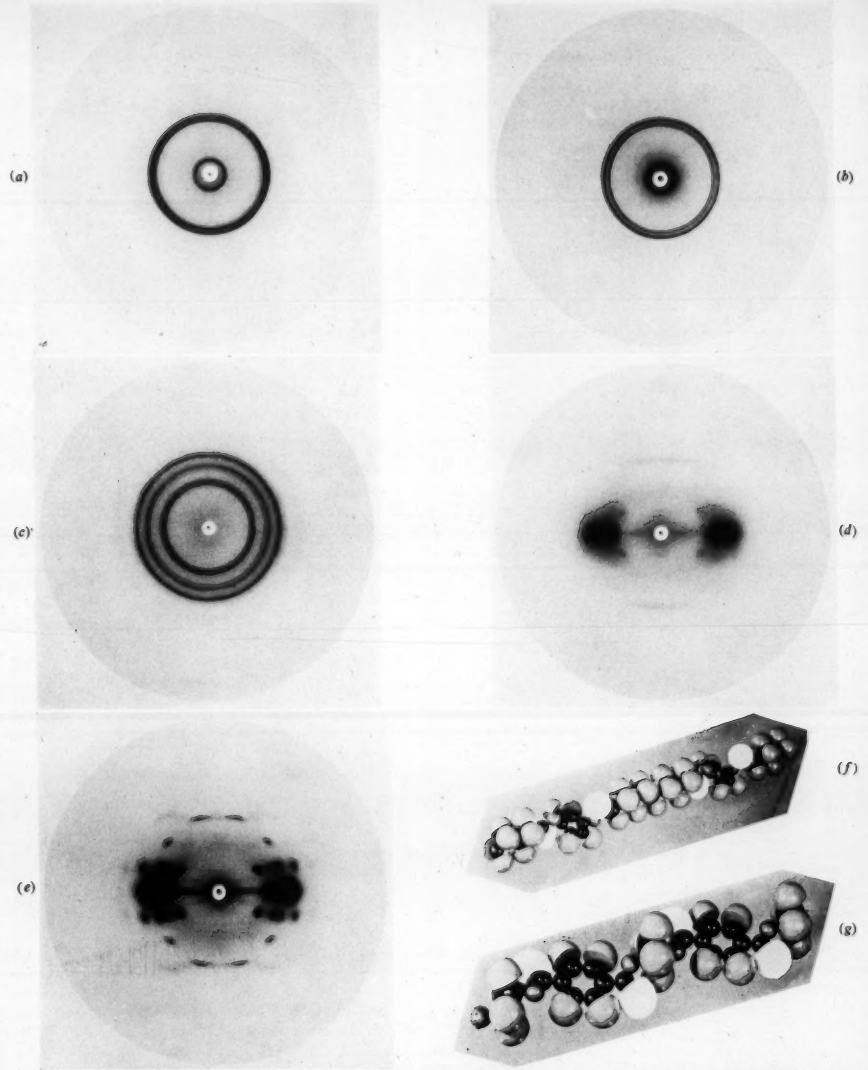


Fig. 4. X-ray diffraction photographs of fibres of polyhydroquinone sebacate and polyethylene terephthalate
(*a*) polyhydroquinone sebacate powder; (*b*) polyhydroquinone sebacate filaments; (*c*) polyethylene terephthalate powder; (*d*) polyethylene terephthalate fibres (Terylene); (*e*) highly crystalline polyethylene terephthalate; (*f*) polyhydroquinone sebacate; (*g*) Terylene

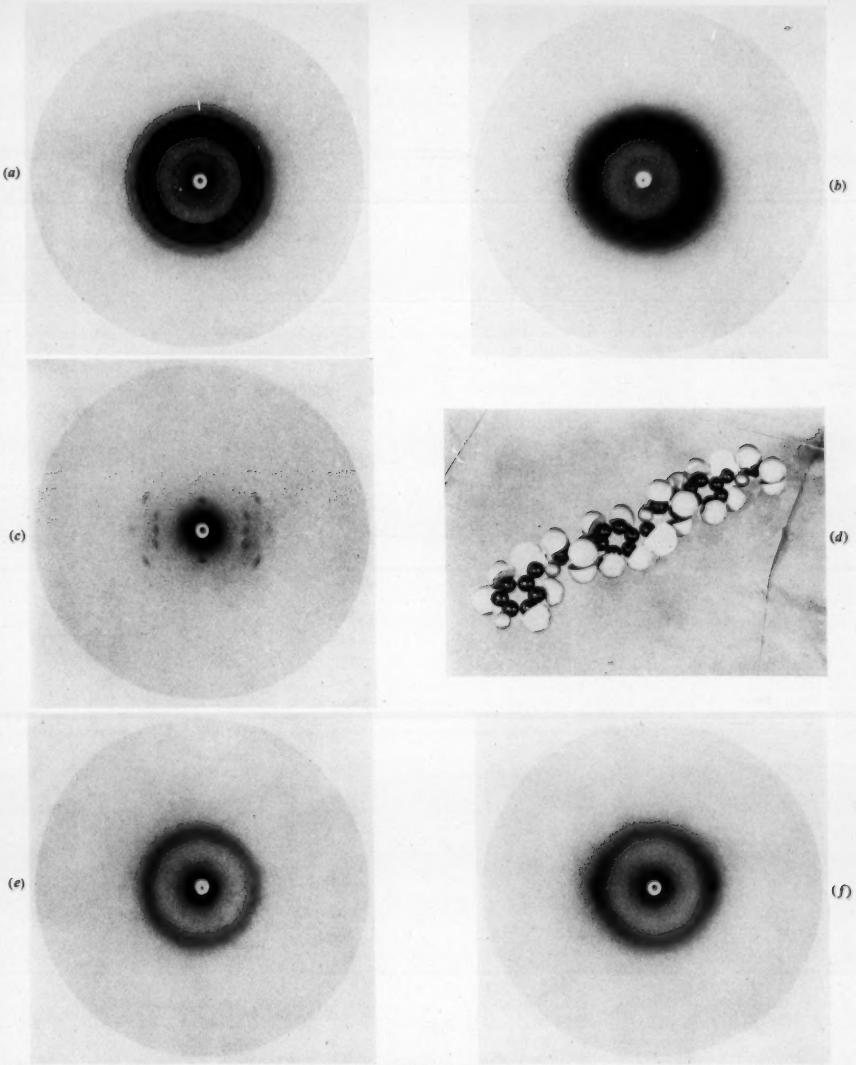


Fig. 5. X-ray diffraction photographs showing the influence of the molecule on the crystallization of polymers after spinning and drawing

(*a*) poly-*p*-*β*-hydroxyethoxy benzoic acid powder; (*b*) spun fibres from poly-*p*-*β*-hydroxyethoxy benzoic acid; (*c*) highly orientated and crystalline fibres of poly-*p*-*β*-hydroxyethoxy benzoic acid made by stretching fibres of 5(*b*); (*d*) poly-*p*-*β*-hydroxyethoxy benzoic acid; (*e*) spun fibres of copolymer of hydroquinone sebacate and *p*-*β*-hydroxyethoxy benzoic acid; (*f*) drawn fibres of specimen of Fig. 5(*e*)

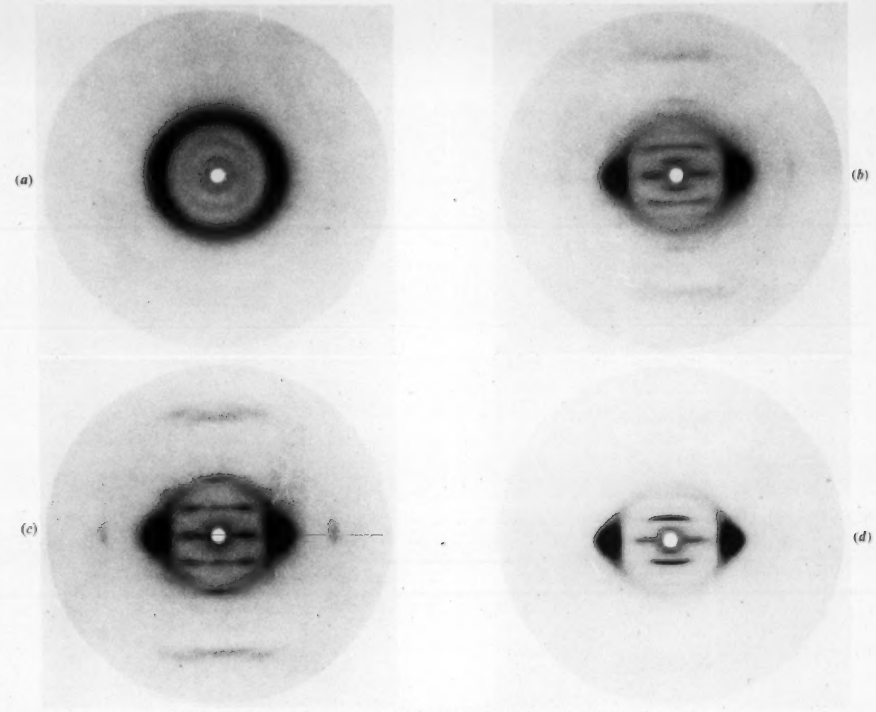


Fig. 6. X-ray diffraction photographs showing the effect of drawing Nylon
(*a*) undrawn Nylon 6·6; (*b*) drawn Nylon 6·6; (*c*) drawn Nylon 6·10; (*d*) drawn Nylon 6

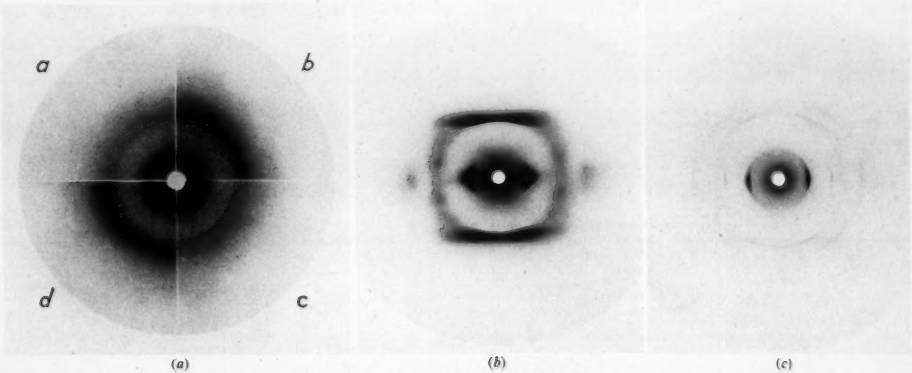


Fig. 9. X-ray diffraction photographs
(*a*) *A* and *C*: α-form of copolymer DL-β-phenylalanine and L-glutamic methyl ester (1 : 1). *B* and *D*: β-form of copolymer.
(*b*) highly crystalline fibres of copolymer of DL-phenylalanine and DL-leucine (1 : 1); (*c*) fibres of L-glutamic γ-methyl ester

to be considered. Examples of these have been found in the polyesters and Nylons,[4] in which the melting-points are in the region of 200° C and higher. Any melt spinning process used, therefore, in fibre formation from these polymers will require an extrusion temperature in excess of this.

THE FIBRE-FORMING POLYESTERS

Fig. 4 shows the X-ray diffraction photographs of fibres of polyhydroquinone sebacate and polyethylene terephthalate (terylene).[5] It can be seen from Figs. 4(*f*) and 4(*g*) that in both cases the molecules can crystallize along a straight line and also appear to have a centre of symmetry in the repeating crystallographic residue. This is probably of very great structural importance in the crystallization of the polymer since there is no differentiation between the parallel and antiparallel arrangements of such molecules. Thus alinement and crystallization along the fibre axis does not depend upon the parallel or antiparallel arrangement of chains as, for example, in Nylon 6. In this polyamide the CO . NH groups break up the centro-symmetrical arrangement of the repeating CH_2 groups found in polythene. In the 6·6 and 6·10 Nylons, however, the centre of symmetry is retained in the molecule as can be seen from the table of polymers. The main factors which might inhibit crystallization in the above polyesters are the staggering of adjacent chains from their positions in a stable crystalline lattice and possible rotation around the molecular axis. In the case of hydroquinone sebacate of the particular batch shown in Figs. 4(*a*) and 4(*b*), the spun polymer appears to have developed a higher degree of crystallinity than that found in the parent polymer. There is, however, no evidence of orientation and the filaments are incapable of being cold drawn. Owing to the high degree of crystallinity developed in the disorientated state, the fibres are of little value for subsequent processing. The Terylene molecule also exhibits a centre of symmetry, but in this case it appears that staggering of the chains, etc., produced on spinning, causes the molecules to assume a less crystalline structure [Fig. 4(*d*)] than in the parent polymer [Fig. 4(*c*)]. As can be seen from Fig. 4(*e*), however, a high degree of crystallization can be achieved in the drawing and final manipulation of the fibre. In the case of this polymer it is possible to produce on spinning an almost amorphous structure, but this will crystallize readily after further manipulation.

From a consideration of Fig. 5, two interesting manifestations of the influence of the molecule on the crystallization of polymers after spinning and drawing are shown. The first three X-ray diffraction photographs show the stages of crystallization in the processes of fibre formation from melt spinning of the self polymer of *p*-β-hydroxyethoxy benzoic acid. This molecule has no centre of symmetry and hence the parallel and antiparallel arrangements of molecules play an important role in the determination of the crystallization of the polymer. On spinning, there is no evidence of crystallization, but this is achieved during the drawing process as can be seen from Fig. 5(*c*). Here, however, the crystallization is not so marked as in the case of the two polymers with centro-symmetrical constituent radicles, and the blackening at the centre of the X-ray diagram indicates the presence of a considerable amorphous phase in the fibres.

Figs. 5(*d*) and 5(*e*) show an interesting phenomenon of the intra-molecular folding of a chain of the copolymer of hydroquinone sebacate and *p*-β-hydroxyethoxy benzoic acid to form an elongated globule which readily packs into a large-scale structure. This large-scale pattern can be identified from the intense low angle spots with a periodicity of about 50 A on the equator of the diagram. The intra-molecular arrangement is almost amorphous, as shown by the diffuse ring at about 4·4 A. When the filament is cold drawn, however, the large angle pattern of the X-ray diffraction photograph becomes diffuse and weak, and the diffuse ring shows an increased intensity on the equator of the X-ray diffraction photograph. Thus it is most likely that the globules are pulling out and the chains alining themselves along the axis of the fibre to give a strengthening of the diffuse reflexion on the equator. The spacing of this reflexion is probably a measure of the mean distance of separation of the individual chains. On orientation an increase in close packing of the chains appears to occur as the spacing of the diffuse spot, 4·2 A, on the equator is less than that of the diffuse ring of the unstretched specimens of about 4·4 A.

THE FIBRE-FORMING NYLONS—INTER-CHAIN HYDROGEN BONDS

Three Nylon specimens have been selected for discussion here: Nylon 6, Nylon 6·6, and Nylon 6·10. These molecules may be considered as polythene chains in which at intervals along the chain the CH_2 groups have been replaced by the CO . NH linkage. This linkage gives rise to the formation of interchain hydrogen bonds which tend to raise the melting-point of the polymer above that of the polythene. The cross linkage by hydrogen bonds between chains has been worked out in detail by Bunn and Garner[6] and other workers,[7] and will not be discussed here. It is, however, of interest to point out that where crystallization occurs in the 6·6 and 6·10 forms of the polymer, the molecules face in the parallel direction while those of the polymer 6 have an antiparallel arrangement in the crystal structure. Crystallization, therefore, can be inhibited to some extent by the molecular arrangement along the fibre axis and some unsaturated C=O or N—H groups are likely to occur in less crystalline parts of the structure for this reason. Also in the 6·6 and 6·10 specimens crystallization appears to occur in sheets with the hydrogen bonds in the plane of the sheet, but these may

Table of synthetic polymers discussed in the text

Material	*Constituent Monomers*	*Resulting Polymer*
Polythene	$CH_2{=}CH_2$	$[-CH_2-]_n$
Nylon 6	$COOH\,.\,(CH_2)_5\,.\,NH_2$	$[-CO\,.\,(CH_2)_5\,.\,NH-]_n$
Nylon 6·6	$COOH\,.\,(CH_2)_4\,.\,COOH$ $NH_2\,.\,(CH_2)_6\,.\,NH_2$	$[-CO\,.\,(CH_2)_4\,.\,CO\,.\,NH\,.\,(CH_2)_6\,.\,NH-]_n$
Nylon 6·10	$COOH\,.\,(CH_2)_8\,.\,COOH$ $NH_2\,.\,(CH_2)_6\,.\,NH_2$	$[-CO\,.\,(CH_2)_8\,.\,CO\,.\,NH\,.\,(CH_2)_6\,.\,NH-]_n$
Terylene	$OH\,.\,(CH_2)\,.\,OH$ $COOH\,.$ $.\,COOH$	$[-O\,.\,(CH_2)_2\,.\,O\,.\,CO\,.$ $CO-]_n$
Polyhydroquinone sebacate	OH OH $COOH\,.\,(CH_2)_8\,.\,COOH$	$[-CO\,.\,(CH_2)_8\,.\,CO\,.\,O\,.$ $.\,O-]_n$
Poly-*p*-β-hydroxethoxy benzoic acid	$OH\,.\,(CH_2)_2\,.\,O\,.$ $.\,COOH$	$[-O\,.\,(CH_2)_2\,.\,O\,.$ $.\,CO-]_n$
Polyglycine	$COOH\,.\,CH_2\,.\,NH_2$	$[-CO\,.\,CH_2\,.\,NH-]_n$
Poly-L-glutamic γ-methyl ester	$COOH\,.\,CH\,.\,NH_2$ $(CH_2)_2$ $COOCH_3$	$\left[-CO\,.\,CH\,.\,NH-\right.$ $(CH_2)_2$ $\left.COOCH_3\right]_n$
Poly-DL-β-phenylalanine	$COOH\,.\,CH\,.\,NH_2$ CH_2 	$\left[-CO\,.\,CH\,.\,NH-\right.$ CH_2 $\left.\right]_n$
1·1 copolymer of DL-β-phenylalanine + L-glutamic γ-methyl ester	Monomers, as above	$COOCH_3$ $(CH_2)_2$ $[-CO\,.\,CH\,.\,NH\,.\,CO\,.\,CH\,.\,NH-]_n$ CH^2

be layered in two ways on top of one another to form the α and β types of packing. The former type of packing in Nylon appears to be the more common form in the 6·6 polymer and its triclinic unit cell contains only one molecular chain. In the latter form, however, there are two molecules in the triclinic unit cell and in this case the sheets of molecules are arranged in pairs. The higher degree of crystallinity of the Nylon 6, compared with the 6·6 and 6·10 forms, can be seen from Fig. 6 and in this former case the adjacent chains are arranged in an antiparallel direction in the crystalline lattice.

The process of cold drawing Nylon 6·6 from spun to orientated fibre is illustrated in Figs. 6(*a*) and 6(*b*), and as has been pointed out earlier, the work required to produce the highly orientated structure can be obtained from the area under the load/extension curve of Nylon (Fig. 1). The load/extension curves of the three finished Nylon yarns are shown in Fig. 7 in which the similarities between the specimens can be noted. As the methods of spinning, etc., are not likely to have been comparable, a detailed comparison of the tenacities and extension at break of the fibres cannot be made with any certainty. It is, however, of interest to note that in this particular

case the highly crystalline Nylon 6 has a greater tenacity than the other two specimens. A comparison between the crystallinity of drawn samples of Nylon 6·6 and polythene can be made from a study of the appearances of the two fibres between crossed polaroid screens during and after drawing (shown in the colour plate). The birefringence of the Nylon sample is obviously much greater than that of polythene in the finally orientated specimens.

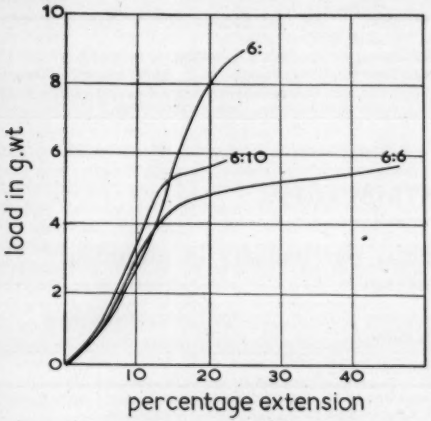
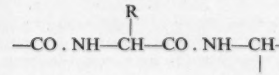


Fig. 7. Load/extension curves of Nylons 6, 6·6, and 6·10 at normal humidities for original filament deniers of 1 (constant rate of loading)

THE POLYPEPTIDE MOLECULES—THE INFLUENCE OF THE α AND β FORMS ON SOLUBILITY AND FIBRE FORMATION

Brief mention can be made here of some work which has recently been carried out in collaboration with Dr. Bamford and Mr. Hanby of Courtaulds' Laboratories, Maidenhead.[8] It concerns the synthetic polypeptide molecules formed by the condensation and self-polymerization of the α amino acids of the type $COOH.NH_2—CH.R$, where R is the side chain of the polypeptide molecule which on polymerization takes the form



This work is only in the early stages of development, but it has been possible to show that polymers such as polypeptides containing potentially hydrogen bonding amino acid residues can be put into solution in certain cases in non-polar organic solvents which are incapable themselves of forming or breaking hydrogen bonds. It has been shown that a single amino acid radical of a polypeptide is capable of taking part in the formation of two hydrogen bonds. When these bonds are interchain in character, the molecule is found to be in the β configuration. Compounds possessing such a structure have in many cases in the past been found difficult to dissolve in any solvent. The polypeptide molecules are, however, capable of folding in quite a definite manner as is demonstrated (Fig. 8), and the fold is referred to as α_{II}. This configuration appears to be common to all the synthetic polypeptides examined in these laboratories with a few exceptions, e.g. polyglycine and probably polyalanine, or others with short side chains. In the

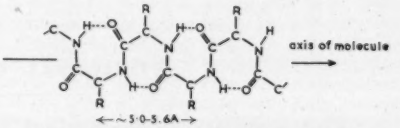


Fig. 8. Diagram of α_{II} synthetic polypeptide ribbon

folded forms, the present weight of evidence appears to be in favour of a molecule with ribbon-like structure in which the hydrogen bonds are intra-chain in character. As can be seen from Fig. 8, the structure involves two amino acid residues per ring, with the ring and side chains approximately in the plane of the polypeptide ribbon. The orientated and highly crystalline α_{II} forms of the synthetic polypeptide of *L*-glutamic methyl ester, and that of the less crystalline copolymer of DL-β-phenylalanine and DL-leucine give the X-ray diffraction photographs shown in Figs. 9(*b*) and 9(*c*). Fig. 9(*a*) compares the X-ray diffraction photographs of the α and β forms of the copolymer *L*-glutamic methyl ester and DL-β-phenylalanine, the β form having been prepared from the α by treatment with formic acid. The differences in solubility in benzene of the two modifications of the same material can readily be demonstrated; the film of the α form passing into solution while that of the β remains swollen but insoluble in the benzene. These polypeptides are likely to be of the greatest importance in the near future as they may form the basis of an entirely new family of synthetic polymers. They all appear to have the same skeleton structure based on the α_{II} fold, but with various properties of solubility, reactivity, etc., controlled mainly by the chemistry of the side chains. In all cases these ribbon-like molecules are capable of giving long straight crystallites with fibre-forming characteristics, and the more they are studied the more is one led to the conclusion that the α_{II} fold of the synthetic polypeptides and the α fold of the keratin-myosin-epidermin-fibrinogen group of proteins are basically similar in character.

Thus, apart from their possibilities in the field of synthetic fibres, it is felt that a fundamental study of their molecular structure and properties may give information capable of application in the wider and more complex problems of the naturally occuring polypeptides.

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ORIGINAL CONTRIBUTIONS

The stability of analytical weights, particularly in chemical laboratories

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Analytical weights of various modern materials, including a large proportion of plated types, have been subjected to accelerated corrosion and stability tests in five representative chemical laboratories. As regards stability of mass there was little to choose between the weights of austenitic stainless steel (25% Cr, 20% Ni), non-magnetic nickel-chromium (80% Ni, 20% Cr), and the plated weights having a nominal thickness of plating of about 0·015 mm (0·0006 in) or more; some weights of specially highly-polished stainless steel were, however, slightly superior. Chromium-plated weights retained their appearance better than any except perhaps those of highly-polished stainless steel and nickel-chromium, but they are open to the objection that the underlying nickel is slightly magnetic. No kind of weight appeared to be much more suited to one laboratory than to another. Lead should not be used for the adjustment of weights whether they are likely to be exposed to corrosive atmospheres or not. The paper also includes information on the behaviour of weights of various materials under good conditions.

It is very desirable that analytical weights should remain unchanged in mass over long periods. They should, therefore, be little affected by the atmospheric conditions under which they are used. The plain polished, lacquered, or thinly-plated brass weights often employed are too frequently unstable even under good working conditions. However, weights of better quality are now available. These present a variety of choice, and it is important to know their respective merits. Manley[1] determined the mass of the removable film formed on various kinds of weights during storage under good conditions, but this is not necessarily an indication of their stability. Some relevant information has been accumulated at the National Physical Laboratory in connexion with its responsibilities for testing national, scientific and industrial standard weights, but little has been known of the stability of weights in the contaminated atmospheres of chemical laboratories, where they are being used increasingly. The investigation which it is the chief purpose of this paper to report was undertaken mainly to obtain a reliable basis for judging the relative merits of weights used under such conditions.

1. PRELIMINARY SURVEY

Changes of mass of analytical weights can be ascribed to four main causes, viz. (1) corrosion, (2) adsorbed films and filmy deposits, (3) wear, and (4) release of occluded gas. The first two result in increase in mass. At the N.P.L. weights are brushed lightly before use to remove loose particles such as dust, but they are not otherwise conditioned. Except where stated to the contrary, the results given in this paper relate to weighings made under these conditions. Wear is an obvious

cause of loss of mass. Release of gas may be responsible for very small but regular decreases of mass, and further reference to this is made later.

Experience with the national standards of mass and platinum fractional weights indicates that weights of platinum, or better still platinum-iridium which is harder, are undoubtedly the most stable under good conditions, and there is no reason to suppose that weights of these materials would not be the best in corrosive atmospheres also. However, their cost limits their use, and recourse must be had to less expensive materials. Weights of non-magnetic nickel-chromium (80% Ni, 20% Cr), a material first introduced for this purpose by the N.P.L., have proved themselves very stable in mass under good conditions. For example, none of the five N.P.L. working standard 100 g weights of this material has changed by more than 0·16 mg since first used, and of the three oldest, made 28 years ago and used almost daily since then, none has changed more than 0·04 mg. All these weights have lost mass, and in the case of the oldest standards the losses must be attributed to wear, which presumably masks any additive effects which may have occurred. The greater reduction in mass of the two more recent standards, despite the fact that these weights have been used very much less in their 26 years of existence, possibly indicates loss of occluded gas. As an example of the more usual increase in the mass of little-used weights, at the National Bureau of Standards, Washington, 100 g weights of the same material remained stable to within about 0·01 mg under good conditions of storage over two years.[2]

Weights of non-magnetic nickel-chromium have been commercially available in this country since 1939. Stainless steels have long had a high reputation in other fields for their resistance to corrosion, and recently weights of austenitic stainless steel (25% Cr, 20% Ni) have also become available following recommendations by the N.P.L., after tests on the magnetic properties of stainless steels of various types.[3] Three highly-polished 100 g one-piece weights of this material made at the N.P.L. in 1945 and kept since then under good conditions of storage have remained constant to within the accuracy of their standardization, viz. $\pm$ 0·01 mg.

Nickel-chromium and stainless steel weights have largely replaced protected or unprotected weights of brass for work of high quality. Weights of plain polished brass would not be expected to stand up to severe atmospheric conditions, but even under good conditions such weights, in addition to tarnishing slowly and more or less uniformly, frequently show signs of incipient corrosion which manifests itself in local pitting. When at all severe this gives the weight a speckled appearance, and noticeable increase in mass is associated with this phenomenon even in its early stages.

Various coating materials are available for the protection of plain brass. Lacquer has been used, but is usually hygroscopic. Gold is probably the oldest of the plating materials, and gilt weights are in general more satisfactory than plain polished weights. Gold plating, however, often contains "pin-holes," and, moreover, gold is both dense and soft; hence gilded weights are particularly liable to changes of mass due to wear. Gold for plating has been largely superseded by chromium, rhodium, platinum, and nickel, the last particularly on weights made abroad. Nickel tends to "fog" severely,[4] and is slightly magnetic (see section 4). Though Ward[5] made high claims for the corrosion resistance and stability of chromium-plated weights in service, no firm data have hitherto been available in this regard. Moreover, the occasional discovery at the N.P.L. of plated weights whose values are changing rapidly for no apparent cause has provided good reason for viewing all plated weights with suspicion, although some platinized standard weights acquired by the N.P.L. in 1922 have proved extraordinarily stable.

As to the behaviour of weights in corrosive atmospheres such as are found in chemical laboratories, there is practically no published information. Reference may be made, however, to the experience of the National Bureau of Standards[2] when nickel-chromium weights were submitted to very severe artificial conditions, including exposure in a closed space to the fumes from a 20% solution of hydrochloric acid and in a chemical laboratory where acid fumes were strong. It was then concluded that such weights were suitable for high-grade analytical sets, though not for the highest classes of standards.

2. ACCELERATED CORROSION TESTS

In order to acquire quickly some knowledge of the resistance to corrosive atmospheres of weights of various materials, a representative group of 100 g weights was exposed for two periods of three months over the working bench in each of five different chemical laboratories around London.

The weights. During the first period each group comprised one weight of each of the first nineteen varieties described in Table 1. The platinum- and rhodium-plated weights were considered, in consultation with the platers, to cover the range likely to be met on the open market. There was some difficulty in obtaining suitable chromium-plated weights, but in view of the claims made for such weights their inclusion in the trials was nevertheless considered essential, and the instructions given to the firm which eventually undertook the plating were based on British Standard 1224: 1945, "Electroplated Coatings of Nickel and Chromium on Steel and Brass." The thicknesses of plating quoted in Table 1 were those specified, but it is not known precisely how closely the platers succeeded in realizing them, and there was evidence that the plating was relatively thin in the neighbourhood of the highly-curved edges of some of the weights.

During the second period the original group of weights

Table 1. *Weights exposed*

	Form	*Surface condition*
Unplated weights:		
Austenitic stainless steel (25% Cr, 20% Ni)	Plain cylinder, approx. 25 mm × 25 mm	Turned
Austenitic stainless steel (20% Cr, 10% Ni)	Plain cylinder, approx. 25 mm × 25 mm	Ground
Non-magnetic nickel-chromium (80% Ni, 20% Cr)	Screw-knob weight, tall form	Polished
Plain brass (Delta metal)	Screw-knob weight, squat form	Polished
Plated brass weights: material and thickness of plating:		
Platinum (0·8μ) on silver (25μ) on copper flash		
Platinum (0·4μ) on silver (25μ) on copper flash		
Platinum (2·5μ)		
Platinum (0·8μ)		
Platinum (0·4μ)		
Rhodium (0·8μ) on silver (25μ) on copper flash		
Rhodium (0·4μ) on silver (25μ) on copper flash		
Rhodium (2·5μ)	Screw-knob weight, squat form	Highly polished
Rhodium (0·8μ)		
Rhodium (0·4μ)		
Chromium (0·6μ) on nickel (30μ)		
Chromium (2·0μ) on nickel (15μ)		
Chromium (0·6μ) on nickel (15μ)		
Chromium (2·0μ) on nickel (10μ)		
Chromium (0·6μ) on nickel (10μ)		
Additional unplated weights, exposed during second period only, in three of the five laboratories:		
Austenitic stainless steel (25% Cr, 20% Ni)	Plain cylinder, approx. 25 mm × 25 mm	Highly polished
Non-magnetic nickel-chromium (80% Ni, 20% Cr)	Plain cylinder, approx. 25 mm × 25 mm	Highly polished

Note.—10μ = 0·01 mm = 0·0004 in approx.

Table 2. *Changes of mass on exposure over three-month periods (average for five laboratories)*

	Mean changes (milligrammes)		
	During first exposure		*During second exposure*
Unplated weights:			
Austenitic stainless steel (25% Cr, 20% Ni), turned finish	0·2		0·3
Austenitic stainless steel (25% Cr, 20% Ni), highly polished*		0·05	
Austenitic stainless steel (20% Cr, 10% Ni), ground finish	1·2		0·8
Non-magnetic nickel-chromium (80% Ni, 20% Cr), polished	0·4		0·4
Non-magnetic nickel-chromium (80% Ni, 20% Cr), highly polished*		0·3	
Plain brass, polished	1·8		1·7
Plated brass weights:			
Chromium (0·6μ) on nickel (30μ) on brass	0·2		0·3
Rhodium (0·8μ) on silver (25μ) on a copper flash on brass	0·4		0·4
Platinum (0·8μ) on silver (25μ) on a copper flash on brass	0·2		0·3
Rhodium (0·4 μ) on silver (25μ) on a copper flash on brass	0·4		0·5
Platinum (0·4μ) on silver (25μ) on a copper flash on brass	0·4		0·3
Chromium (2·0μ) on nickel (15μ) on brass	0·3		0·4
Chromium (0·6μ) on nickel (15μ) on brass	0·3		0·2
Chromium (2·0μ) on nickel (10μ) on brass	0·4		0·3
Chromium (0·6μ) on nickel (10μ) on brass	0·5		0·5
Rhodium (2·5μ) on brass	1·2		1·4
Platinum (2·5μ) on brass	1·0		1·2
Rhodium (0·8μ) on brass	1·1		1·5
Platinum (0·8μ) on brass	1·2		1·2
Rhodium (0·4μ) on brass	1·2		1·4

* First exposed during the second exposure of the remaining weights.

in each of three laboratories was supplemented by more highly-polished weights of 25/20 austenitic stainless steel and non-magnetic nickel-chromium (see foot of Table 1).

Housing and location. In each laboratory the weights were housed during exposure in a two-tier container designed to permit free ventilation while preventing the entry of gross dust. The weights were placed symmetrically so as to provide closely similar conditions of exposure for each. The containers were made of aluminium, anodized to avoid corrosion, and the weights were placed on several thicknesses of acid-free tissue paper.

Each of the five containers, with its complement of weights, was suspended slightly above head level over a working bench in one of the five chosen laboratories.

Basis of assessment of corrosion. As changes of mass were the main concern in this investigation the values of the weights were determined on the following occasions:

(*a*) on receipt of the weights from the manufacturers;
(*b*) immediately prior to the first exposure period;
(*c*) between the two exposure periods;
(*d*) after the second exposure period.

Their values were also determined after the weights had been housed for 24 h in a desiccator (relative humidity 10%) immediately following weighing (*a*), because instability in weights can sometimes be detected in this way, though in this instance only one of the weights changed by as much as 0·1 mg. Before each weighing the weights were brushed with a camel-hair brush. All the weighings were carried out at the N.P.L. to an accuracy within $\pm$ 0·02 mg by comparison under good conditions with standard weights of proved stability.

3. OBSERVED RESULTS

Appearance. The appearance of most of the weights suffered markedly on account of the severity of the test conditions. The chromium-plated weights were the least affected in appearance. This may have been due in part to the fact that the underlying deposit, nickel, changes much less in appearance on atmospheric corrosion than does silver or brass, the metal immediately beneath the rhodium and platinum. The 25/20 stainless steel and the nickel-chromium weights were also little affected, though these did show some local pitting. The ground-finished 20/10 stainless steel weights rusted heavily, and those of brass tarnished severely. The highly-polished stainless steel and nickel-chromium weights which were introduced for the second period of exposure only, showed little deterioration. The bases of all the weights were almost completely unaffected, and there was a most noticeable reduction in corrosion towards the bottom of the flanks, but elsewhere corrosion was often marked in the neighbourhood of edges where, owing to the high curvature, excessive amounts of metal had probably been removed unavoidably when buffing after plating.

Changes of mass. Table 2 gives the mean changes for each type of weight. The table is based only on those weights which had been stable over at least three months prior to exposure; eleven plated weights, to which reference will be made later, gained appreciably during this period and their subsequent changes were therefore excluded from the analysis. Most of the weights increased in mass during exposure. Some of those of nickel-chromium and stainless steel decreased between the last two weighings, probably owing to the loss of corrosion products, but Table 2 is based on average changes, irrespective of sign. As a matter of principle, to avoid giving undue importance to the results from any one laboratory, the observed changes in the individual weights in a given laboratory were adjusted on the basis of their sum and the corresponding sums in the other laboratories to bring the total change in each laboratory to the average level. The adjustment, however, made no difference to the final conclusions. The relative severity of the five laboratories as indicated by the mean changes of mass was as follows:—

Table 3. *Relative severity of laboratories*

Laboratory	*Mean change during first exposure*	*Mean change during second exposure*
A	0·4 mg	1·2 mg
B	1·5	0·2
C	0·2	0·1
D	1·4	1·3
E	2·8	1·0

A continuous record of the humidity in each laboratory revealed no unusual conditions. Except on one day in each of two laboratories the average humidity remained below 70%, a level above which the tendency towards corrosion markedly increases.[6]

The relative changes in mass are of more concern than their absolute magnitude, which was very large in some laboratories owing to the severity of the conditions there. Table 2 shows that the weights of 25/20 stainless steel, 80/20 nickel-chromium, and the more heavily-plated weights experienced the least change of mass, and there is little to choose between them. The highly-polished 25/20 stainless steel was the least affected, and indeed none of the three weights of this material and finish changed by as much as 0·1 mg though exposed in the three laboratories which affected the other weights the most. The ground-finished 20/10 stainless steel weights deteriorated very badly during the first test, and it was not thought worth while to prepare highly-polished weights of this material, particularly as it is not so good magnetically as the 25/20 variety.[3]

As regards plated brass weights, it is clear from Table 2 that their stability depends more on the total thickness of plating than on the material. A coating of a few microns (about 0·0001 in) of platinum or rhodium provides little protection, but when backed by a substantial thickness of underplating, coatings of these metals, and of chromium, are very resistant. A total thickness of about 15μ (0·0006 in) of protective plating material would appear to be called for. According to the table, platinum on silver is very slightly more effective than rhodium on silver, and chromium on nickel is just better than either. However, this may not present an entirely fair comparison because the thicknesses of plating quoted are nominal, not measured, and much must depend on the particular distribution of the plated metal over the surface. Moreover, any advantage possessed by chromium tends to be offset by the magnetic properties of the underlying nickel (see below), and even if platinum is in fact slightly more resistant to corrosion than rhodium, it is softer and therefore more liable to wear. It would therefore be unwise to infer that one type of heavily-plated weight is definitely superior to another.

The results given so far apply, of course, under the average conditions encountered, because the analysis is based on the average change for each particular type of weight, but it is of interest to know whether the stability of any particular kind of weight was outstandingly good or bad under the particular conditions in one or other

Table 4. *Examination of the dust accumulated on the paper disks on which the weights rested during exposure*

Laboratory	*Exposure*	*Tier*	*Weight of dust (mg)*	*Sulphide*	*Sulphate*	*Chloride*	*Nitrate*
A	1st	upper	0·5				
		lower	0·5	s	s	tr	v.s
	2nd	upper	0·4	s	tr	s.tr	s
		lower	0·5	s	s	v.s.tr	v.s.tr
B	1st	upper	1·3				
		lower	1·5	v.s	s	d.tr	v.s
	2nd	upper	2·8	tr	tr	d.tr	s.tr
		lower	1·9	tr	tr	d.tr	d.tr
C	1st	upper	0·0	Quantity insufficient for analysis			
		lower	0·0				
	2nd	upper	0·1	tr	s.tr		
		lower	0·1			d.tr	d.tr
D	1st	upper	5·5				
		lower	6·8	s	s	s	v.c
	2nd	upper	0·2	tr	tr	s.tr	m
		lower	1·2	s	m	m	m
E	1st	upper	2·0				
		lower	3·3	s	s	s	s
	2nd	upper	2·9	c	m	s.tr	tr
		lower	1·5	m	m	m	tr

v.c = very considerable amount
c = considerable amount
m = moderate amount
s = small amount
v.s = very small amount

tr = trace
s.tr = small trace
v.s.tr = very small trace
d.tr = doubtful trace

The diameter of each paper disk was 25 cm.

Table 5. *Effect of lead-loading on stability under good atmospheric conditions*

Identification number of weight	*Approximate amount of lead present (g)*	*Increase in mass (mg)*	*Period (months)*
3C5	0	0·0	6
3P4	0	0·05	6
3R4	0	0·1	14
2P3	1·5	0·1	6
1P5	2·5	0·2	6
5C2	3	0·2	4
4P3	4	0·4	6
2P1	4	0·4	6
5P1	5	0·8	6
2R2	5	0·9	6
1P3	5	7·4	6

of the laboratories. While it was not found practicable to assess reliably the quality of the atmospheres in the respective laboratories over the long period involved, some indication of the atmospheric conditions can be inferred from Table 4. No reliable correlation was found between the different chemical characteristics of the individual laboratories and the changes in the particular weights exposed in them. This was further illustrated by the fact that on arranging the different types of weights in order of merit on the basis of their changes in mass, substantially the same order was arrived at whether the arrangement was based on the *average* change of each particular type of weight, or the *greatest* change among the weights of a given type.

An incidental outcome of the tests was a clear demonstration that lead should not be used for the adjustment to nominal mass of screw-knob weights. Tin was used for this purpose in most of the weights, but some of them were so deficient in mass before final adjustment that lead was used on account of its higher density. The lead was in the form of shot, tightly rammed home, and in some instances a large quantity was necessary. The strong correlation between the amount of lead used and the increase in mass of those weights which proved to be unstable prior to exposure is illustrated in Table 5, which includes for purposes of comparison some stable weights taken at random. Lead is readily corrodible, and full conversion into the basic carbonate $2PbCO_3 . Pb(OH)_2$ is accompanied by a 25% increase in mass, so that the presence within a weight of very little lead may cause significant changes in mass. It is not known why the weight 1P3 (Table 5) changed so much more than any of the rest, but it is noteworthy that the lead within it was found, on inspection after the trials, to be much more liberally covered with powder than that in 2R2. It is possible that the increase was not entirely due to the lead, and this recalls instances within the experience of the N.P.L. in which other plated weights, entirely free from loading material, have changed very appreciably without discernible cause. In the present instance, however, the makers had been asked not to omit the normal process of boiling the weights and knobs separately in distilled water to remove all remaining traces of plating salts. Hence it is unlikely that such a large change as 7·4 mg is substantially due to such residues. To a large extent, therefore, the cause remains obscure. As already stated, these unstable weights, eleven in number, were excluded from the analysis of the results.

4. MAGNETIC PROPERTIES OF NICKEL-PLATED BRASS WEIGHTS

Any slight advantage as regards stability and maintained good appearance possessed by weights plated

with chromium on a nickel undercoating tends to be offset by the magnetic properties of the nickel, because weights, pan-arrests and balance pointers may quite unsuspectedly be in a state of magnetization. For example, after temporary magnetization in a field of approximately 800 oersteds of a number of the weights used in the stability experiments, the residual magnetic moments per unit volume were found to range from 0·16 to 0·33 c.g.s. units. (These values appeared not to be related to the nominal thickness of the nickel layer, although the permeabilities also determined were broadly consistent with it.) The residual intensities of magnetization may be compared with those reported by Gould.[3] They are considerably greater than the largest (0·03) for brass weights, but of a smaller order than many for materials which were judged to be unacceptable magnetically (18% Cr, 8% Ni stainless steel, for example). While they are not so large as to condemn the use of nickel on plated weights, they do tend to offset any slight advantage which chromium-plated weights may have as regards stability and maintained good appearance. It would not be satisfactory, however, to omit the nickel undercoating and increase the thickness of chromium appropriately, as the corrosion resistance of thick chromium plating deposited with a lustrous surface is generally inferior.

5. CONCLUSIONS

(1) Under good atmospheric conditions, analytical weights of austenitic stainless steel (25% Cr, 20% Ni) and of non-magnetic nickel-chromium (80% Ni, 20% Cr) can be relied upon to be extremely stable. Good quality platinized brass weights have been known to be extraordinarily stable and it is probable that rhodium-plated weights can be equally stable; in any particular case it should not be assumed without good evidence, however, that a given plated weight is satisfactorily stable for normal use, and the absence from within it of corrodible adjusting material is not sufficient evidence of stability. Plain polished or lacquered brass weights are unsuitable for laboratory standards. As for nickel-plated and gold-plated weights, there are a number of superior alternatives.

(2) In the corrosive atmospheres encountered in these experiments there was little to choose between the weights of austenitic stainless steel (25% Cr, 20% Ni), non-magnetic nickel-chromium (80% Ni, 20% Cr), and the plated weights having a nominal thickness of plating of about 0·015 mm (0·0006 in) or more; the highly-polished stainless steel weights were, however, slightly superior. Brass weights coated with a "flash" of platinum or rhodium were not very superior to weights of plain polished brass.

(3) Among the plated weights, the appearance of those plated with chromium over nickel deteriorated the least, but any superiority in stability of mass over that of weights having the same total thickness of plating of platinum on silver or rhodium on silver is only slight, and is offset by the magnetic properties of nickel.

(4) No reliable correlation was found between the different chemical characteristics of the individual laboratories and the changes in the weights of the various materials exposed in them.

(5) Owing to its high corrodibility, lead should on no account be used for the adjustment to nominal mass of screw-knob weights.

(6) While it is always advisable to avoid sharp edges on weights and to provide a fairly broad seating, this becomes most important when preparing weights for plating, owing to the difficulty of buffing in the neighbourhood of sharp edges without removing excessive amounts of metal.

6. ACKNOWLEDGMENTS

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Efficiency and mechanism of barium getters at low pressures

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The gettering rates of barium getters for oxygen, nitrogen, hydrogen, carbon-monoxide, carbon-dioxide and water vapour are measured in the pressure range between 5×10^{-8} and 5×10^{-5} mm of mercury. Values for the decay in gettering rate during use of the getter are given. It is shown that the gettering effect at these low pressures is almost entirely due to the take-up of atoms or metastable molecules produced by the impact of electrons on the stable gas molecules. Ions play a negligibly small part in the gettering process. The position of the getter with respect to the electronic discharge influences the getter efficiency due to two causes: re-formation of stable molecules and flow resistance between discharge and getter. Examples for the two effects which affect different gases in different ways are given.

In a previous publication[1] a method for determining the efficiency of getters at low pressures has been described. This method, in the meantime, has been used for a detailed investigation of barium getters. The efficiency of these getters has been determined as a function of a number of different parameters and an attempt has been made to find out the mechanism on which the gettering process is based.

EXPERIMENTAL DETAILS

The measurements have been carried out by using a new type of ionization gauge which has recently been described by Metson.[2] This gauge is designed in such a way that the photoelectronic current which is produced by soft X-rays impinging upon the collector electrode, is suppressed. The gauge, therefore, has no radiation level by which the measurement of low pressures is limited, and pressures down to 10^{-8} mm of mercury and less can be measured accurately. Two subtypes of this suppressor gauge have been used, one supplied with a tungsten cathode and the other with an oxide cathode (see Fig. 1). The calibration of these gauges which was undertaken for a number of different gases by means of a Knudsen gauge will be described in a separate publication. During measurements with the oxide cathode gauge the cathode was kept at a high temperature ($\simeq 1\,250°$ K) and it was verified by a special experiment that the gettering action of the cathode itself is negligibly small at such temperatures. The barium getters employed for the measurements were *KIC* getters* which were located in one of the three positions *A*, *B* or *C* shown in Fig. 1. Normally two getters on either side of the tube were used. Before evaporating the getters on to the glass wall they were outgassed at a dull red heat. The evaporation, during which the pressure in the tube was of the order of 10^{-5} mm, was undertaken either by eddy current heating or by directly passing current through the getter wire. After evaporation the getter covered an area of between 4 and 6 cm^2 on either side of the tube.

The pumping unit, used for these measurements, consisted of a rotary pump and two diffusion pumps in series operated with silicon oil. The use of liquid air traps in between diffusion pumps and manifold was proved unnecessary. Only a water-cooled baffle was employed, and a pressure of 10^{-7} mm was readily obtained in the manifold after the initial bake at 400° C. Before starting the measurements, the components of the ionization gauges were outgassed by eddy current heating and electron bombardment in the usual way.

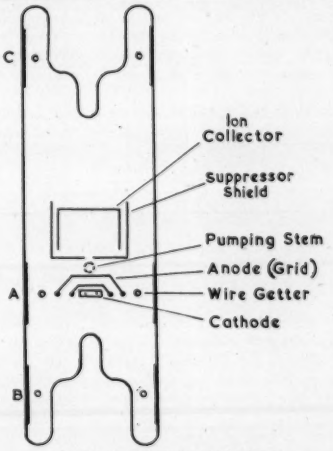

Fig. 1. Experimental tube

It has been attempted to extend the measurements to most of the gases which are known to be residual in electronic valves. According to recent mass spectrometric measurements by Plumlee and Smith,[3] oxygen, carbon monoxide, carbon dioxide and water vapour are the main constituents of the residual gas; hydrogen certainly must be added to these. In addition to the five gases mentioned, nitrogen was examined which, although playing no important part in the residual gas, showed some characteristic properties with respect to the gettering process. Hydrocarbons which are also of interest have not yet been included. The gases oxygen, nitrogen, hydrogen, carbon dioxide were taken from steel cylinders and, apart from drying by phosphorus pentoxide, no purification was attempted. Water vapour was obtained from distilled water, while carbon monoxide was

* See Reference (1), page 228.

prepared by heating a mixture of calcium carbonate and zinc powder (Weinhouse[4]). In this latter case the gas was passed through a liquid air trap to avoid a possible contamination by CO_2.

GETTERING RATES OF DIFFERENT GASES

The pressure p_g in the tube containing the getter (see [1], Fig. 2) was measured as a function of the manifold pressure p_{man} of the pump. It was observed that the two gases, oxygen and nitrogen, behaved quite differently. The ratio between the two pressures p_{man} and p_g was high for oxygen but low for nitrogen, showing that the gettering rate G, as obtained from equation (3) of [1], was large for oxygen but small for nitrogen. Furthermore, the pressure p_g measured above the getter for constant manifold pressure p_{man} was fairly constant for oxygen but increased with time for nitrogen. Finally, if a plot of log p_g against log p_{man} was drawn (see Fig. 2) a straight line was obtained with good approximation for oxygen, while for nitrogen only the points measured

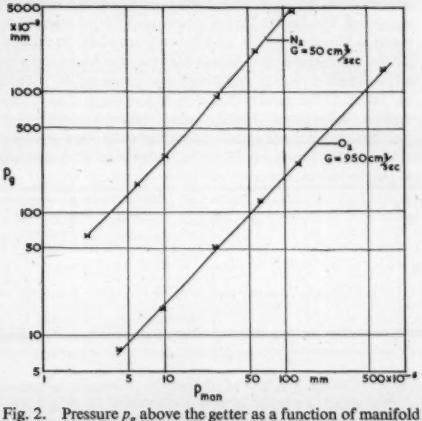

Fig. 2. Pressure p_g above the getter as a function of manifold pressure p_{man} for oxygen and nitrogen (getter in central position *A*, see Fig. 1)

for low pressures lay on a straight line and this line curved towards higher pressures when the manifold pressure exceeded 10^{-5} mm. The slope of the straight line obtained for oxygen was always about 45°, thus indicating that the gettering rate of oxygen is nearly independent of pressure.

When further gases were examined it was found that they could be grouped into one of two classes of which oxygen and nitrogen are representatives. All the gases containing oxygen (carbon monoxide, carbon dioxide, water) were found to belong to the same class as oxygen. Their gettering rates are high (> 700 cm³/sec), are nearly independent of pressure and remain fairly constant over several minutes. In the second class, however, to which hydrogen belongs in addition to nitrogen, the gettering rates are low ($\leqslant 300$ cm³/sec), and decrease with increasing pressure and with time. Average values of the gettering rates measured for different gases have been given in Table 1. While oxygen and nitrogen have been measured extensively (sixteen values for oxygen and nine values for nitrogen), only a few measurements (one or two) have been undertaken for the other gases. These latter values, therefore, cannot claim a high accuracy. In order to give some idea of the attainable accuracy it may be noted that the values for oxygen vary between 740 and 1 350 cm³/sec with a standard deviation from the average of 40 cm³/sec.

Table 1. *Average values of gettering rates G for different gases*

Central position (*A*), $I_A = 10$ mA. Initial values (no decay)

Type of gas	O_2	CO	CO_2	H_2O	H_2	N_2
G in cm³/sec	1 030	1 250	3 000	2 300	250	80
Measured at p_g	$2{\cdot}5 \times 10^{-7}$	$2{\cdot}0 \times 10^{-7}$	$2{\cdot}0 \times 10^{-7}$	$1{\cdot}5 \times 10^{-7}$	$2{\cdot}0 \times 10^{-6}$	$2{\cdot}0 \times 10^{-6}$

DECAY OF GETTERS

The decrease in gettering rate which is normally observed for nitrogen within a fraction of a minute also occurs with oxygen if the pressure p_g above the getter is sufficiently high and if the period during which the getter is exposed, is extended to more than a few minutes. This decay of getter efficiency which is produced by the take-up of gas is of particular practical importance and has therefore been investigated in detail. In these experiments the manifold pressure p_{man} has been kept constant while the variation of the getter pressure p_g was measured as a function of time. Naturally, when the gettering rate decays, the pressure p_g rises and the values obtained for the decay therefore refer to a pressure above the getter which varies during the experiment. This variation, which only affects the quantitative, but not the qualitative part of the results, had to be tolerated since the attainment of a constant pressure p_g proved to be hardly possible with the existing set-up.

Some of the results obtained are shown in Table 2 which gives the initial and final values of gettering rate G measured before and after a certain decay time. The variation of G as function of time is shown for oxygen by Fig. 3. Different getter areas were investigated.

Table 2. *Decay of gettering rates*

Type of gas	*t min*	p_g *mm*	G_{O2} cm³/sec	G_{N2} cm³/sec	Q $\mu\times$litres
O_2	0	$1{\cdot}5 \times 10^{-5}$	1 130	—	—
	30	$3{\cdot}0 \times 10^{-5}$	580	—	21
O_2	0	$6{\cdot}0 \times 10^{-6}$	1 300	—	—
	75	$5{\cdot}0 \times 10^{-5}$	35*	—	53
N_2	0	$2{\cdot}5 \times 10^{-4}$	900	65	—
	15	$5{\cdot}0 \times 10^{-4}$	300	15	$\simeq 6$

G_{O2} measured at $p_g = 2 \times 10^{-7}$, but value with asterisk measured at 2×10^{-5}.

G_{N2} measured at $p_g = 1{\cdot}5 \times 10^{-6}$.

Considering the larger getter area first, the gettering rate rises slightly at the beginning and starts falling only after a period of about 10 min has elapsed. The getter can be operated for more than half an hour before the gettering rate has decreased to half the initial value. The smaller getter area shows the same behaviour, principally,

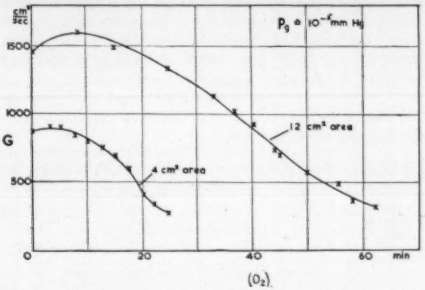

Fig. 3. Gettering rate G of oxygen as a function of time during which the getter is used at about 10^{-5} mm pressure

but the fall in gettering rate starts at an earlier time and takes place more rapidly. The behaviour of nitrogen during decay, as shown by Fig. 4, is quite different. Instead of an initial rise, the gettering rate falls rapidly at the beginning and then fades out to very small values.

From the values given in Table 2 the total quantity Q of gas taken up by the getter can be calculated. If equation (2) of [1] is used we have

$$Q = Gp_g t = \frac{Fp_{man}t}{1 + F/G} \simeq Fp_{man}t \qquad (1)$$

If the pressure p_g varies considerably during the experiment (e.g. in Fig. 3) the right hand side of the above equation is preferably used since this side contains the manifold pressure p_{man} which is kept constant. The values calculated for Q are given in the last column of Table 2. These values can be compared with the total quantity of gas which can be taken up by a barium getter according to Ehrke and Slack.[5] Bearing in mind that the weight of the getters used here was a few mg, Ehrke and Slack's value for oxygen (15 μl/mg) is of the same order as the value $Q \simeq 50$ litres obtained in the second experiment of Table 2. It may be noted that the corresponding quantities Q calculated for nitrogen are considerably smaller. In experiments such as the second one in Table 2, the consumption of the getter by the gas can actually be seen; the getter deposit gradually disappears, beginning from the boundary where the deposit is thinnest. No such effect, however, was observed with nitrogen.

It must be pointed out that the decay in gettering rate, as described above, is only observed if the ionization gauge, associated with the getter, is kept working during the exposure to the gas. If, however, the ionization gauge is switched off during the exposure, the observed rate of decay is very much lower. An exposure to 10^{-5} mm oxygen without drawing current in the gauge (for about 30 min) normally gives a slight increase between the G-values measured before and after the exposure, but certainly no decrease. Only by exposing to a pressure of as much as 5×10^{-4} mm could a slight fall in gettering rate by about 10% be obtained under these conditions. It can be concluded from this result that the existence of the electron current in the ionization gauge is necessary for obtaining the high gettering rates which have been measured. If this electron current is non-existent, the gettering rate, and consequently the quantity of gas taken up and the decay produced by this quantity, are evidently very much smaller.

With the experimental lay-out employed here, for which ionization gauges are used, the gettering rate itself cannot be determined without co-existent electron current. The conclusion, however, that this gettering rate is very low, compared with the rate obtained with electron current, is confirmed by some recent measurements by Haase.[6] Haase employed for his measurements a Knudsen gauge which needs no electron current for its operation. He found that no gas was taken up by a barium getter at pressures between 10^{-8} and 10^{-6} mm if the getter was evaporated and used under normal conditions. Only after taking special precautions, for instance evaporating the getter at a pressure as low as 10^{-8} mm, could the take-up of small quantities of gas be detected.

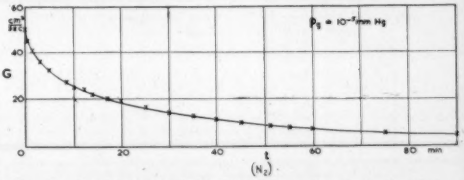

Fig. 4. Gettering rate G of nitrogen as a function of period during which the getter is used at about 10^{-5} mm pressure

MECHANISM OF GETTERING AT LOW PRESSURES

The above considerations show that normal gas molecules are taken up at a negligibly small rate at the low pressures used. The high gettering rates which have been measured must be due to the take-up of products of interaction between the gas molecules and the electrons passing from cathode to anode of the ionization gauge. Such a gettering process for which the existence of an electronic discharge is essential has been investigated for magnesium getters by Lukirsky and Ptitsin[7] and by Reimann[8] and has been called "discharge gettering." These latter investigations, however, have only been undertaken at much higher pressures (between 10^{-3} and 10^{-2} mm) where the gettering due to take-up of normal molecules is not negligibly small.

A further question now arising is: which products of interaction between gas molecules and electrons are taken up by the getter, and in particular, are these products charged (e.g. ions) or uncharged (e.g. atoms)? The latter question can be answered by varying the potential of the getter and by measuring the electric current flowing into the getter. To achieve this the glass bulb of the getter tube was supplied with a sealed-in platinum wire which made contact with the getter. The gettering rate was then measured in the usual way for different potentials applied to the getter, the potential applied to the anode of the ionization gauge being + 100 V. The result of these measurements is given in Table 3 which shows that there is only a very slight variation of the gettering rate as a function of getter voltage. It will be noticed that the absolute value of the maximum voltages applied to the getter is higher than the anode voltage accelerating the electrons. Therefore, in one or the other of the experiments, either positively or negatively charged particles are entirely prevented from reaching the getter. Hence it can be concluded that the gettering effect is mainly produced by uncharged particles.

This result is confirmed by measurements of the electric current flowing into the getter. This current which was measured with the getter at − 10 V has the direction of a positive ion current. Before conclusions can be drawn from the measured magnitude of this current, a theoretical relation between this current and the rate of gettering must be derived. Since 1 cm³/sec at $p = 1$ mm contains $6{\cdot}02 \times 10^{23}/760 \times 2{\cdot}24 \times 10^{4} = 3{\cdot}54 \times 10^{16}$ molecules, a gettering rate of G cm³/sec at a pressure p_g corresponds to

$$n_g = 3{\cdot}54 \times 10^{16}\, p_g\, G \text{ molecules/sec.} \qquad (2)$$

Table 3. *Gettering rate G as a function of potential V_G applied to the getter*

Potential accelerating the electrons = 100 V

	Values for oxygen
V_G	G
+ 125 V	970 cm³/sec
+ 60	970
0	970
− 125	1 050

On the other hand a current I^+ of singly charged positive ions, flowing into the getter, consists of $I^+/1{\cdot}60 \times 10^{-19} = 6{\cdot}25 \times 10^{18}\, I^+$ ions. If, therefore, the fraction of ions, contributing to the gettering effect, is denoted by α, we have, provided the molecules are singly ionized:

$$\alpha \times 3{\cdot}54 \times 10^{16}\, p_g\, G = 6{\cdot}25 \times 10^{18}\, I^+ \qquad (3)$$

$$\alpha = 176 \frac{I^+}{p_g G} \qquad (4)$$

Table 4 gives the ionic currents which were measured, the respective pressures p_g and gettering rates G, and the calculated values of α. The result shows that the contribution of ions to the gettering effect is extremely small.

Table 4. *Fraction α of ions contributing to gettering effect*

Gas	*I^+, amps*	*p_g, mm Hg*	*G^*, cm³/sec*	α
O_2	$1{\cdot}9 \times 10^{-8}$	$1{\cdot}5 \times 10^{-6}$	700	3×10^{-3}
N_2	$1{\cdot}8 \times 10^{-8}$	$1{\cdot}7 \times 10^{-5}$	40	5×10^{-3}
CO	$2{\cdot}3 \times 10^{-9}$	6×10^{-7}	1 500	$0{\cdot}4 \times 10^{-3}$

* The gettering rates quoted here are slightly lower than normal since they refer to a getter deposit on one side of the glass bulb only.

The experiments show that the particles bringing about the gettering effect at low pressures are neither normal stable molecules nor ions. The two remaining possibilities are metastable molecules and atoms. Gases in which part of the normal molecules have been converted into metastable molecules and atoms by means of an electric discharge are well known to the chemist and are called "active gases" on account of their higher activity in chemical reactions. Active oxygen and hydrogen have been widely examined (Geib[9]) and it is generally assumed that their active constituent is made up by atoms only. With nitrogen, however, conditions are less simple and the assumption of metastable molecules seems to be necessary for explaining the observed phenomena (Kneser[10]). The assumption that atoms are responsible for discharge gettering has been made by Reimann[8] in his experiments at higher pressures.

It will now be attempted to give a few quantitative relations on which further experiments may be based. The following symbols will be used:

- n_g number of molecules taken up by the getter per second
- n_{ra} number of molecules made reactive in the discharge per second
- n_{rf} number of non-reactive molecules re-formed from reactive ones per second
- N_{ra} number of reactive molecules formed per electron at $p = 1$ mm (efficiency of formation)
- β coefficient of re-formation
- I_A anode current of ionization gauge
- ϵ electronic charge,

and for the special case of atoms being produced as reactive particles:

- N_a number of atoms/cm³
- V volume of gettering vessel
- C number of collisions between atoms per cm³ and sec
- δ diameter of atoms in A
- $\bar{t}$ average life of atoms
- $\bar{l}$ average travelling distance of atoms
- $\bar{v}$ average velocity of atoms.

The following equations are easily verified:

$$n_g = n_{ra} - n_{rf} \qquad (5)$$

$$n_g = n_{ra}(1 - \beta) \text{ with } \beta = n_{rf}/n_{ra} \qquad (6)$$

$$n_{ra} = N_{ra} \times \frac{I_A p}{\epsilon} \qquad (7)$$

From (2) is derived

$$G = 2{\cdot}82 \times 10^{-1} \frac{n_g}{p} \qquad (8)$$

and in connexion with (6) and (7)

$$G = 176 N_{ra} I_A (1 - \beta) \tag{9}$$

An estimate of the number n_{rf} of re-formations can be given for the case of atoms where re-formation can only occur due to collisions between the atoms. Then, if assuming that every collision leads to re-formation, we have (Joos[11]):

$$\frac{n_{rf}}{V} = C = \tfrac{1}{2}\sqrt{(2)}\pi \times 10^{-16}\delta^2 N_a^2 \bar{v} \tag{10}$$

and with

$$N_a = \frac{2n_{ra}\bar{l}}{V} = \frac{2n_{ra}\bar{l}}{\bar{v}V}$$

$$\beta = 2\sqrt{(2)}\pi \times 10^{-16}\frac{n_{ra}(\delta\bar{l})^2}{\bar{v}V} \tag{11}$$

If for $n_{rf} = 0$ and $n_{ra} = n_s$: $G = G_0$,

then

$$\beta = 31{\cdot}4\,\frac{(\delta\bar{l})^2}{\bar{v}V} G_0 p \tag{12}$$

Using the known values for oxygen $\delta = 1{\cdot}3$, $v = 4{\cdot}4 \times 10^4$ cm/sec, and with $V = 100$ cm^3 we obtain:

$$\beta = 1{\cdot}2 \times 10^{-5} G_0 \bar{l}^2 p \tag{12a}$$

which gives for $G_0 = 1\,000$ and $p = 10^{-5}$ mm

$$\beta = 1{\cdot}2 \times 10^{-7}\bar{l}^2 \tag{12b}$$

According to the last equation, the coefficient of re-formation is only noticeable ($> 0{\cdot}1$) if the average travelling distance $\bar{l}$ of the atoms exceeds 10^3 cm. Provided $\bar{l}$ is smaller than this rather large value, β is negligible and then, according to (9), the gettering rate G becomes independent of pressure. In the case of oxygen, therefore, where such an independence of pressure is found experimentally, the assumption of atoms as being responsible for the gettering effect, is consistent with the experimental results, so far obtained.

Before we proceed to some further experiments which were carried out in order to check the above equations, it may be pointed out that the efficiency N_{ra} of formation of reactive particles is very high. From (9) we obtain for $I_A = 10$ mA and $G = 1\,000$ ($\beta = 0$) $N_{ra} \simeq 500$. This compares with a maximum value of only about 10/cm for the formation of ions from molecules of the common gases (Arnot[12]).

DEPENDENCE OF GETTERING RATE ON DISTANCE FROM ELECTRIC DISCHARGE

If re-formation of non-reactive molecules plays an important part in the gettering process, the observed gettering rates will depend on the time during which this re-formation can take place. Let us assume that such a case exists and that the distance between the electric discharge where the reactive particles are formed, and the getter where they are bound is increased. One will then expect that, owing to the increase in the number of re-formations, the gettering rate will decrease with increasing distance. This influence of position of the getter on the magnitude of the gettering rate has been studied experimentally.

In these experiments the getter was not only located in the centre position A (see Fig. 1) where it is closest to the discharge but also at the extremities of the gettering tube, at B and C. At first the getter was deposited at C where it was expected to be least effective, and the appropriate gettering rates were measured. Subsequently B was deposited and measured, and finally A was dealt with in the same way. The results so obtained are shown in Fig. 5 in the usual plot of $\log p_g$ against

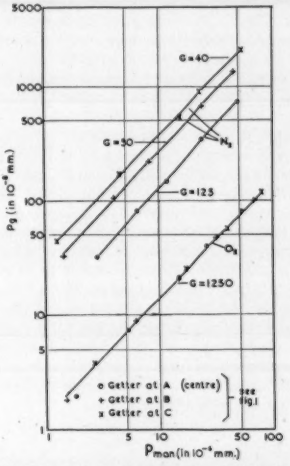

Fig. 5. Pressure p_g above getter as a function of manifold pressure p_{man} for different getter positions

$\log p_{man}$. For oxygen no difference was observed between the different getter positions. All the measured points lie on one straight line, thus indicating that the gettering rate is independent of the distance of the getter. For nitrogen, however, three different lines were obtained for the three positions and the gettering rate decreases from 125 for the closest position to 40 cm^3/sec for the most distant one. It follows from these results that re-formation is insignificant for the gettering of oxygen. The assumption that the particles responsible in this case are oxygen atoms is supported further by these measurements. With nitrogen, however, the observations are not consistent with such an assumption. Re-formation obviously plays an important part in the gettering of this gas. A tentative conclusion will be that this gettering process is due to metastable molecules. The re-formation coefficient of such molecules may be considerably higher than for atoms because re-formation in this case is given by a relaxation time and not only due to occasional collisions with other particles of the same kind.

The above result that the gettering rate of oxygen is independent of the distance of the getter is only valid on one condition. The atomic gas which is produced in the discharge must not be impeded mechanically from getting in contact with the getter. The degree of impedance is measured by the flow resistance of the parts of the gettering vessel which connect the discharge space with the region where the getter is located. This flow resistance is negligibly small in the experiment described above, but it can well be of considerable importance; for instance, in electronic valves, when the discharge space is more or less screened from the getter by electrodes or insulating parts of the valve system.

The existence of such an influence was proved by some special experiments using a combination of two normal gettering vessels X and Y which were connected by a tube of about 10 cm length and 10 mm diameter as shown in Fig. 6. The getter was deposited at the central

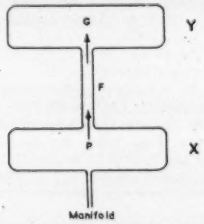

Fig. 6. Experimental arrangement for studying the influence of flow resistance on gettering rate

position in vessel Y, and the pressure p_g measured in both vessels Y and X. The measurement in Y is in no way different from the normal measurement of the gettering rate G. If, however, the pressure is measured in X, the atoms which are produced there have to be pumped through the connecting tube before they reach the getter. In this latter case the getter in Y can therefore be visualized as a pump with an inherent speed G which pumps atoms through the connecting tube and exercises in X a certain pumping speed P ($P < G$). This pumping speed is related to the gettering rate G and to the known flow resistance F^* of the connecting tube by the known equation

$$P = \frac{G}{1 + G/F^*} \qquad (13)$$

(see Dushman[13]).

The value of P can be calculated from the above equation and can be determined experimentally from the pressure measurement in X. G can be obtained from a corresponding measurement in Y. The values so determined from two experiments are seen from Table 5. There is good agreement between the observed and calculated values of P. It may be noted that, in the case of nitrogen, the pumping speed measured in X is extremely low, less than 10% of the normal gettering rates. This again shows that for nitrogen re-formation is much more important than the influence of flow resistances.

Table 5. *Measured and calculated values of pumping speed P in experiments with 2 connected gettering vessels* (see Fig. 6)

	(*Values for oxygen*) I	II
Gettering rate G	520	915 cm³/sec
Flow resistance F^*	1 000	750 cm³/sec
Pumping speed P:		
Measured	330	410 cm³/sec
Calculated	342	412 cm³/sec

DEPENDENCE OF GETTERING RATE ON MAGNITUDE OF ELECTRON CURRENT

The variation of gettering rate G with the electron current of the ionization gauge is shown in Table 6 for oxygen and nitrogen. As to be expected, G falls with decreasing current I_A, but there is no proportionality between G and I_A.

Table 6. *Gettering rate G as a function of anode current I_A in ionization gauge*

I_A mA	G in cm³/sec Oxygen	Nitrogen I	Nitrogen II
20	1 300	78	—
10	1 080	66	62
5	970	—	53
2·5	—	—	44
1	—	—	33

This result is only consistent with the simple theory outlined above [cf. equation (9)] if the coefficient of re-formation β depends on pressure and if this coefficient is not negligibly small. As shown above, such an assumption is reasonable for nitrogen but not for oxygen where β was found to be negligible. In the case of oxygen, therefore, the theory must be refined. Possibly the oxygen atoms responsible for the gettering effect are not only formed directly by collisions between molecules and electrons but also by secondary processes.

It appears to the author that these processes can hardly be explored in more detail with the equipment used so far in which the production of the reactive particles is inherently connected with the pressure measurement in the ionization gauge. It seems to be feasible to separate the two processes by using for the pressure measurement a gauge which is sufficiently sensitive but is not based on the existence of an electron current. The Knudsen gauge seems to be the only known type which satisfies these requirements. By using such a gauge, it would be possible to determine the critical potentials at which the gettering effect sets in, and to find out possible correlations between these critical potentials and the dissociation potential of the gas molecules and other molecule constants. In order to be suitable for such measurements, however, the Knudsen gauge must be well degassable. Such a gauge has been described by Klumb and Schwarz[14] and it is

intended to construct a gauge of similar design for further investigations on getters.

CONCLUSIONS

The experiments have shown that the gettering rates do not decrease with decreasing pressure and that with gases such as oxygen the initial gettering rate of about 1 l/sec can be maintained for periods of 10 to 20 min at pressures as high as 10^{-5} mm. In normal electronic valves, therefore, having a volume of 50 cm^3 or less, it should be possible to obtain extremely low pressures if the getter is used in the right way. One main problem arising is that of position of the getter. This, apparently, is most important for gases which are taken up with lower rates than oxygen and for which, as the example of nitrogen shows, the take-up is particularly sensitive to variations in position.

As the position of the barium and of similar flash getters must of necessity be confined to the cooler parts of the valve, attention turns naturally to the coating getters which, not being prone to evaporation, do not suffer from such limitations. Investigation of these getters is more complicated, since an additional parameter, the temperature of the getter, has to be taken into account. It is hoped that more information on these getters can be reported soon.

ACKNOWLEDGMENTS

Acknowledgment is made to the Engineer-in-Chief of the General Post Office for permission to make use of the information contained in this paper. The author also wishes to express his appreciation to Mr. R. E. Hines for his skilled assistance in constructing the experimental tubes and to Mr. R. E. Thorne who assisted in the experimental work.

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Measurement of the curvature of stress trajectories in photoelastic models

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[*Paper received* 3 *January*, 1951]

The use of Jessop's extension of the Lamé–Maxwell equations for separating the principal stresses in a three-dimensional photoelastic problem depends for its accuracy on a precise knowledge of the curvature of the stress trajectories crossing the line along which the integration process is carried out. A method of measuring these curvatures more accurately than has been possible before is described and an example is given of the application of the method to a particular frozen stress problem.

The frozen stress method of photoelastic stress analysis enables one to examine the stress distribution in the interior of three-dimensional engineering components. Only the principal stress differences are given directly from the stress pattern in slices cut from a frozen stress model, but Jessop[1] has shown that the Lamé–Maxwell equations can be extended so as to effect the separation of the principal stresses in a plane of symmetry. This involves a step-by-step integration along a line of principal stress, from a point where the stress is known, and the process requires a knowledge of the curvature of the trajectories of the two other principal stresses which cut the line at right angles along its length. These curvatures can be measured to a satisfactory degree of accuracy with the aid of a photometric technique developed at the National Physical Laboratory.[2]

In general, the integration will be carried out along a stress trajectory which is known to be straight and, in Fig. 1, AB represents a portion of such a line which is intersected at 0 by a small part of an orthogonal stress trajectory represented by the circular arc CD. A chord EF is drawn and the angles made with it by the tangents at its extremities are θ. The curvature of the arc, $1/\rho$ is $\sin\theta/a$. θ is given by the isoclinic angles at E and F and $2a$ can be accurately determined, but it is not easy to measure θ at exactly equal distances from AB. Fig. 2 shows, therefore, the case where E' and F' lie at distances a_1 and a_2 from AB, the

corresponding angles being θ_1 and θ_2. The curvature now becomes:—

$$\frac{1}{\rho} = \frac{\sin\theta_1}{a_1} = \frac{\sin\theta_2}{a_2}$$
$$= \frac{\sin\theta_1 + \sin\theta_2}{a_1 + a_2}$$

Since θ_1 and θ_2 must be small for the arc CD to approximate to the curve of the stress trajectory, a close approximation for the curvature is:—

$$\frac{1}{\rho} = \frac{\theta_1 + \theta_2}{a_1 + a_2}$$

The instance of a particular stress distribution problem will be quoted to illustrate the use of the above method.

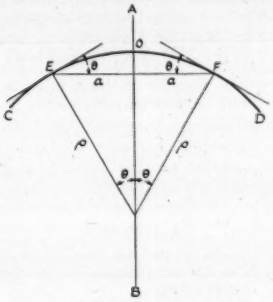

Fig. 1. Curvature measurement

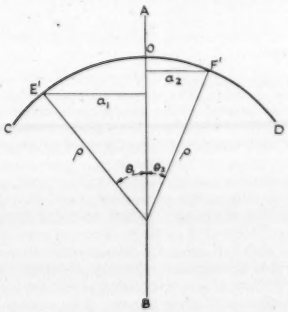

Fig. 2. Curvature measurement, more general case

The values of the three principal stresses were required over the whole of the reduced section of a round bar with a circumferential *V*-notch, when loaded in tension (Fig. 3). Since the case is one of axial symmetry, it suffices to integrate for the radial stress along the line *AB* from the root of the notch, where it is zero, to the centre of the bar, in order to obtain the complete stress distribution over the shaded section in the plan view.

The necessary information is obtained from a frozen stress model from which an axial slice is cut and as much as possible of a transverse slice through the notch plane. Values of the principal stress differences can be determined along the line *AB* in the two slices, preferably using the photometric technique already referred to.[2]

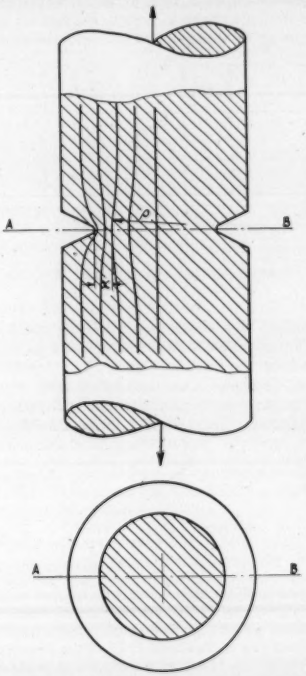

Fig. 3. Diagram showing round bar with circumferential *V*-notch in tension

In addition, the curvatures of the stress trajectories crossing the line *AB* in the two slices must be known. In the transverse slice it is evident that the trajectories are circles. In the axial slice, however, they take the form indicated diagrammatically in the upper part of Fig. 3, where ρ is the radius of curvature.

The method of measurement adopted was to mount the axial slice in an immersion cell containing liquid of the same refractive index as the model material, in such a way that it could be traversed at right angles to *AB* by accurately known distances. The slice was in a field of plane polarized light in a photoelastic bench and the isoclinic angle was determined at the extremities of the

chord in the trajectory of which the curvature was to be measured. The angles were determined by rotating the polarizer and analyser together till the dark shadow was centrally disposed over the point examined, this condition being determined photometrically, as described elsewhere.(2) In the present case it was found that, provided the angles did not exceed 1° (when the departure of the curve from a circular arc became apparent) the accuracy of measurement of curvature depended on

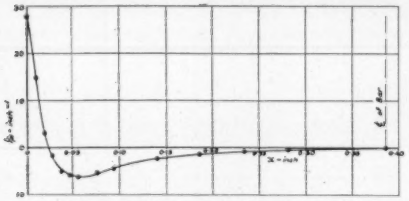

Fig. 4. Relationship between curvature ($1/\rho$) and distance from notch root (x) (see Fig. 3)

the closeness with which the angle scale could be read. Thus, if the angles could be determined to $\frac{1}{10}$ degree, the curvature was known within $\pm 5\%$. Some improvement on this degree of accuracy was possible by drawing a smooth curve through the points and taking values from the curve. Fig. 4 shows a series of results in which the curvature, $1/\rho$, is plotted against distance from the root of the notch, x, the end point at the notch root being inserted from the known radius of the notch.

These results were used to evaluate the three principal stresses along *AB* and the usual check was applied, viz. comparison of the integrated longitudinal stress with the load originally applied to the model. The fact that the agreement was within $3\frac{1}{2}\%$ is regarded as satisfactory.

It may be added that to gain the full advantage of the photometric method of measuring curvatures, it is essential to arrange some form of coupling between the polarizer and analyser of the photoelastic bench and possibly also a vernier attachment for the measurement of small angles of rotation. The improved accuracy which can be realized with this method, which is applicable to two-dimensional photoelastic stress analysis as well as to frozen stress work, should enable much greater reliance to be placed on photoelastic stress determinations based on graphical integration methods.

ACKNOWLEDGMENT

The work described above has been carried out as part of the research programme of the National Physical Laboratory, and this paper is published by permission of the Director of the Laboratory.

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Correspondence

A photographic method for displacement/time recording

Use of an ordinary light source reflected in a small bright ball-bearing to give a point source, the movement of which is recorded as a trace on a photographic drum, was described as something new by Professor F. M. Bruce in your issue of November 1950.(1) It may interest your readers to know that this principle has been used by us for a number of years for recording creep curves, as published by us some time ago.(2) Certain refinements in the technique are recorded in our publication, which are, in general, advantageous. These are: (*a*) the mounting of the sphere on a plane mirror, so arranged as to throw the illuminating beam well away from the recording drum, thus ensuring a good dark background, and (*b*) the automatic registration of time and displacement scales to form a grid as the displacement proceeds. This eliminates errors otherwise introduced by distortion of the paper during the development process, errors which are only reduced by Professor Bruce's method of marking out the axes.

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E. N. da C. Andrade
A. J. Kennedy

The Royal Institution,
London, W.1.

The paper referred to by Professor Andrade and Mr. Kennedy should certainly have been quoted in the references that I gave, and I must thank them for bringing it to my notice. The various reflectors described in my paper were evolved over a considerable period, each having its particular application, but in all cases the velocity of movement was appreciable, and the amplitudes ranged upwards from several millimetres. The use of a plane mirror to provide a dark background does not appear to be applicable to such work, but I have used an equivalent effect by adjusting the relative angles of the light source and camera, with the mechanism link itself acting as a plane mirror. I have taken records with a series of stationary reflectors and with timing marks, and checked that distortion due to photographic processing was not measurable by the methods I was using, but I agree that the grid referred to would be necessary in work of greater precision and requiring measurements by a travelling microscope.

F. M. Bruce

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New Books

Progress in Nuclear Physics. Edited by O. R. FRISCH, O.B.E., F.R.S. (London: Butterworth-Springer Ltd.) Pp. viii + 224. Price 45*s.* net.

Professor O. R. Frisch is editor of a new series of progress reviews on Nuclear Physics which will appear annually. Nuclear physics has by now established itself as a separate branch of physical science and even within that field specialization is developing; for example, on particle accelerators, particle detectors, nuclear spectroscopy, high energy physics. This series should help to keep workers within the field of nuclear physics informed on developments over the whole field.

The first volume presents articles on cyclotrons by T. G. Pickavance; high-voltage direct-current generators by R. L. Fortescue; photographic emulsion techniques by J. Rotblat; theories of beta decay by T. Skyrme; the spectroscopy of beta and gamma decay by P. E. Cavanagh; the Szilard–Chalmers process by H. A. C. McKay; and the scattering of neutrons by crystals by J. M. Cassels.

Six of the articles are concerned with the new machines and techniques of nuclear physics and two with the description and theory of the experimental results. They are all written by active workers in the separate fields and provide an authoritative account of the present position of the various subjects. They are more specialized than the well-established Physical Society reports on Progress in Physics and for that reason the two series are not likely to overlap seriously.

J. D. COCKCROFT

Physics in Medical Radiology. By S. RUSS, C.B.E., D.Sc., F.Inst.P., L. H. CLARK, M.Sc., Ph.D., F.Inst.P., and S. R. PELC, D.Ph. (London: Chapman and Hall Ltd.) Pp. viii + 296. Price 25*s.* net.

Since the first edition of this book was published, twenty-two years have elapsed. In view of the important advances which have been made in the techniques and apparatus of medical radiology during those years, it is surprising how much of the original book has stood the test of time and is incorporated in this second edition. A critical comparison of the two editions shows, however, that much new material has been added and that a changing emphasis over the years has resulted in much recasting of the original material. The first five chapters deal with the elementary physical principles of electricity, in motion and at rest, necessary to an understanding of the processes leading to the production of X-rays. These chapters have changed little since 1928. It is, however, when we consider those parts of the book dealing with radiation dosage and its instrumentation that one becomes aware of the rapid progress that has been made in this field during the last quarter of a century. Planck's quantum theory and the Compton effect have been promoted from rather less than a page to a full new chapter. As would also be expected, the chapters on radioactivity and atomic structure are treated much more fully in this edition. It is likely that some of the more timid medical radiology students will be disturbed by portions of the mathematical arguments used in these sections, for example, the development of the Compton scatter equation. In other respects, however, notably where the concept of de Broglie waves is introduced and explained in two paragraphs, the authors have accomplished a remarkable feat in depicting difficult material in extremely simple terms. The section on photography has been rewritten and a short chapter on techniques using radio-isotopes has been added. It is pleasing to note that the elements of the statistical treatment of results obtained in counting is included in a work of this kind. The book ends with several chapters on X-ray tubes, high-voltage generators and auxiliary apparatus, and with an appendix on X-ray and radium protection.

Although possessing an adequate subject index, a serious defect is the complete absence of a bibliography. The student wishing to read original papers mentioned in the text is left with no clue as to their origin. The book is well produced, however, and the diagrams, with a few glaring exceptions, are particularly clear. As an introduction to the subject, particularly for radiological diploma students, the book is to be recommended.

W. A. LANGMEAD

Drying in the Heavy Clay Industries. By H. H. MACEY, F.Inst.P., M.Inst.F. (London: His Majesty's Stationery Office.) Pp. vii + 188. Price 7*s.* 6*d.* net.

The research worker who sets out to investigate the physical mechanism of some industrial process invariably finds that measurements, which from the text-book appear to be the essence of simplicity, are difficult to make in industrial practice with any assured degree of accuracy. This book provides an excellent object lesson in the way in which such difficulties may be countered by an experienced worker with regard to such fundamental measurements, for instance, as air humidity and air velocity.

The book is divided into three sections, as follows: (1) hygrometry and the determination of thermal efficiency, (2) dryers used in the heavy clay industry, (3) survey of thermal efficiency in practice. While the last two sections are primarily of interest to clay technologists, the first should have a wider appeal. It contains a comprehensive and practical account of methods available for the measurement of atmospheric humidity and also a critical review of existing hygrometric tables and charts. In both cases a praiseworthy feature is the very full bibliography appended. A minor criticism that might be offered here is that a reproduction of a typical psychrometric chart with some account of the operations that can be performed with it would have made for completeness. As the author points out, however, psychrometric charts are usually designed primarily from the standpoint of relative humidity, whereas in drying problems absolute humidity is the more logical basis of expression.

In an appendix is presented a set of new hygrometric tables drawn up by the author. They are based upon the fact that when air is passed over a free water surface under adiabatic conditions its wet-bulb temperature remains substantially constant; this follows from the fortuitous circumstance that for air-water vapour mixtures under most ordinary conditions the wet-bulb temperature is almost identical with the adiabatic saturation temperature. The tables are convenient in use, extend to a wet-bulb temperature of 60° C and should be amply accurate for industrial purposes.

F. W. THOMAS

Physics. By S. G. STARLING, B.Sc., A.R.C.Sc., F.Inst.P., and A. J. WOODALL, B.Sc., Ph.D. (London: Longmans, Green and Co. Ltd.) Pp. xvi + 1301. Price 42*s.* net.

The appearance of a new text-book of which Mr. S. G. Starling is one of the authors will naturally be a matter of interest to teachers of physics. His *Electricity and Magnetism for Degree Students*, first published in 1912 and which has gone through numerous editions, will be remembered by many generations of students. The aim of the new publication, a comprehensive text-book of physics by Mr. Starling and Dr. A. J. Woodall, is to cover the requirements of candidates taking physics as a subject in a Pass or General Honours Degree or as a subsidiary subject in a Special Honours Degree. The authors hope that the book will be of value to the physics specialist in his first year, as well as to the general reader of the subject.

It can well be described as a weighty tome, as the whole subject is dealt with in one volume extending to 1300 pages. The reader will at first feel rather appalled by its weight and size, but further perusal will allow this to be forgiven and forgotten, and the impression will grow that we have here a valuable work of reference all under one cover. (My wife asked why I was reading Mrs. Beeton's *Household Management*—the description, apart from size, may in several ways prove to be surprisingly apt.)

The subject is taken up at about the intermediate standard and the transition from the more elementary work, which must be assumed, to the more advanced arguments is effected smoothly. Numerous cross-references are given between different parts of the subject and this is particularly helpful in these days when we do not want physics divided into watertight compartments. A number of separate books by different authors could not do this so successfully.

Although it in no way claims to be a text-book of practical physics, experimental devices are well described, and this will increase its usefulness to the working physicist who often has to deal with relatively simple problems with which he is at the moment not familiar. This aspect of the book will appeal to both students and practitioners of applied physics.

By avoiding any claim to carrying the subject to the full Honours Degree standard, the authors have been able to place their emphasis on what is generally regarded as classical physics—this, of course, includes its own recent developments. Sufficient "modern physics" is included to achieve the authors' specific purpose, but this occupies a small part of the book. This has enabled them to reach a relatively high standard in most of the work.

The majority of the exercises at the end of the book are taken from London University examinations, and this suggests that the authors have these examinations particularly in mind. It will no doubt have a special appeal for colleges taking the London external degrees where the need for a new text-book of this kind has for some time been felt. In a small physics department, where the whole subject must be taught by a few members of the staff, it will be very welcome. Most university teachers will continue to have their own views as to how different parts of the subject can best be developed, but they also will find it useful for reference.

In considering a work of this kind it would be ungallant to emphasize minor points of disagreement. For example, the old-fashioned "elasticity" has not been extended so as to include plasticity and the ductile properties of metals. But the work is of such importance that in later editions the authors will probably consider how best to bring such classical portions of physics up to date. We can, however, readily agree with the authors who say in their preface that the writing of the book has been no light task and congratulate them on having brought the undertaking to a successful conclusion.

The book is well printed and pleasant to read, with numerous diagrams. Some of the tables at the end seem to be rather an anti-climax. For example, is it necessary to show how to convert 1 lb (troy) into drams (troy), pennyweights and scruples? T. L. IBBS

Laboratory Physics. By J. H. AVERY, M.A., and A. W. K. INGRAM, M.A. (London: Messrs. William Heinemann Ltd.) Pp. vii + 439. Price 16*s.* net.

This book sets out a course of laboratory work in physics suitable for the later years in schools and extending to the first year at the university. It contains a wealth of practical detail and sufficient theory to introduce and explain the experiments. There are about two hundred experiments described and they are grouped under sections on mechanics, properties of matter, heat, light, sound, magnetism, electricity and electronics. Each part begins with a section on the standard procedure relevant to that part. The authors have incorporated the results of many years' experience in teaching physics and other teachers will greatly profit from this careful record. The apparatus needed is listed at the beginning of each experiment and the theory is then discussed and the procedure and manner of presenting the results is described. The Introduction discusses errors of measurement and the graphical representation of results. There are a number of exercises and problems at the end of the book. The late Dr. Alexander Wood says in a Foreword: "The pitfalls are scheduled and enough help given to assist the student without relieving him of the necessity of thinking for himself. The new experiments, especially those using valves, will meet a demand which can no longer be evaded, and will be welcomed by teachers who have been waiting for this new departure." The authors believe that the student can profit from making much of the special apparatus for himself, and have therefore included brief instructions on useful techniques such as soldering, glass blowing, etc.

There are about two hundred and fifty clearly drawn diagrams. The book can be warmly recommended as fulfilling a real need for a practical manual for the Sixth Form and College Physics Laboratory. G. VAN PRAAGH

Metal Spectroscopy. By F. TWYMAN, F.R.S., F.Inst.P. (London: Charles Griffin and Co. Ltd.) Pp. vii + 569. Price 50*s.* net.

Mr. Twyman's claim to be the elder statesman of spectrochemical analysis in this country is acknowledged and his earlier book, *The Spectrochemical Analysis of Metals and Alloys*, must have been consulted by all who have had any concern in this field. The growth in status of the subject during the past ten years provides the soundest of reasons for the appearance of an extensively revised version of the book, now under a new title.

The success of the application of physical principles to

chemical analysis by spectrography has been so complete that comparatively few physicists are nowadays concerned with it, for the technique is so simple to operate that it can be trusted entirely to those analytical chemists who need to use it. It is as well, therefore, to remind readers of this *Journal* that metal spectroscopy is a method of chemical analysis, developed for industrial use over the past thirty years, and reaching maturity in the last ten, which competes successfully over a wide field on one or more grounds of speed, sensitivity, accuracy and running costs with what used to be regarded as orthodox chemical methods. The importance of the method in securing control of quality over the whole field of industrial metallurgy could hardly be exaggerated and is outstanding among the more modest contributions of physical science to our material civilization.

The technique of spectrochemical analysis calls on three fields of physical science—electrical discharges, optics and quantitative photography. Only from the optical sector is the contribution substantially perfect, and it need hardly be recalled that it is in this sector that Mr. Twyman's personal effort has been outstanding. It is natural that a description of spectrographs and microphotometers should take an early place in his book, after an historical introduction and a discussion of the elements of atomic spectrum theory which closely follow the corresponding chapters of the earlier volume. This description includes brief reference to commercial spectrographs in which photoelectric recording is substituted in place of photography and to the use of electronic methods not only for this purpose but also in photometry and absorptiometry.

If methods of exciting spectra were as well understood as are the methods of dispersing and measuring them, one more chapter would have completed the work. However, the remarkable simplicity of the rules of discharges which apply for analytical purposes cannot obscure the fact that, for want of understanding, the spectrographer must use his art in choosing as best he may the electrical conditions of discharge, the electrode shape and geometry and the methods of preparing the material for analysis. This accounts in large measure for the bulky literature which the author of a book such as this ought to sift and seldom does. The particular value of this book in comparison with most others is that, while dealing faithfully with the principles and apparatus of the method, Mr. Twyman was devoted two-thirds of his space to the all-important "know-how" of the subject; and he has, for this purpose, called upon the services of contributors with special knowledge in particular branches. Chapter 12, "Spectrochemical Analysis in Works Practice" (pp. 322–492), which gives procedures for many metals and their alloys and is a work of reference in itself, contains many such contributions, of the greatest value, from British industrial laboratories, while the chapters by Mr. Walsh on "Light Sources" and Mr. Milbourn on "Electrodes" are important for their thoughtful attempt to draw out of wide fields of empirical variation the essential common features of the variants in these fields.

This book will find a place on the shelves of every practising spectrographer and will be valued by teachers and students of the subject alike; and it will be consulted by others who wish to know how spectrography can help them resolve their analytical problems.

G. L. J. BAILEY

Progress in Photography 1940–1950. Editor-in-Chief: D. A. SPENCER, Ph.D., D.I.C., F.R.I.C., F.I.B.P., Hon.F.R.P.S. (London: The Focal Press Ltd.) Pp. 463. Price 42*s.* net.

This is an important and ambitious work, filling a real gap in the literature of photography. With the rapid spread of photographic techniques in industry and science, the need for such a progress review was already evident long before the war, nothing of the sort having appeared in English (other than the brief annual surveys produced in this country and the U.S.A.) since the publication of *Photography as a Scientific Implement*, by Blackie in 1923. The only other comparable work was Stenger and Staude's *Fortschritte der Photographie*, of which three successive volumes have appeared, in 1938, 1940 and 1944. The enormous impetus afforded by the war to the development of new apparatus and materials and their large-scale civil and military application, and to the accompanying progress in fundamental studies, has made such a review more than ever desirable.

The present volume, a collective work containing contributions from both sides of the Atlantic, is of wider scope than either of the works mentioned above since it includes also reviews of the materials and equipment of the amateur and the professional photographer concerned with picture-making in the ordinary sense. The Editor-in-Chief has thus had to face some difficult problems of balance and of the depth of treatment to be accorded to the more specialized technical subjects. He has sought to solve these problems by a careful sectional organization of the work and he has in the main succeeded; but in the reviewer's opinion some of the scientific and technical sections would have profited by a more thoroughgoing and detailed treatment: indeed, this is not an English "Stenger and Staude." In the circumstances, great importance attaches to the sectional bibliographies, some of which are thoroughgoing but some perfunctory.

In a work covering such a wide field it is perhaps invidious to single out individual articles from the many which achieve the standard set by its general aims, but mention should be made of the articles on "New Developments in America," by G. E. Matthews and W. F. Walker (pp. 15–32), on "Methods of Document Recording," by K. Stuart Smith (pp. 147–59), on "Photogrammetry," by G. Poivilliers (pp. 161–79), on "Photography of Motion," by J. H. Waddell (pp. 200–11), on "Accuracy of Reproduction obtainable by Straightforward Three-Colour Processes," by R. A. Morris (pp. 235–51), and on "Radiography," by L. Mullins (pp. 263–75). The section on the "Camera as a Tool of Research" (pp. 301–62) gives an excellent account of work in many fields of interest to the physicist. Later sections on standardization and on the growth of the photographic industry will be of wider appeal.

The book is on the whole well produced. The half-tone illustrations are excellent, but many of the line diagrams are of poor quality. The definition of reflexion density given on p. 87 is wrong, and the treatment of the fractional-gradient speed criterion on pp. 90–2 is unconvincing. There are, unfortunately, many misprints, particularly in the spelling of proper names, e.g. Lippman for Lippmann (p. 83), Batelle for Battelle (p. 116), Blitz for Biltz (p. 100), Trillatt for Trillat (p. 275), and no fewer than three in the short paragraph and footnote dealing with the Royal Photographic Society (p. 447). The reviewer's main typographical criticism is the very inadequate printing of even the simplest

mathematical material, with errors such as those occurring in the equations on pp. 91, 96 and 120.

None the less, this work deserves a sincere welcome. We shall look forward to its successors which, as Dr. D. A. Spencer remarks in the introduction, will be able to deal at greater length with a limited number of fields in turn.

L. V. CHILTON

Centenary Proceedings of the Royal Meteorological Society. (London: Royal Meteorological Society.) Pp. 279. Price 21*s.* net.

This volume records the centenary celebrations of the Society held at Oxford and London in the spring of 1950. Here are recorded the papers presented at the seven symposia and the discussion on them. Their titles are: Radiation and its effect on the troposphere and lower stratosphere; Physics of clouds and precipitation; The structure of weather systems; The general circulation; Climatic change; Meteorology and the community; Atmospheric turbulence.

The volume has been prepared under the editorship of Professor P. A. Sheppard and is well worthy of the occasion it celebrates.

How to Write Technical Books: with some Pertinent Remarks about planning Technical Papers and Forms. By J. GLOAG. (London: George Allen and Unwin Ltd.) Pp. x + 159. Price 12*s.* 6*d.* net.

Any book which seeks to improve modern technical writing is to be welcomed; and this little volume is a useful contribution. It is "concerned with the planning, writing, presentation and publication of technical books. The penultimate chapter deals with the planning, writing and presentation of technical and semi-technical documents, report and memoranda; the last suggests how the language and presentation of forms might be improved."

This experienced author manages to squeeze into the limited space a brief account of his experiences in writing books, some personal views, and a few platitudes as well as snippets of technical information about printing. All this goes to make a readable book. It is a matter of opinion whether some of it might not more usefully have been replaced by *additional* hints and tips on the actual preparation of the script, on corrections of galley proofs and on all those things for which an author is personally responsible.

The chapter on the preparation of technical papers is too brief to be of much help to inexperienced authors writing for a journal such as this. It is surprising that no reference is made to the pamphlets issued by most learned societies for such people.

The book is beautifully printed and produced.

H. R. LANG

The Year's Photography, 1950–51. (London: Royal Photographic Society.) Pp. 16 and 105 plates. Price 6*s.* net.

In this edition, for the first time, an attempt has been made to represent all sections of The Royal Photographic Society's Annual Exhibition, which covers every kind of photography. Unfortunately, cost and other considerations prevent the full use of colour and stereoscopy in a work of this character, and these sections are represented by text only. Two examples of colour photography from the 1949 Exhibition have, however, been included as frontispieces. The section on scientific photography contains 13 plates and Mr. V. G. Macdonald contributes a brief article on them. The plates include an interference photograph and a Toepler–Schlieren photograph.

Journal of Scientific Instruments

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Improvements on the design of ultrasonic lamination detection equipment. By R. S. Young and H. R. Clayton.

An anisotropic drafting device. By L. Bainbridge Bell.

A multiplier-photocell attachment for use with prism spectrographs. By J. M. Naish.

A null reading mercury cistern manometer. By K. R. Honick.

A variable-length radio-frequency transmission line section. By K. R. McAlister.

An apparatus for the production of pure deuterium from heavy water. By M. M. Winn.

An improved electron diffraction camera. By P. W. Allen, H. Mackie and L. E. Sutton.

A two-colour infra-red radiation pyrometer. By A. F. Gibson.

NOTES AND NEWS

New Instruments

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New Books

Practical spectroscopy—Misure Elettriche.

British Journal of Applied Physics

Original Contributions accepted for publication in future issues of this Journal

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The boundary conditions at a mist-water interface. By P. M. Davidson.

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SPECIAL ARTICLE

The role of vegetation in meteorology, soil mechanics and hydrology*

By H. L. PENMAN, M.Sc., Ph.D., F.Inst.P., Rothamsted Experimental Station, Harpenden, Herts

Plants transpire large quantities of water at a rate primarily dependent upon weather conditions, and calculable from easily measured weather elements. They therefore play a dominant part in the water and heat balances of the earth's surface, and affect land drainage, underground storage and river flow. Calculation of the transpiration rate can be made the basis of controlled irrigation to produce maximum growth without waste of water, and it is shown that the principle can be applied equally successfully to a field, to a catchment area and to the British Isles as a whole. When the plants grow in a clay soil, shrinkage may occur as the soil dries, with damaging results on roads or buildings carried on the soil.

In a previous survey[1] the dependence of natural evaporation on weather was examined in detail, but, because of concentration on the mathematics and physics of the problem, it was not possible to give much attention to the practical application of the results. The present survey will remedy this defect. The basic principle is simple and will be outlined in the next section without giving details of the theories and experiments on which it is based. It amounts to answering "What happens to the sunshine?" and its simplicity arises from the close interdependence of two primary needs of the plant world—a supply of water and a supply of radiant energy. When plants get both they make very inefficient use of them, for most of the energy is used in evaporating nearly all of the water, the amounts taking part in photosynthesis being absurdly small. To the physicist this is a fortunate circumstance because it makes possible the drawing up of an energy balance sheet in a way that leaves evaporation as the only unknown, i.e. natural evaporation can be estimated from suitable weather measurements.

WHAT HAPPENS TO THE SUNSHINE?

The three main components of incoming solar radiation are indicated in the top left corner of Fig. 1, and below, the day regime is represented approximately to scale. After a loss by reflexion, the remaining energy is degraded in various ways. There is a complex interchange of long wave radiation, upward from the earth and downward from the clouds and the water vapour in the atmosphere: only the net outflow is represented in Fig. 1. There are two sinks for sensible heat, one in the soil, and the other in the air which is warmed by turbulent transfer of heat from the ground. Vegetation provides two sinks, a weak sink in taking up energy for photosynthesis and a very much stronger sink where the energy is consumed as latent heat of vaporization. By night, conditions are simpler, for there are only three components. Back radiation is maintained and this must draw its energy from the soil and the air. For a 24 h balance the day and night balances must be combined,

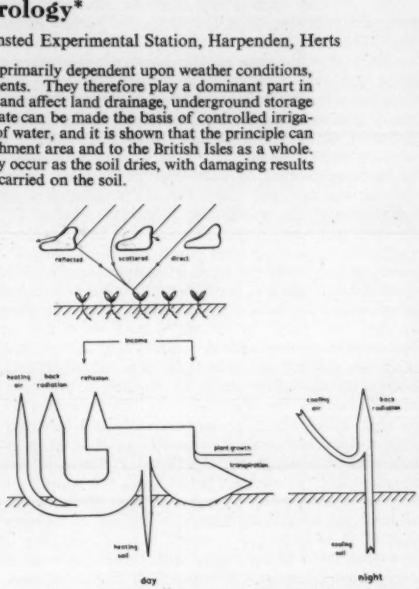

Fig. 1. Energy balance for average summer conditions

and as the air and soil components change sign between day and night their net effect is small. Table 1 gives

Table 1. *Energy balance May to September*, 1949

Energy sink	*Evaporation equivalent (in)*	%
Plant growth	0·4	1
Heating soil (S)	0·7	2
Heating air (K)	1·4	4
Reflexion (r)	7·2	20
Back radiation (R_B)	12·3	34
Transpiration (E)	14·0	39
Total (R_C)	36·0	100

a picture of average conditions over four months in the summer of 1949, the right-hand column showing the fraction of incoming energy taken up by each sink. The second column gives the evaporation equivalent for the whole period: taking the latent heat of vaporization as 590 cal/g, the evaporation equivalent is 1 in evaporated = 1 500 cal/cm^2. (The total income represents about 2½ million kWh/acre.) Accepting these figures as being correct in order of magnitude, the first two sinks can be

* Based on a lecture given on 20 October, 1950, to the Manchester Branch of The Institute of Physics.

ignored in most attempts to estimate evaporation from energy balance: heating of the air requires about one-tenth of the energy needed for evaporation, so it must be kept in.

The problem then is to evaluate incoming solar radiation, reflexion, back radiation, and heating of the air. The first is not difficult, as suitable radiation recorders exist, but their number is limited. Fortunately there is a very close relation between radiation and duration of bright sunshine, and in the absence of marked bias in the distribution of sunshine during the day the incoming intensity can be estimated for periods of a week or longer (frequently for single days) from sunshine figures supplemented by knowledge of latitude, season, and the solar constant. Measurement of reflexion is not quite so easy. In calculations Schmidt's value of 5% for open water and Ångström's value of 20% for green vegetation are used. Recent measurements by Pasquill[2] have confirmed that the latter is reasonable. Measurement of back radiation is difficult. Empirical formulae, based on such few measurements as exist, have been given by Ångström[3] and by Brunt.[4] Both show the net outflow as increasing with air temperature, with dryness of the air, and with clearness of the sky. In calculations Brunt's equation (the easier to handle) is preferred, using duration of bright sunshine to estimate clearness of the sky. Heat transfer to the air is estimated indirectly using a transport constant dependent upon wind speed in a way determined by experiment. Pasquill[2] has suggested that this constant is too small, but it could be 50% in error without having a serious effect on the resultant evaporation estimate. In the end it becomes possible to estimate the potential evaporation from a fresh green area of vegetation with an adequate water supply from knowledge of duration of bright sunshine, mean air temperature, mean vapour pressure and mean wind speed, supplemented by seasonal factors and constants obtainable from standard sources.

As a contrast to this diagram of the balance for an average summer day in a particular year, Fig. 2 shows the annual cycle for the Stour catchment area based on 10-yr monthly means.[5] This is given without any detailed explanation other than the following: (*a*) All energy is in evaporation units (1 mm = 59 cal/cm²) and the symbols are as in Table 1. (*b*) The income $R_I = R_c(1 - r)$, i.e. is the residue after allowance for 20% reflexion except in July, August and September when the factor was increased to 25% to allow for extra reflexion from ripening crops. (*c*) The balance was drawn up in a way that left heat transfer to the air as the only unknown, and it is encouraging to find that its annual cycle is in phase with Best's[6] measurements of temperature gradients (ΔT_a) over similar crop and soil. It will be noted that it is only in summer that evaporation is the major sink for solar energy, and that estimation of evaporation in winter cannot be as precise as in summer, but fortunately the same fractional efficiency is not needed in winter because the absolute amount is so small. About 90% of the year's evaporation takes place in the six months from April to September, amounting in southern England to about 18 in: for the winter six months the total is about 2 in.

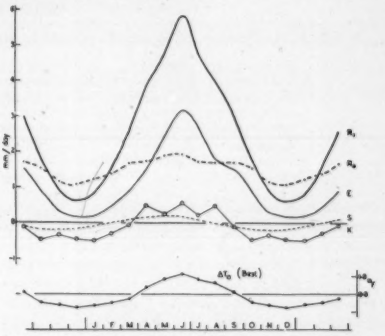

Fig. 2. Energy balance for grassland—based on ten-year monthly means. $R_I - R_B = E + S + K$ (1 mm/day = 59 cal/cm²/day)

SOIL MOISTURE DEFICIT AND RIVER FLOW

Out of this demonstration of the important part played by vegetation in the meteorology of the air-soil boundary two ideas emerge to be carried forward. First, vegetation transfers large quantities of water from soil to air: and, second, these quantities can be estimated from weather data. The former has important consequences for soil and plant; the latter offers the possibility of interpretation, prediction and control.

The two important sets of soil conditions of practical interest are usually found side by side in most river valleys in which the stream is merely an exposed portion of the underground water-table. Close to the stream the water-table (well level) is near the surface of the ground, and in such an area plants draw their water from a layer of soil in hydrostatic contact with the water-table. This layer, in turn, is re-charged by upward movement from the water-table, so that maintained steady drying conditions at the surface usually produce a steady fall in the level of the water-table. Away from the stream the water-table is so far below the surface that plant roots cannot reach the layer in hydrostatic contact with it, and in such places all water withdrawn by plants must come from the soil without any re-charge from below, i.e. the soil in the root zone gets progressively drier without having any effect on the water-table. It is convenient to refer to the amount of this drying as the "soil moisture deficit" or, more simply, the "deficit," the level of zero deficit being that at which the soil can hold no more water against gravity. This is an arbitrary level, but has the practical advantages that it is a state fairly easily recognized in the field because it is the state in which

through drainage first starts and at which it comes to an end, and it is a state that can be reproduced under controlled laboratory conditions. The study of natural evaporation is largely a study of short and long period changes in deficit: in agriculture it is important because when it becomes too big plant growth stops; in hydrology it is important because while it exists there is no appreciable movement of rainwater down to the water-table to replenish underground stores; and in soil mechanics it is important because of the changed physical properties of the soil, including actual shrinkage of some clay soils.

The dry weather flow of the stream is maintained by lateral drainage from the catchment. Away from the stream this causes a steady fall in the water-table, a fall not affected by surface wettings and dryings as long as there is a deficit in the soil above, but near the stream this underground current passes through a region in which deficits are negligible. Here, in effect, transpiring plants short circuit the current and the more active the transpiration the less the water reaching the stream, i.e. stream levels ought to fluctuate in anti-phase with transpiration. Observed day-to-day fluctuations in dry weather flow of streams are almost certainly a result of corresponding changes in transpiration rates, in turn resulting from day-to-day changes in weather, and a clear demonstration of the effect has been obtained on one of the streams of the Coweeta catchment in Asheville,[7] North Carolina, where continuous recording of stream level showed a marked diurnal oscillation, the level being higher at sunrise than at sunset. Although of no great practical importance, the phenomenon is a useful confirmation of the ideas from which it can be predicted, for it is known that transpiration rates go through a daily cycle in phase with meteorological factors that control it. In this same area near a stream there is a converse effect of a general state of near-zero deficit. There is little lag in response to rain, so that summer rainstorms produce a rapid local rise in the water-table and hence in stream level, and these short-lived summer spates come and go without in any way affecting the steady fall in well level at places away from the river bank.

CONTROL OF THE DEFICIT BY IRRIGATION

The important effects of plant transpiration are found where deficits can be built up. As the deficit increases the soil reaches a state of dryness at which extraction of water by the plant ceases to be easy, the transition being sharp in some soils and gradual in others, i.e. there is a value of the deficit beyond which the transpiration rate no longer is as great as the potential rate calculable from weather data. At this stage the growth rate will be reduced and very probably even before, so that in the interests of food production it is desirable to maintain the water supply at a level sufficient to maintain full transpiration, and an even greater supply may be needed to maintain full growth. Although guesses can be made at the value of the permissible upper limit, the only safe answer is to be obtained experimentally, crop by crop, and a start has been made in experiments run by the British Sugar Corporation on the irrigation of sugar beet, experiments carried out on commercial farms where the farmers are already using irrigation systems. Ignoring fertilizer aspects of the experiments, there have been either three or four groups of plots: (1) a control plot, left to natural rainfall; (2) a "farmer's" plot which he irrigates when his experience suggests that it is needed; (3) and (4) one (or two) plots which are irrigated on the basis of weather data collected on the field. The equipment set up includes screened temperature and humidity recorders, an anemometer and a raingauge. Sunshine records are obtained from the nearest official observatory.

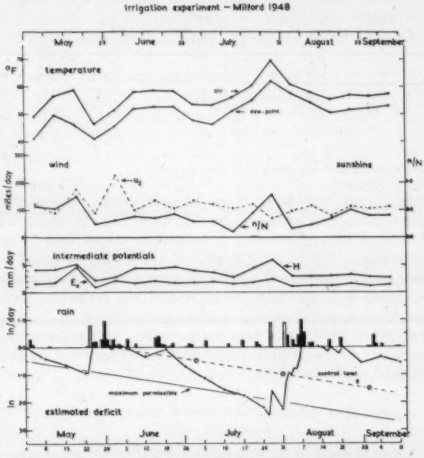

Fig. 3. Physical data for irrigation experiment, 1948
[*Reproduced from Quarterly Journal Royal Meterological Society*

Data are collected weekly and sent to Rothamsted Experimental Station, and the balance of evaporation and rainfall for the week estimated. Fig. 3[8] gives the physical history of the 1948 experiment, showing the week-by-week values of the four main weather elements and the two intermediate potentials calculated in estimating the week's evaporation. Weekly rainfall (closed column) and irrigation (open column) are also shown. The dotted line joining open circles shows the level of deficit aimed at throughout the season: below it, a full line represents the lower limit of tolerable deficit, the spacing being determined by the amount of water that could conveniently be applied in one day's work. The trend of deficit is shown by the full line joining full points. This season was not perfect, for on two out of three occasions rainfall was heavy after irrigation, and at the end of August it was predicted that irrigation would have no major effect on crop yield. This was so, as Table 2

reveals, but the weather was more helpful for the two experiments carried out in 1949, a magnificent year for this kind of experiment, and only marred by a severe virus disease infection that cut down all yields enormously. Briefly the story of the table is as follows. Treatment *O* is the control—natural rain only. Treatment *F* and treatment *J* are farmer's plots. (Note the great difference between the two in 1949. In terms of irrigation water, *J* had 4 in irrigation and *F* had 13 in irrigation.) Treatments *R* and *M* are based on weather data, and a guess at the order of magnitude of maximum permissible deficit: as it has happened, the guess has been good, for in all three experiments the weather-controlled treatments have given maximum yield. At Kesgrave the application of 6 in of water (treatment *M*) pushed up the yield of sugar by 60%.

Table 2. *Irrigation of sugar beet*

Yield of sugar (cwt/acre)—mean of all fertilizer treatments

Year	Site	Treatment	Deficit on 31 August (in)	Yield
1948	Milford	*F*	− 1·6*	64
		R	0·5	66½
		O	1·4	65½
1949	Milford	*F*	− 3·0*	41½
		R	1·5	44½
		M	3·0	43
		O	(7·0)	34½
1949	Kesgrave	*M*	2·8	49½
		J	4·3	46
		O	(6·5)	31

* Treatment *F* had too much water in both years. The negative deficit represents the estimated loss as drainage.

DEFICITS PRODUCED WHEN WATER SUPPLY IS LIMITED

The calculation of soil moisture deficit from weather data is only valid when there is adequate water available to meet the potential demand, a condition deliberately produced in the irrigation experiments except for treatments *O*. For these the estimates of Table 2 are much less certain: hence the brackets around them. The normal condition in large areas of the world, including south-east England, is that summer water supply is not sufficient to maintain maximum transpiration and growth. The plant's first main response to water shortage is to extend its root system, so tapping a deeper layer of soil, but there are limits to the rate and extent of this growth and a stage is reached at which the plant must be satisfied with what it can draw from the soil remote from the roots, and because water movement in a drying soil is so very, very slow, the transpiration rate falls off very abruptly to about one-tenth of what it could be. From laboratory data on the later stages of the drying of soil under constant drying conditions it has been possible to construct a synthetic curve connecting actual transpiration with potential transpiration,[9] so that seasonal changes in soil moisture deficit can be estimated from weather data even beyond the stage at which soil moisture becomes a limiting factor in transpiration rate. To construct the curve it is necessary to know how much readily available water can be held within the depth of rooting, and, fortunately, there is sufficient known about the rooting habit of most plants to be able to make an intelligent guess at this "root constant." The concept of a "root constant" is speculative, and has been introduced as a guide to future research rather than as a statement of what is true, but it is encouraging to find that no choice of the best value of the constant made on physical grounds has proved to be biologically unreasonable. Given the weather data, including rainfall, and knowing the dominant plant type, it should now be possible to make a quantitative study of the water balance of any cropped area. One example was given in the earlier survey, where it was shown that by calculating from the last date in spring when there was zero deficit (drains just stopped running) it was possible to predict the first autumn or winter running of field drains to within a few days. As the technique is successful for a single field, it should work for a whole catchment.

WATER BALANCE OF STOUR CATCHMENT AREA

Based on data supplied by a number of authorities a test has been made on the Stour (Suffolk and Essex) catchment month by month from 1933 to 1948.[10] The catchment lies mainly on the chalk of the north-east Chilterns, its area is about 330 square miles and it is roughly elliptical, the extensions of the long axis passing through Cambridge and Felixstowe. Monthly rainfall and river flow figures were available, and monthly values of potential transpiration were calculated from weather data recorded at Cambridge and Felixstowe. Estimates of actual transpiration were made by assuming the area to be in three parts: (*a*) an area near water-courses in which plants took their water direct from the water-table; (*b*) an area of deep-rooted vegetation for which a root constant of 8·0 in was used; and (*c*) an area of shallow-rooted vegetation for which a constant of 5·0 in was used (based on other experience of chalk soils). The water balance for any period is: rainfall is equal to the sum of evaporation, run-off, and increase in storage, the last representing the change in the quantity of water in the chalk below well level. As this was not measured it was made the unknown in the equation, rainfall and run-off being measured directly and evaporation estimated from weather data. Although this meant that an ideal direct check on the calculations was impossible, an adequate indirect check was obtained from well-level records at a site on the top of the same chalk ridge about 40 miles to the west of the catchment area. It was a very deep well, so a phase lag was to be anticipated, and it was sufficiently far away for its rainfall regime to be significantly different in some years, but in spite of these defects it was judged good enough to use. Fig. 4 shows

the plot of observed well level, and of estimated storage calculated from an arbitrary datum of 10 in on 31 March, 1933. Although the two curves are out of phase—as expected—it is obvious that the main features of the annual cycle and the long term trends have been adequately reproduced in the estimated storage curve. Study of the diagram will reveal a discontinuity between the upper and lower halves, the relative positions of the

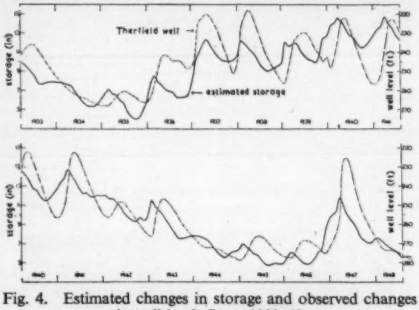

Fig. 4. Estimated changes in storage and observed changes in well-level: Stour 1933–48
[*Reproduced from Journal Institution of Water Engineers*

storage and well-level scales having been displaced by 10 ft on the well-level scale. The reason for the shift was a desire to free assessment of the goodness of fit in the second period from the effects of possible errors in the first, errors which are cumulative and include known differences in rainfall at the well and in the catchment during the first period. Some months after the completion of Fig. 4 a report came from the well site at Therfield stating that the measuring line had shrunk by 13 ft since it was last checked in 1937: the method of plotting virtually assumes that 10 ft had gone by 1941, but because it is not known how the 13 ft are to be spread over the period 1937–48, the diagram has been left untouched.

Two kinds of mean can be taken from the data. If the 15-yr monthly means are plotted, so giving a picture of the annual cycles, the two curves have much the same shape but the well curve lags about two months out of phase with estimated storage. If the lag is interpreted as the time taken for water to percolate 250 ft, then the average rate is about 4 ft/day, a very reasonable value for chalk. If the average annual values are plotted, based on a storage year from April to March, and a well year from June to May, so giving a smoothed picture of the long term trends already apparent in Fig. 4, a striking discontinuity is revealed round about 1938, from which it can be inferred that most, if not all, of the change in the line had taken place within a year of the 1937 check. Further analysis on an annual basis reveals an important relationship on which the next section is based: the 15-yr mean value of potential evaporation, as calculated, was 21·5 in/annum, whereas the actual annual mean, as estimated and as actually measured by difference of rainfall and run-off, was 20·0 in. The change affected by use of the synthetic curve is obviously not very great, and as these values are for the driest part of the British Isles, it seems reasonable to suppose that elsewhere a similar estimate of potential transpiration (without correction for large deficits) should come to within 5 or 10% of the actual evaporation. So, having stepped up from field to catchment area, the next stage is to consider the whole of the British Isles.

EVAPORATION OVER THE BRITISH ISLES

From four basic weather elements, regularly recorded in the Monthly Weather Report, it is possible to calculate potential transpiration: from measured values of rainfall and run-off it is possible to get a direct estimate of the actual transpiration. Weather data for 100 stations were collected and average annual evaporation was estimated for each from the long term averages: these values were mapped without correction, knowing that in south-east England they would be over-estimates of

Fig. 5. Average annual evaporation over the British Isles
[*Reproduced from Quarterly Journal Royal Meterological Society*

actual evaporation. From official and private sources mean annual values of rainfall and run-off were obtained for 40 catchment areas, ranging in size from 4 to 4 000 square miles: in the absence of deep percolation and changes in storage the mean difference for each catchment should be the mean annual evaporation. Plotted on the same map (Fig. 5) the two sets of data agree extremely well.[11] Theoretical values are in upright figures, not underlined: direct estimates are in sloping figures, underlined to show order of size of area, a dashed line for less than 10 square miles, one full line for 10 to 100, two lines for 100 to 1 000 and three lines for over 1 000 square miles. There are too many minor sources of error to permit discussion of accuracy and it must suffice to state that agreement within 10% would have been acceptable: it is pleasing to find that the general agreement is within 5%. As the two sets of estimates have nothing in common (air temperature, vapour pressure, wind and sunshine in one: rainfall and run-off in the other) their agreement is strong presumptive evidence that both are true and that this physical basis of estimating evaporation can be applied to parts of these islands where direct values do not exist.

The map can be regarded as a contribution to the climatology of the British Isles, or as an indication of the average annual water requirement for agriculture, or as showing the average annual loss that water engineers must allow for in planning impounding operations. None of these will be discussed here, and the only application to be made is one that could have followed the description of the irrigation experiments.

EFFECT OF VEGETATION ON CLAY SOILS

It will be noted that over south and south-east England the annual evaporation in an average year is 17 to 20 in, and as about 90% takes place from April to September, the potential summer evaporation is about 15 to 18 in, i.e. in dry summers when the seasonal rainfall may be only 6 to 9 in, the potential deficit that could be built up in the six months is about 9 in. This is the deficit that would be reached under trees, but under grass with its shorter root system the limit would be about 6 in. The withdrawal of this water affects many properties of the soil, irrespective of soil type, but for clay soils there is a further particular effect of considerable practical importance, and as such soils cover much of south-east England the effect is widespread. When these clay soils are saturated (i.e. at zero deficit) the porespace is full of water: as they dry the micro-porespace remains full of water, i.e. as water is removed it is not replaced by air, but, instead, the clay particles come closer together. This shrinkage of the aggregates necessarily involves the production of cracks between the aggregates and evidence of this can be found on almost any lawn in a dry summer. If the shrinkage is isotropic (it probably is nearly so) there will be an equal movement vertically, and for a deficit of 6 in. in a dominantly clay soil the vertical movement of the surface should be about 2 in. At the Building Research Station gauges have been designed to measure the vertical movement, and in Fig. 6 is a plot of the movements at several depths under grass-covered soil during the dry summer of 1947.[12] Extrapolation to find the surface movement gives a maximum value of about 2 in, which is what the meteorological and soil physics of the process predicts that it ought to be. A

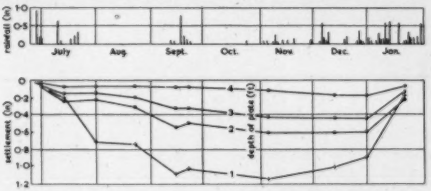

Fig. 6. Vertical movements of clay soil at Garston, 1947 [*Crown Copyright reserved*

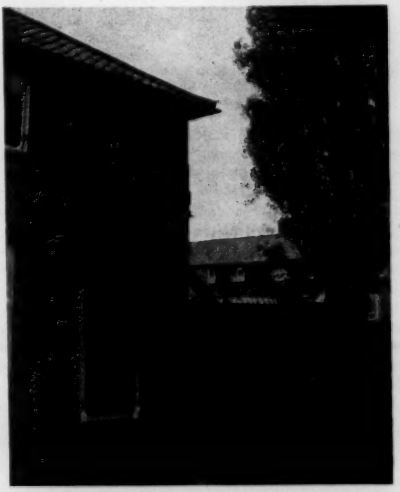

Fig. 7. Damage to a house, caused by shrinkage of clay soil, Edgware, 1947 [*Crown Copyright reserved*

house or a road on such a soil will not suffer if the shrinkage is the same everywhere under the foundation, but the mere act of covering the ground eliminates transpiration inside the area without stopping it round the edges, where there may be a lawn or flower bed. But many houses and roads have the aggravating circumstance of neighbouring trees, particularly poplar trees, which throw out a long shallow horizontal root system that may be 100 ft in length. So under the middle of

the house or the crown of the road there is little seasonal change in the moisture content and little change in volume: at the edges there is a seasonal cycle of alternate shrinkage and swelling, and eventually, of course, under this differential stress something gives way. Fig. 7 shows the effect on a house, and during 1947 and 1949 similar structural damage occurred in many buildings whose foundations were not deep enough to be below the main drying action of tree roots. The effect on roads may take the form of cracking (especially round manholes) or of tilting of slabs of road material.[13] On clay soils it is obvious that the amenity value of trees in gardens and by roadsides may often be obtained only at a rather high cost, particularly if the trees are poplars.

CONCLUSION

Although the survey has dealt almost exclusively with the part played by plants in the transfer of water across the earth-air boundary, it should be sufficient to demonstrate the great importance of vegetation as a factor in open air physics, pure and applied. Much remains to be done and the problems are sufficiently complex to present a worthwhile challenge in fundamental physics, the successful acceptance of which would be of great benefit to agriculture, meteorology, hydrology, soil mechanics and several other branches of applied science.

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ORIGINAL CONTRIBUTIONS

A theory of stresses in glass butt seals

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A series of equations are derived which may be used for the calculation of the stresses in glass butt seals and three component graded seals between materials of differing expansion coefficients. Stress determinations were made by the photoelastic method and good agreement was found between the theoretical and experimental values.

The theory set out in this paper deals with the magnitude and the distribution of stresses in glass seals in which two tubes of materials of different expansion coefficients are sealed end to end at a high temperature and then cooled down to a lower temperature at which both materials are rigid. In practice both tubes may be glass or one may be metal. The theory is later developed to deal with seals consisting of more than two components. The graded seal shown diagrammatically in Fig. 1 is an example of this. In a graded seal the end tubes of widely different coefficients are joined by a series of short lengths of glass of intermediate expansions.

THE STRESSES IN SIMPLE BUTT SEALS

Fig. 2 shows a butt seal in which two cylinders, 1 and 2, are sealed end to end at A. The tubes are made from materials of different expansion coefficients and it is assumed that they are of semi-infinite length; the cylinders have the same mean radius R and wall thickness t. The sealing is carried out at a high temperature after which the seal is cooled; it is assumed that there is a temperature T_1 above which the seal is stress free and below which any difference in the rate of contraction of the two materials causes stresses to be set up in the tubes.

The seal is cooled to a temperature T_2 below T_1, and

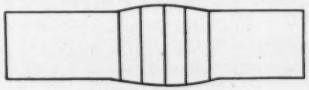

Fig. 1. Graded seal

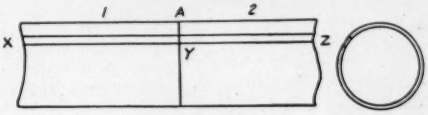

Fig. 2. Butt seal

it is required to find the stresses in the seal at this temperature, T_2. It is assumed for simplicity that the expansion coefficients α_1 and α_2 of the two materials, 1 and 2, are constant over the temperature range T_1 to T_2.

Since the tube is symmetrical about its central axis the stress distribution will also be symmetrical about this axis and the problem may be simplified to a consideration of the equilibrium of the strip *XYZ* (see Fig. 2). At the temperature T_1 the line representing the centre line of the strip *XYZ* will occupy the position *CD* shown in Fig. 3. If the tubes had not been sealed together the

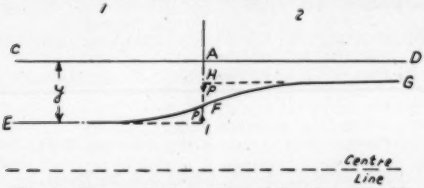

Fig. 3. Movement of centre line of glass butt seal with change of temperature

centre line of the strip in tube 1 would have moved to the position *EI* at the temperature T_2 and in tube 2 it would have moved to the position *HG*. The stresses caused by the expansion difference cause deflexions of the centre lines, of η_1 in tube 1 and η_2 in tube 2, so that the actual position of the centre line is represented by the full line *EG*. If the ratio, t/R, of the tube wall thickness to the tube radius is small, then the hoop strain e_θ due to the stresses is uniform through the wall of the tube and is given by $e_\theta = \eta/R$. The corresponding hoop stress p_θ is given by

$$p_\theta = \frac{E\eta}{R} \tag{1}$$

where E is the Young's Modulus. Thus if the variation of η with the distance from the joint can be found, then by using equation (1) it is an easy matter to find the variation of the hoop stress p_θ with the distance from the joint.

THE STRESS AT THE JOINT

If both tubes have the same Young's modulus then at the joint $IF = FH$ or $\eta_1 = \eta_2$. It is easy to see that:

$$\eta_1 + \eta_2 = R\alpha_1(T_1 - T_2) - R\alpha_2(T_1 - T_2)$$

or $\eta_1 + \eta_2 = R\delta$, where $\delta = (\alpha_1 - \alpha_2)(T_1 - T_2)$

Thus: $$\eta_1 = \eta_2 = \frac{R\delta}{2}$$

and $$p_{\theta_1} = -p_{\theta_2} = \frac{E}{R}\frac{R\delta}{2} = \frac{E\delta}{2} \tag{2}$$

Now the value of Young's modulus of glass varies very little with its composition. Thus equation (2) may be used to find the value of the hoop stress at the joint in a glass to glass butt seal. A typical value of E for glass may be taken as $0{\cdot}6 \times 10^6$ kg/cm^2. Thus if δ is known for a particular combination of glasses the value of the hoop stress in a butt seal made from these glasses may be calculated. It is not an easy matter, however, to determine the value of δ. The difficulty lies in determining the value of T_1 and also in the fact that the contraction curves of the glasses are affected by the cooling rate. These points are discussed fairly fully by A. W. Hull and E. E. Burger[1] and also in a recent paper by F. W. Martin.[2]

It is possible, however, to obtain a value of δ for a given combination of glasses 1 and 2, by making from these glasses a bead seal of the type shown in Fig. 4. The theory of the stresses in this type of seal was worked out by Poritsky[3] and it was shown by G. D. Redston and J. E. Stanworth[4] that experimental measurements of stresses in these seals were in good agreement with the values calculated from Poritsky's equations. Redston and Stanworth measured the hoop and radial stresses in bead seals at the joint, i.e. at points such as *X*, the stresses being measured photoelastically, using an optical bench described by them in earlier publications.[5, 6] The plane polarized light beam was passed through the seal parallel to its axis and the retardation introduced by the stresses was measured at the joint. The retardation at the joint is directly proportional to the sum of the hoop and radial stresses there. Its value is given by the equation

$$R = 2Cl\left[\frac{E\delta}{1 + \alpha + \alpha\beta}\right]$$

where C is the photoelastic constant of the glass, E is the Young's modulus, $\alpha = a^2(1 - \sigma)/b^2$ and $\beta = b^2/a^2 - 1$. a, b, and l are seal dimensions indicated in Fig. 4; σ is the Poisson's ratio of the glass which is assumed to be 0·3.

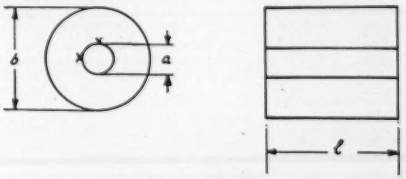

Fig. 4. Bead seal

If the retardation measurements are made in a bead seal by the above method, then the value of δ may be calculated for the two glasses used in making the seal. This value of δ may then be used to calculate the stress expected in a butt seal made from tubes of the same two glasses. This calculated stress may then be compared with the stress as determined photoelastically on actual butt seals. The latter determination is carried out quite simply by immersing the seal in a liquid of approximately the same refractive index as the glass, a plane polarized light beam passing through the seal parallel to the joint as shown in Fig. 5. The retardation at the joint is given by the equation:

$$R = 2Ctp_\theta$$

where C is the photoelastic constant of the glass. There are, it is true, other stresses at the joint in addition to the

hoop stresses p_θ. Each of the strips *XY* and *YZ* behaves as a bending beam and therefore there will be shearing stresses and axial bending stresses present in these strips. The shearing stresses should not contribute to the retardation since they are directed along the light beam. The axial bending stresses, also, should have no effect since they will be tensile on one side of the neutral axis in the strip and compressive on the other side, the stresses

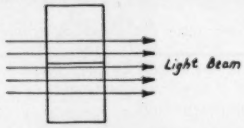

Fig. 5. Photoelastic method of determining stress in glass butt seal

on one side of the neutral axis will cancel out the effect of the stresses on the other side. Thus only the hoop stresses contribute to the retardation. In the following table the experimental values of the hoop stresses at the joint in the butt seals are compared with the values calculated from equation (2).

Sealing combination	*δ from bead seal measurements*	*Hoop stress at the joint*	
		Theoretical	*Experimental*
1	3·92	117·6	111·0
2	3·30	99·0	115·5
3	2·18	65·4	83·4
4	2·49	74·7	76·0

Each value is the mean of measurements made on three seals. The agreement between the theoretical and experimental values is as good as can be expected. Bead seals made entirely from glass are not easy to produce and as a result they are rarely symmetrical. This disturbs the ideal stress distribution so that the value of δ obtained from measurements on such a seal are not very accurate.

The Distribution of Stresses away from the Joint. The radial deflexion of the centre line of the strip *YZ* from its "free" position *HG* is $-\eta_2$. (The negative sign is used in accordance with the usual sign convention that compressive strains and stresses are negative and tensile strains and stresses are positive.) The radial strain is given by $e_\theta = -\eta_2/R$ and the corresponding hoop stress $p_{2\theta}$ by the equation

$$p_{\theta_2} = -\frac{E_2\eta_2}{R} \tag{3}$$

where E_2 is the Young's modulus of the material of tube 2. The stress p_{θ_2} may be replaced by a force of $P_{\theta_2} = p_{\theta_2}t$ per unit length of the strip acting on the side faces of the strip as shown in Fig. 6(*a*). These forces will have a radial component of P_{r_2} (see Fig. 6(*b*)), which is given by the equation

$$P_{r_2} = 2P_{\theta_2}\sin\frac{\delta\theta}{2}$$

where $\delta\theta$ is the angle subtended by the strip at the centre line of the tube. If the width of the strip, *w*, is small, then

$$\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2} = \frac{w}{2R}$$

therefore

$$P_{r_2} = P_{\theta_2}w/R = p_{\theta_2}wt/R \tag{4}$$

Combining equations (3) and (4) we have

$$P_{r_2} = -\eta_2E_2wt/R^2 = -K_2\eta_2 \tag{5}$$

where K_2 is a constant for the tube.

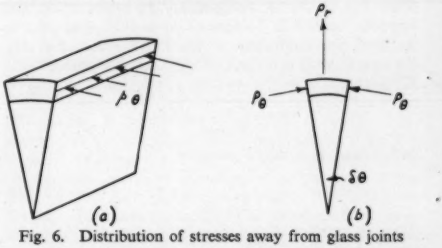

Fig. 6. Distribution of stresses away from glass joints

Thus the radial reaction P_{r_2} exerted by the rest of the tube on the strip *YZ* is directly proportional to the deflexion η_2.

The standard engineering theory (Timoshenko and Lessels[7]) of stresses in a beam resting on an elastic foundation is based on the assumption that the upthrust per unit length on the beam is proportional to the displacement of the beam from its original position. It is possible, therefore, to treat the strips *XY* and *YZ* as beams resting on an elastic foundation formed by the rest of the tube and thus the standard results may be applied directly to this problem. Fig. 7 shows a semi-

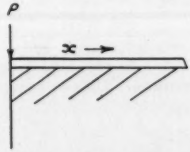

Fig. 7. Illustrating standard theory of stresses in a beam resting on an elastic foundation

infinite rectangular beam attached on its under surface to an elastic foundation. The beam is loaded at one end, the load being *P*. Then according to the standard theory the deflexion produced by this load is given by the equation

$$\eta = \frac{P}{2\beta^3ET}e^{-\beta x}\cos\beta x \tag{6}$$

where *x* is the distance from the loaded end of the beam, *E* is the Young's modulus of the material of the beam,

I is the moment of inertia of the beam section about its horizontal centre line, and β is given by the equation

$$\beta = \left(\frac{K}{4EI}\right)^{\frac{1}{4}} \tag{7}$$

where K is the "modulus of foundation" of the foundation. The modulus of foundation is simply the proportionality constant relating the upward reaction exerted on the beam per unit length to the deflection η. Equation (6) may be directly applied to describe the behaviour of the two strips XY and YZ. It may be seen from Fig. 3 that it is necessary to apply equal and opposite forces P to the ends of the two strips in order to maintain the continuity of the mean radius line EG. By equation (6) this force P will cause deflexions η_2 in YZ given by

$$\eta_2 = \frac{P_\theta^{-\beta_2 x}}{2\beta_2^3 E_2 I}\cos\beta_2 x \tag{8}$$

and deflexions η_1 in XY given by

$$\eta_1 = \frac{P_\theta^{-\beta_1 x}}{2\beta_1^3 E_2 I}\cos\beta x \tag{9}$$

(Note x is taken as positive in going away from the joint in both directions.)

In equation (8) β_2 is given by

$$\beta_2 = \left(\frac{K_2}{4E_2 I}\right)^{\frac{1}{4}}$$

From equation (5) it is seen that

$$K_2 = E_2 wt/R^2$$

Similarly:

$$\beta_1 = \left(\frac{K_1}{4E_1 I}\right)^{\frac{1}{4}}$$

and

$$K_1 = E_1 wt/R^2$$

If w is small the strip may be considered to be rectangular, then

$$I = wt^3/12$$

The sum of the two deflexions η_1 and η_2 at $x = 0$ must equal $R\alpha_1(T_1 - T_2) - R\alpha_2(T_1 - T_2)$ if the mean radius line is to be continuous. Thus we have:

$$\frac{P}{2\beta_2^3 E_2 I} + \frac{P}{2\beta_1^3 E_1 I} = R(\alpha_1 - \alpha_2)(T_1 - T_2) = R\delta$$

where

$$\delta = (\alpha_1 - \alpha_2)(T_1 - T_2)$$

Hence

$$P = 2R\delta I\left(\frac{\beta_1\beta_2^3 E_1 E_2}{\beta_1^3 E_1 + \beta_2^3 E_2}\right) \tag{10}$$

On combining equations (3), (8), (9) and (10), the equations are obtained giving p_{θ_1} and p_{θ_2}, the hoop stresses in the two tubes.

$$\left.\begin{aligned} p_{\theta_1} &= \delta\left(\frac{\beta_2^3 E_1 E_2}{\beta_1^3 E_1 + \beta_2^3 E_2}\right)e^{-\beta_1 x}\cos\beta_1 x \\ p_{\theta_2} &= -\delta\left(\frac{\beta_1^3 E_1 E_2}{\beta_1^3 E_1 + \beta_2^3 E_2}\right)e^{-\beta_2 x}\cos\beta_2 x \end{aligned}\right\} \tag{11}$$

If it is assumed that materials of both tubes have the same value of Young's modulus (this is a very good approximation to the truth in glass-to-glass seals) then equations (11) may be simplified to the following:

$$\left.\begin{aligned} p_{\theta_1} &= \frac{E\delta}{2}e^{-\beta x}\cos\beta x \\ p_{\theta_2} &= -\frac{E\delta}{2}e^{-\beta x}\cos\beta x \end{aligned}\right\} \tag{12}$$

It may be seen from the above equations that when $x = 0$, $p_{\theta_1} = -p_{\theta_2} = E\delta/2$. This agrees with equation (2) which was derived by quite simple methods.

The values of the bending moments M at distances x from the joint are given by

$$M_1 = -E_1 I\frac{d^2\eta_1}{dx^2} = -\frac{P}{\beta_1}e^{-\beta_1 x}\sin\beta_1 x$$

and

$$M_2 = -E_2 I\frac{d^2\eta_2}{dx^2} = \frac{P}{\beta_2}e^{-\beta_2 x}\sin\beta_2 x$$

If it is assumed that $E_1 = E_2$, then from equation (10)

$$P = RIE\beta^3\delta \tag{13}$$

and

$$\left.\begin{aligned} M_1 &= -RIE\beta^2\delta e^{-\beta x}\sin\beta x \\ M_2 &= RIE\beta^2\delta e^{-\beta x}\sin\beta x \end{aligned}\right\} \tag{14}$$

The shearing forces F at distances x from the joint are given by

$$F_1 = \frac{dM_1}{dx} = Pe^{-\beta_1 x}(\sin\beta_1 x - \cos\beta_1 x)$$

and

$$F_2 = \frac{dM_2}{dx} = -Pe^{-\beta_2 x}(\sin\beta_2 x - \cos\beta_2 x)$$

Again, if it is assumed that $E_1 = E_2$, these equations simplify to

$$\left.\begin{aligned} F_1 &= RIE\beta^3\delta e^{-\beta x}(\sin\beta x - \cos\beta x) \\ F_2 &= -RIE\beta^3\delta e^{-\beta x}(\sin\beta x - \cos\beta x) \end{aligned}\right\} \tag{15}$$

From the equations (12), (14) and (15) the stresses may be calculated at any point in a glass-to-glass butt seal. Photoelastic measurements of stress were carried out to check the theory. The methods of measurement were the same as those used for the measurement of the hoop stresses at the joint, but in this case the retardation introduced by the stresses was measured at various distances from the joint.

As has been pointed out before, the shear stresses and the bending stresses do not contribute anything to the retardation; it is due simply to the hoop stresses and is related to them by the equation

$$R = 2Ctp_\theta \tag{16}$$

where R is the retardation, t is the wall thickness of the tube and C is the stress-optical coefficient of the glass. Combining equations (12) and (16) the following equation for R is obtained:

$$R = \pm CtE\delta e^{-\beta x}\cos\beta x \tag{17}$$

In Fig. 8 the full curve shows the variation of $e^{-\beta x}\cos\beta x$ with βx. The points show the experimentally determined values.

The experimental values have been fitted to the theoretical curve at $x = 0$. There is a fairly good agreement between the theoretical and the experimentally determined stress distributions. From Fig. 8 the experimentally determined stresses appear to fall off more rapidly than is to be expected theoretically. It is doubtful whether this difference is significant. The retardation measurements involve determining the darkest position in an interference fringe, the width of which is of the order of 1 mm and this width is equivalent to a value of βx of 0·3. It is possible, therefore, that a large part of the discrepancy between the theoretical and experimental values may be due to a systematic error in determining the position of the centre of the fringe.

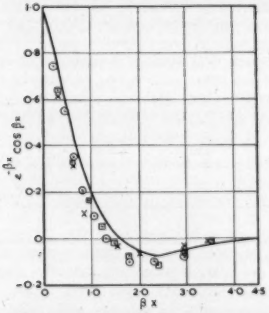

Fig. 8. Comparison between the theoretical curve for the variation of hoop stresses with distance from the joint and the experimental values determined on tubes having the following dimensions

× 19 mm diameter, 2 mm wall, $\beta = 0{\cdot}285$
○ 36 mm diameter, 2–2½ mm wall, $\beta = 0{\cdot}211$
⊡ 25 mm diameter, 1–1½ mm wall, $\beta = 0{\cdot}285$

It is a matter of some practical importance to know the magnitude of the maximum tensile stress in a glass seal. It may be shown easily by calculations based on equations (12) and (14) that in this seal the maximum stress is the hoop stress p_θ at the joint. The maximum stress due to the bending moment M occurs at a value of βx of about 0·8 (or about 3 mm from the joint in a typical seal). The magnitude of this maximum bending stress is about one half of the hoop stress at the joint for seals of normal dimensions.

THREE COMPONENT SEALS

The above theory of the butt seal may be applied to the type of seal in which two or more joints are spaced at relatively small distances from one another along the tube. One of the simplest types of seal of this type is that shown in Fig. 9. A short tube 2 made of one glass is sealed between two other tubes, 1 and 3, both made of another glass.

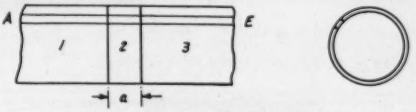

Fig. 9. Three component butt seal

Here also the problem may be reduced to a consideration of the equilibrium of the thin strip *AE*. In Fig. 10, *AJ*, *KL* and *IE* represent the positions at the temperature T_2 which the three sections of the strip would occupy if they were not joined together. If it is assumed that the three sections of the seal have the same radius at the temperature T_1 then the distance $JK = IL = R\delta$. Continuity has to be restored by the application of loads to the ends of strips *J*, *K*, *L* and *I*. The displacements due to the restoring forces of the seal at *JK* are given by the line *ABCD*, and those due to the seal at *IL* by the curve *HGFE*. The total displacement at any point is given by the sum of these two displacement curves. Thus at the

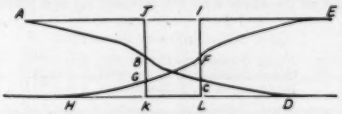

Fig. 10. Application of the theory to three component seals

joint *IL* the upward displacement of the point *L* is the sum of *FL* and *CL*. From equations (8) and (13),

$$CL = \frac{R\delta}{2}e^{-\beta a}\cos\beta a$$

and

$$FL = \tfrac{1}{2}R\delta$$

where a is the length of the central section of the seal. Therefore η_2, the upwards displacement of the point *L*, is given by:

$$\eta_2 = R\delta/2(1 + e^{-\beta a}\cos\beta a) \qquad (18)$$

The downward displacement η_3 of the point *I* is the difference between *IF* and *CL*. Thus

$$\eta_3 = \frac{R\delta}{2}(1 - e^{-\beta a}\cos\beta a) \qquad (19)$$

By combining equations (18) and (3), the following equation is obtained for the hoop stress at the joint in tube 2:

$$p_{\theta_2} = \frac{E\delta}{2}(1 + e^{-\beta a}\cos\beta a) \qquad (20)$$

The stress p_{θ_3} at the joint in tube 3 is given by

$$p_{\theta_3} = -\frac{E\delta}{2}(1 - e^{-\beta a}\cos\beta a) \qquad (21)$$

Thus the final equations for the stresses at the joints are

$$\left.\begin{aligned} p_{\theta_1} = p_{\theta_3} &= -\frac{E}{2}(1 - e^{-\beta a}\cos\beta a) \\ p_{\theta_2} &= \frac{E\delta}{2}(1 + e^{-\beta a}\cos\beta a) \end{aligned}\right\} \quad (22)$$

where p_{θ_1} is the stress at the joint in tube 1.

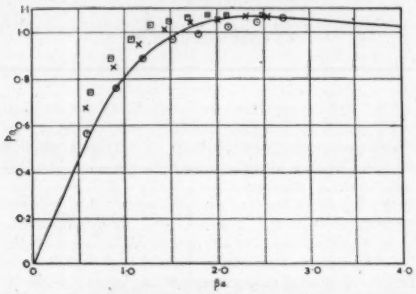

Fig. 11. Comparison between the theoretical curve for the variation of the stress with the distance p_{θ_1} in a three component butt seal and the experimental values determined on tubes of the following dimensions

○ 30 mm diameter, 1–1½ mm wall, $\beta = 0{\cdot}303$
× 19 mm diameter, 2–2½ mm wall, $\beta = 0{\cdot}285$
⊡ 36 mm diameter, 2–2½ mm wall, $\beta = 0{\cdot}211$

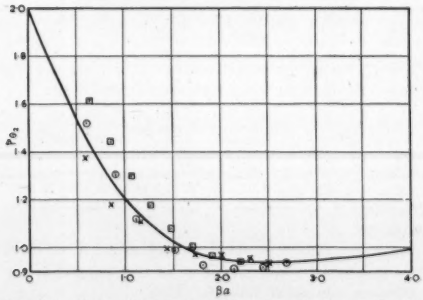

Fig. 12. Comparison between the theoretical curve for the variation of the stress p_{θ_2} with the distance a in a three component butt seal and the experimental values determined on tubes of the following dimensions

○ 30 mm diameter, 1–1½ mm wall, $\beta = 0{\cdot}303$
× 19 mm diameter, 2–2½ mm wall, $\beta = 0{\cdot}285$
⊡ 36 mm diameter, 2–2½ mm wall, $\beta = 0{\cdot}211$

The theoretical curves showing the variation of p_{θ_1}, p_{θ_2} and $p_{\theta_2}/p_{\theta_1}$ at the joint as functions of βa are given in Figs. 11, 12 and 13. The values of βa are in millimetres. The experimental values are determined on C11–C40–C11 seals of the type shown in Fig. 9. The experimental points are fitted to the theoretical curves at a large value of βa. The agreement between theory and experiment is seen to be good.

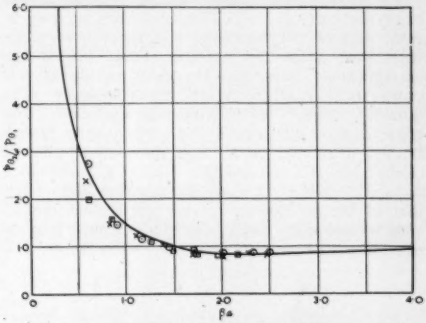

Fig. 13. Comparison between the theoretical curve for the variation of the stress ratio $p_{\theta_2}/p_{\theta_1}$ with the distance a in a three component butt seal and the experimental values determined on tubes of the following dimensions

○ 30 mm diameter, 1–1½ mm wall, $\beta = 0{\cdot}303$
× 19 mm diameter, 2–2½ mm wall, $\beta = 0{\cdot}285$
⊡ 36 mm diameter, 2–2½ mm wall, $\beta = 0{\cdot}211$

CONCLUSION

Equations have been derived for the stresses in a simple butt seal and also in a simple type of seal in which two joints are present which are spaced a small distance apart. These equations have been tested by photoelastic measurements of stresses and good agreement has been found between the theoretical and experimental values.

ACKNOWLEDGMENTS

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The use of the Geiger-Müller counter X-ray spectrometer in an X-ray laboratory, with special reference to automatic recording

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The advantages and limitations of Geiger counter techniques in a laboratory applying X-ray methods to a variety of problems are discussed and some simple examples are described. These illustrate the convenience of its use in obtaining semi-quantitative results very rapidly and in following the effects of various treatments. The accuracy of the method is considered in relation to the statistical effects associated with the operation of the Geiger counter and the influence of damping in the recording system.

The addition of Geiger–Müller counting devices to X-ray equipment has placed at the disposal of the investigator a means of directly and rapidly estimating X-ray diffraction intensities which he has not possessed since the early ionization chamber technique was generally discarded in favour of the photographic record. Several years' experience with the new technique at the National Physical Laboratory has made it evident that Geiger–Müller counter apparatus has a definite part to play in an X-ray laboratory and that for dealing with certain kinds of problem it has marked advantages over the photographic procedures.

The following example is typical of the more straightforward forms of application. A request was received for the examination of six metallic specimens, all somewhat dirty or oxidized, each of which was stated to consist of an iron base with a plated surface layer, probably nickel. The problem was to determine as rapidly as possible whether all the specimens were in fact plated with nickel. One of the specimens was immediately mounted on a Geiger–Müller counter X-ray spectrometer, and on the assumption that nickel diffraction lines would be found, a manual traverse was made over the range $2\theta = 70° - 40°$ at a rate of approximately 1° in 10 sec, using unfiltered iron radiation. The pen of the automatic recorder was switched on at the same time, but its chart remained stationary, and so long as no diffraction line was encountered, the pen suffered a small deflexion corresponding to background intensity and remained stationary upon the chart except for slight movement due to the natural fluctuations of the counts emerging from the Geiger–Müller counter tube. As soon as a diffraction line was encountered, however, an increasing deflexion was observed, and the effective position and intensity of the line was then determined in a few seconds by successively narrower bracketing traverses across the peak and the angle written down. The chart was then moved forward by hand to a new position and the angular survey continued. In this way eight lines were discovered and their approximate intensities obtained in less than 10 min, after which a few minutes' calculation sufficed to show that 4 of the lines corresponded to the (111) and (200) lines (α and β) from nickel, and the remaining 4 to the corresponding lines from nickel oxide. The other specimens were examined in the same way, with some reduction in the time required for each due to the information already gained from the first specimen.

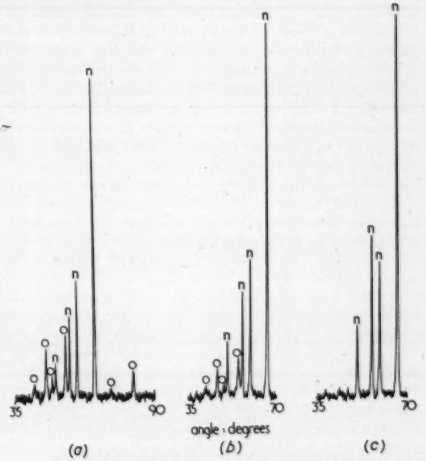

Fig. 1. X-ray Geiger-Müller counter record (about $\frac{1}{3}$ full size) of a nickel-plated metal specimen.

n = nickel. o = nickel oxide. (*a*) As received showing nickel oxide. (*b*) After abrasion showing reduced nickel oxide. (*c*) After further abrasion removing nickel oxide.

From 5 of the specimens the same evidence of nickel and nickel oxide was obtained, but in the sixth case no lines at all were observed, though it was noted that the background intensity approached twice the normal. The suggestion was therefore made that the exceptional surface coating would be found to be chromium, since this element would strongly absorb and scatter both the α and β wavelengths of iron, and chromium is a common plating metal. This suggestion was subsequently confirmed by a chemical test.

It is to be noted that roughly quantitative results were being obtained from the first sample within 1 or 2 min of the enquiry being presented. The complete survey extended to about $\frac{3}{4}$ h, a time which compared favourably with that required for the simpler forms of chemical test which might have been applied, and very favourably with normal photographic procedures, in which the time

required for the development of six films would alone approach ½ h.

In Fig. 1*a* there is shown a record obtained from the first specimen by traversing the Geiger–Müller tube automatically at a rate of 1° in 36 sec (the maximum available) and running the chart at a speed of 0·25 in in the same time. Both methods show the very much enhanced value of the (200) diffraction line of the electro-deposited layer. The two procedures have supplied essentially the same information, so that the decision as to which is the most profitable method in cases of this kind is to be determined both by considerations of expedience and by the application of experience to the particular features of a problem. It may sometimes prove desirable to transfer to the mechanical survey after a first manual observation, or vice versa.

In Figs. 1 *b* and *c* are shown the traces obtained after successive attempts to remove nickel oxide from one of the specimens by means of abrasive, the left-hand curve indicating that at this stage a small amount still remains. The curves of Fig. 1 illustrate another feature of direct X-ray observation, in that it is possible to combine successive modifications in the condition of a material with immediate X-ray examinations.

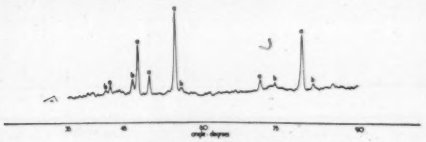

Fig. 2. Automatic Geiger-Müller counter record of iron oxides (about ⅛ full size)
a = iron oxide (FeO) b = iron oxide (Fe_3O_4)

Fig. 2 illustrates a survey made upon a sample of nominally pure iron oxide (FeO), the traversal of the angular range $2\theta = 90° - 35°$ at a rate of 1° in 144 sec requiring about 2 h. The existence of a proportion of the higher oxide Fe_3O_4 is apparent. The time required for this survey approximated to the normal exposure time by the photographic method.

It is evident from these examples that there is a very considerable advantage to be gained from the Geiger–Müller counter technique if there is involved the measurement of the intensity or some other property of a single X-ray line or of several lines. Thus a specific test made at the known positions for the nickel lines would have been accomplished in less than 60 sec. In much the same way it was found that when making a survey of a sample which displayed orientation vagaries over its surface, readings could be obtained from successive regions at a rate of 30/h, differences of 5% being observed with ease. In another investigation involving the measurement of line-breadths, line contours were plotted in about 30 min.

When the problem is of an elementary kind it may be unnecessary to make use of any formal measurement. The simplest of observations, applied almost instinctively by the regular user, is undoubtedly the sudden impression on the ear when there is a significant change in the rate at which pulses are reaching a mechanical counter. Similar responses may be obtained with devices like the electronic "magic eye," and visual observation of the altered pulsing rate of the neon tubes of a scaling system, of the changing rate of movement of a mechanical counter, or of the first response of the pointer of a rate-meter are other obvious sources of spontaneous information.

INTRINSIC FEATURES OF THE GEIGER–MÜLLER COUNTER TECHNIQUE

The above discussion has shown that by using the Geiger–Müller counter devices there will result a very considerable gain in facility when certain types of problem are being investigated. It is now necessary to consider two factors which are mainly responsible for attaching limits to the practical scope of the method. The first limitation is imposed by the finite size of the aperture (usually a slit) through which the diffracted X-rays pass into the Geiger–Müller counter tube. The pulses emitted by the tube are related to the sum total of the intensities of the X-rays passing through different parts of the aperture, and obviously provide no indication of any variation across its area. If an X-ray line is continuous the only effect is to cause a slight broadening and modification in shape, exactly as the finite area of a photometer slit modifies slightly a photographic record. This is the condition when the sample is a fine powder or a fine grained solid. If, however, the X-ray line is discontinuous due to a larger particle or grain size in the sample, the influence upon the observations may be considerable, and in the limiting case when only a few X-ray spots are produced instead of an approximation to a continuous line, there is a distinct probability that there will be no indication of the existence of diffraction rings. In such circumstances a photographic record will most probably still provide the essential evidence.

Rather more complicated conditions may arise when technological materials are being examined. After heat and working treatments they may display deviations from the fine grain continuous X-ray line condition which are not so much simple grain growth effects as the development of fine detail within the X-ray lines. Apart from this the lines remain more or less continuous. Detail of this kind can provide much useful information to the experienced observer, but cannot be expressed in any quantitative fashion. It will obviously not be recorded by a Geiger–Müller counter survey.

It is evident that the Geiger–Müller technique is to be regarded essentially as an appropriate method for application to materials which are fine-grained, or which can be made to produce continuous lines by rotation, oscillation or traversing of the specimen in order to bring a multiplicity of grains into the reflecting position.

While the effect of simple grain growth may be remedied in this way, the fine detail effects will not in any case be observed. It follows in general that a material in an entirely unknown condition should not be examined in the first place by the Geiger–Müller technique without the qualifying knowledge that, in spite of positive information derived from the record, evidence may be missed which would have been recorded by the photographic technique.

In practice, experience of the properties of the working materials is of considerable assistance in determining which should be the mode of preliminary attack in any given instance.

The second limitation imposed on Geiger–Müller counter measurements is related to the errors of observation. Although the actual number of pulses emitted by a Geiger–Müller tube may be counted accurately by electronic methods, intrinsic errors arise from their statistically random distribution around the average rate, the probable error, that is the error below which lie 50% of any large number of observations, being 0·67 times the square root of the average number of counts in the time concerned or, for most practical purposes, 0·67 times the square root of the observed counts in an observation. An ultimate limit to the counting rate is set by the characteristics of the Geiger–Müller tube itself, being of the order of 10^4 counts/sec. This figure is hardly a practical limit, if only because sensitivity disappears as it is approached, and it is usual to work at considerably lower rates. With standard X-ray equipment there is no difficulty in obtaining counts of more than 10^3/sec at the maxima of strong lines, but for conservative estimation the figure of 5×10^2/sec may be selected as a somewhat arbitrary working maximum. In the same way a count of 10/sec may be chosen as a minimum count obtained from a rather low background intensity. With these rates the 4 000 counts required to give a probable error of about 1% are obtained in about 10 sec and 5 min respectively. It is evident that there are definite limits to the scope of Geiger–Müller counting mechanisms, determined in the first place by the characteristics of the counter tube, and secondly by the necessity for maintaining the time schedule of operations within reasonable limits. Thus to increase a given accuracy tenfold requires a hundredfold increase in the observational time scale if the counting rate remains the same.

AUTOMATIC RECORDING

The errors of observation cannot be less than those connected with the randomness of the pulses and with the number received during a given time interval. Consequently there can be no gain in accuracy by the introduction of additional circuits and the recording of observations by means of a ratemeter or an automatic recorder. Thus the advantages of an automatic recording are connected intrinsically with convenience of operation. Automatic recording adds to the Geiger–Müller counting technique an elementary merit of the photographic method, in that the production of the record need absorb only a small proportion of the operator's time.

The most direct automatic record of discrete counts would be that obtained by a teleprinter enumeration of the numbers of pulses obtained during equal intervals of time, provision being made for moving the counter tube by a pre-set amount after each period. Alternatively there might be entered as ordinates on a chart successive lengths proportional to the number of counts received during equal intervals of time, both counter tube and chart being moved on by predetermined amounts between readings. Both these methods would give the essential results of direct counting, the second method giving also the profile of the pattern. However, the normal interpretation attached to the term "automatic recorder" involves an extension of the ratemeter principle, the pen of the recorder providing a measure of the average counting rate. The desired smoothing of the random fluctuations is secured, as with a ratemeter, by the introduction into the circuit of a resistance-capacity unit with a suitable time-constant. The Geiger–Müller tube and the recorder chart travel continuously and synchronously. It is with this type of recording mechanism that the curves already depicted have been obtained. The qualities of adequate sensitivity, consistency in operation, and linear relationship are satisfied by commercially available equipment. It may be noted that when it is desired to record both strong and weak diffraction lines during a single run, a relationship other than linear, e.g. logarithmic, offers advantages.

It is now appropriate to discuss errors of observation in their relation to an automatic recording process.

If counts arrive at a steady (statistically) average rate which produces a corresponding average deflexion of the recorder pen, the area beneath a given length of the trace is a measure of the pulses received during that period, and the error is given by the difference between the actual and the average areas. More simply, for like periods the error is measured by the difference between the average ordinate, and may be considered in relation to the probability of error estimated statistically. Since the errors are normally small, they may be described conveniently in terms of a small unit of measurement on the chart, say 1/100th of the full scale width.

If c represents the number of counts/sec to give full-scale deflexion, r the average recorded ordinate measured in terms of the chosen units of which the chart width contains s, a and l the angular and linear rates respectively of spectrometer and chart, and A and L the ranges traversed in time t, then

Probable error (numerical)

$$= 0{\cdot}67\sqrt{\frac{crt}{s}} = 0{\cdot}67\sqrt{\frac{crA}{sa}} = 0{\cdot}67\sqrt{\frac{crL}{sl}}$$

Probable error (scale deflexion)

$$= 0{\cdot}67\sqrt{\frac{sr}{ct}} = 0{\cdot}67\sqrt{\frac{sra}{cA}} = 0{\cdot}67\sqrt{\frac{srl}{cL}}$$

The probability that the observed error will exceed any given multiple of the probable error is listed in Table 1.

Table 1. *Probability of occurrence of observed errors greater than P.E.*

Observed error	*Probability of occurrence outside this limit* %
1 P.E.	50
2 P.E.	17·73
3 P.E.	4·30
4 P.E.	0·698
5 P.E.	0·0745
6 P.E.	0·0052

When applying these probabilities to X-ray observations the above figures are divisible by two, as we are concerned with positive deviations from the average level. It is evident, for example, that there is a probability of about 300 : 1 that an excess of counts greater than 4 P.E. is not due to background fluctuations and is a real diffraction effect. For a deviation of 6 P.E. the figure is about 40 000 : 1.

The trace of the background in Fig. 2 may be used as an illustration. There $s = 100$ divisions, $r = 20$ divisions, $c = 250$ counts, $a = 1{\cdot}0°$ in 144 sec, $1 = 0{\cdot}0015$ in/sec and

$$\text{P.E.} = \frac{1{\cdot}9}{\sqrt{t}} = \frac{0{\cdot}158}{\sqrt{A}} = \frac{0{\cdot}074}{\sqrt{L}} \text{ divisions.}$$

The variation of the Probable Error in divisions with the time of observation, t, is shown in Table 2.

It is clear that if the recorder pen were able to respond to the variations occurring within 1 sec or less, the fluctuations about the average background reading would be so large and so frequent as to make it impracticable to estimate the smaller overall deviations reckoned over significant intervals. The much smaller spread of the background record is the result of incorporating in the system a time constant of about 34 sec.

INFLUENCE OF TIME-CONSTANT

For the steady state in which changes in counting rate are due to statistical fluctuations only, Schiff and Evans have established the relationships between time-constant and anticipated errors in a ratemeter containing a resistance-capacity unit, having a time-constant RC defined by the product of the resistance R and the capacity C. They show that the Probable Error of a single momentary reading from a ratemeter (e.g. recorder) is $0{\cdot}67(2\times RC)^{-\frac{1}{2}}$, where x is the expected number of pulses in unit time, basing their argument on the assumption that the influence of each pulse dies away exponentially as determined by the time-constant RC. Consequently the single reading has the same accuracy as the counting of pulses for a period twice the length of the time-constant, this being a measure of the "memory" of the resistance capacity system. When the average of ratemeter readings over a period of time is compared with the number of direct counts in the same period, the probable error of the average is found by this estimation always to be less than the probable error of the direct count, and to be the same as the probable error of a direct count taken over a period somewhat longer than the period over which the ratemeter average was taken. The effective extension of the period varies from $2RC$ when $t = 0$ to $1{\cdot}5RC$ when t approaches infinity. Thus the reduction in probable error is important when the time intervals under consideration are short, and negligible when they are long; it is about one-third when the period over which the observations are averaged is equal to the time-constant.

We may now consider the case of X-ray diffraction peaks superimposed on a background counting level. When examining a diffraction pattern it is natural to proceed by first noting the strongest lines, then those of medium or rather low intensity, and finally those deviations from the background which are difficult to assess with certainty. This is not the logical process, and the search should be for significant fluctuations above the average background rather than for supposed line shapes of limiting intensity. A weak line which gives a significant increase above the average may have little resemblance to the normal shape, because of the additive effects of subsidiary background fluctuations.

Referring to the trace of Fig. 2, the effective width of the X-ray lines of the pattern is about 1°, corresponding to a chart displacement of 0·22 in, 1° being scanned in 144 sec, so that with a time-constant of 34 sec the effective time of observation is $(144 + n \times 34)$ sec, where n has a value between 1·5 and 2·0. The choice of $n = 2{\cdot}0$ gives the most stringent condition. Thus the Probable Error for a single observation is

$$\frac{1{\cdot}9}{\sqrt{(144 + 2 \times 34)}} = 0{\cdot}13 \text{ divisions}$$

and the probabilities for multiples of this error are listed in Table 3.

Table 3. *Probability of Background Errors*

Error	*Average displacement from background in divisions*	*Probability*, %
1 × P.E.	0·13	25·0
2 × P.E.	0·26	8·8
3 × P.E.	0·39	2·15
4 × P.E.	0·52	0·35
5 × P.E.	0·65	0·037
6 × P.E.	0·78	0·0026

Table 2. *Variation of P.E. with time of observation, t*

t seconds	1	4	16	30	60	120
P.E. divisions	1·9	0·95	0·475	0·35	0·245	0·175
P.E. as a percentage of the average recorded ordinate	10%	5%	2·5%	1·7%	1·2%	0·85%

If a probability of 1/300 is adopted as an arbitrary criterion, corresponding to a mean displacement of 0·52 divisions (4 × P.E.), a number of peaks of low intensity are found to be significant and can be allocated to the diffraction patterns of FeO and Fe_3O_4.

Several other positive deviations which may be significant are also to be found. In particular, high values at about 40·5°, although giving no definite line shape, coincide in position with the strongest line of the α-Fe_2O_3 pattern, and consequently it would seem reasonable to postulate that the sample contains a small proportion of this oxide of iron. To check the actual existence of this or any other line it is necessary to make several more traverses over a limited range containing the particular angles concerned in order to determine whether the incidence of the deviation is greater than its calculated probability.

From observations of this kind upon the traces from background radiation, it is evident that assuming the strong lines of a pattern to correspond to full-scale deflexion, lines of about 2% of the maximum intensity are almost certain to be picked out from the background. The conditions provided by the X-ray equipment make this estimation of sensitivity definitely conservative. One outstanding difficulty has not been mentioned in the above arguments, namely that if a limiting deviation from a weak line coincides with a *low* background, there may result a normal background and thus no evidence of the existence of the line.

Criteria having been established for the existence of a line (e.g. a probability of 300 : 1 or some other value chosen to suit the requirements of the work) consideration may be extended by normal arguments to the expectation of discovering one or more false maxima in a survey covering any given angular range, and of repeating any one observation if corroborating runs are made.

It is necessary to take note of the modification in the Probable Error due to observations which involve two superimposed counting rates which is the normal case in X-ray work. The Probable Error of the sum or difference of the counts, at say the peak position of a line, is then $0{\cdot}67\,(n_1 + n_2)^{\frac{1}{2}}$, where n_1 is the count obtained from the background during a given time interval and n_2 the count from background plus line. Thus when line and background are equally strong, six times as many counts are required to achieve a given accuracy as would be the case if the background were missing. The respective corrections are small in the two limiting cases when the line is either very strong or very weak compared with the background. In assessing the possible existence (not the accuracy of observation) of very weak lines, only the background count need be used to establish the criteria.

EFFECT OF TIME-CONSTANT ON LINE-SHAPE AND POSITION

A ratemeter mechanism is normally provided with the means of varying the time constant to suit the experimental conditions. In Figs. 3, 4, and 5 there are shown three sets of curves constructed artificially by imposing millivolts from a potentiometer upon the input terminals of the recorder. The first curves show the expected modifications in the appearance of a flat-topped peak when four different time-constants are used. The second set shows the modifications of an artificially contrived triangular peak which is an approximation to an X-ray line shape. The third set shows the corresponding modifications when neighbouring peaks are traversed. In all cases the uppermost curve was obtained with a time constant so short that it was closely representative of the true variation of the electromotive force supplied by the potentiometer. The remaining time-constants were 3, 10, and 25 sec.

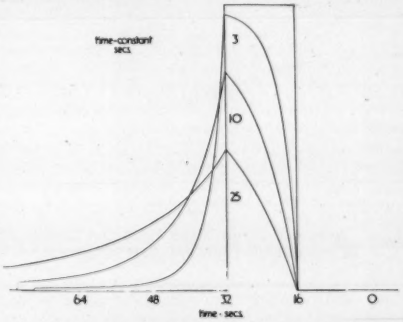

Fig. 3. Automatic record (about ¼ full size) showing effect of time-constants on a rectangular "line shape"

Using the elementary fact that the driving force which moves the pen is proportional to the difference between the ordinates of the true and modified curves at any moment, several interesting deductions may be made. If the straight sides of the true curves were prolonged indefinitely, the modified curve would eventually rise at a similar rate, the difference between them being proportional to the time-constant, or with a given time-constant to the slope of the line. Secondly, a point of inflexion must occur on the modified curve immediately below the maximum ordinate of the true curve. Thirdly, the maximum value of the modified curve must lie at the point where the two curves cross, and if in the case of neighbouring peaks a minimum value appears on the modified curve it must be at the point where the rising edge of the second peak is crossed. There will also be a point of inflexion immediately above the true minimum between the curves. Lastly, the modified curve will be an exponential decay curve beyond the region where the true curve joins or closely approaches the background. These criteria are all specific for ideal conditions of operation and apparatus. In practice they will be satisfied closely enough to provide useful information

about the possible deviations from true line shape or peak position. There are also practical limitations, for although it is theoretically possible to mark the true peak position of a line by observing the point of inflexion on the rising modified curve, this can only be done accurately if the line is drawn on a large horizontal scale. This introduces practical difficulties if a large number of lines is involved.

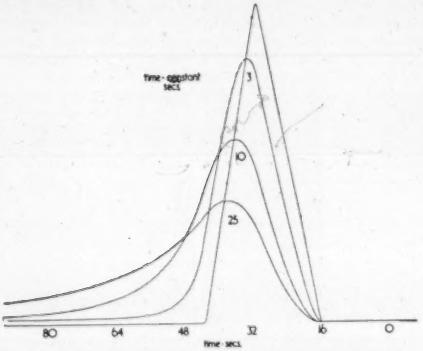

Fig. 4. Automatic record (about ½ full size) showing effect of time-constants on a triangular "line shape"

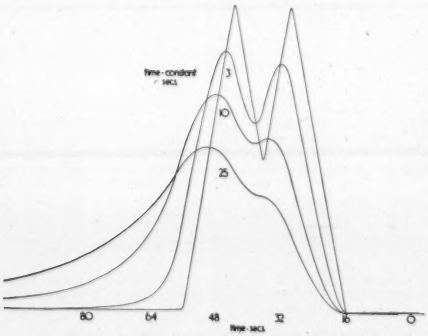

Fig. 5. Automatic record (about ½ full size) showing effect of time-constants on neighbouring triangular "line shapes"

A further modification attaches to the curves obtained from neighbouring peaks. It is clearly shown in Fig. 5 that with a large time-constant there occurs an enhancement of the intensity of the second line relative to the first. This is of practical importance, and when the relative intensities of close neighbours are to be assessed it is necessary to ensure that angular rate and time-constant are so chosen that a correct relationship between the peak values is preserved.

Fig. 6 illustrates the practical significance of these considerations, traverses being made across an X-ray line from a crystalline sample, which was strong enough to give substantially full-scale deflexion (*ca* 250 counts/sec), using angular and chart rates of 1° and 4·5 in in 36 sec. There is a close resemblance to the curves obtained by the use of a potentiometer, with the added complication of excessively large fluctuations when using the shortest time-constant. The average curve drawn through the trace with this short time-constant is found to be a close approximation to the true line shape; on the other hand, with maximum damping, though the trace is smoothed very adequately, there is excessive distortion of the line shape and reduction in intensity. The curves also show the relative efficiencies with which a neighbouring weak

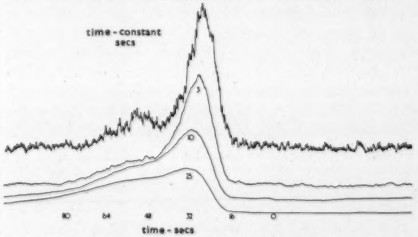

Fig. 6. Automatic record (about ½ full size) showing effect of time-constants on an X-ray line

line due to an impurity in the specimen is recorded when it follows closely after the main peak. With minimum damping the existence of a secondary peak is made evident, but the range of fluctuation is of the same order as its maximum value, while with maximum damping the extra intensity has been lost almost completely in the smoothed curve. Evidently for these working conditions the best value for the time-constant is near to 3 sec.

SUMMARY

The advantages arising from the introduction of Geiger–Müller counter techniques into the X-ray diffraction field are facility in operation, the securing of direct physical observations of a quantitative nature, and economy in time. Elimination of the photographic and photometric procedures is not only significant because of the simplification of any one X-ray examination, but also because it increases the likelihood of using the X-ray method as a practical tool in company with other physical and chemical tests. In circumstances where the X-ray examination has been known to supply useful information in the laboratory but has been precluded from more practical application by its complexity and by the hiatus between the photographic

exposure and the critical measurement, it may now prove possible to use the method as a short-term testing process.

By the application of elementary statistical considerations it is possible to maintain a check on the performance of the X-ray and recording equipment, and to assess the sensitivity and accuracy to be expected from a recording apparatus. In practice, with established conditions of operation of an X-ray and Geiger–Müller counter equipment and some working experience with it, a sufficiently good assessment of the essential features of a recorded pattern is obtainable from visual inspection alone.

ACKNOWLEDGMENTS

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The electron energy distribution in helium

By S. H. DUNLOP, M.Sc., and Professor K. G. EMELEUS, M.A., Ph.D., F.Inst.P., Queen's University, Belfast

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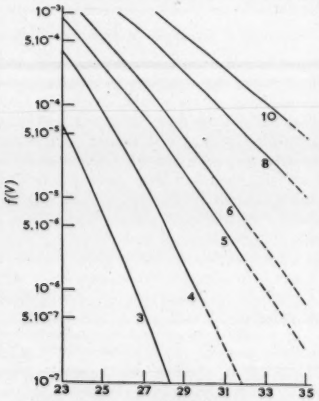
Smit's data for the energy distribution amongst electrons drifting through helium when the ratio of the electric field to gas pressure is small have been extended to higher values of electron energy by development of his formulas. Attention is called to the possible inaccuracy of some collision data on which the calculations are based.

In order to calculate quantities such as the rates of excitation and ionization of atoms by electrons drifting through a gas in an electric field, it is necessary to know the fraction $f(V)dV$ of the electrons in unit volume which have energies between V and $V + dV$ electron volts. It is possible to measure $f(V)$ directly in a low-pressure plasma, but no direct method is known for measuring it under conditions such as exist, for example, in an ionization chamber. It can, however, be calculated in the latter case if sufficient information is available about the cross-sections of the atoms concerned for various types of electron collision. This has been done by Smit[1] for helium, using smoothed collision data obtained by Maier-Leibnitz.[2] Smit has given a series of graphs (Fig. 7 of his paper) showing $f(V)$ as a logarithmic function of V, for values of the ratio X/p of the field in volts/cm to the gas-pressure p in mm of mercury at 0°, from 3 to 10. These do not extend to high enough values of V or small enough values of $f(V)$ to permit of the calculation of Townsend's ionization coefficient α/p, and it was necessary to extend them for this purpose.[3] The extrapolations are shown in Fig. 1. To obtain the graph for $X/p = 6$, a complete calculation of $f(V)$ for all values of V down to zero was performed by Smit's method, using the formulae derived by him for the ranges 0–5, 5–19·8, 19·8–23·6, and above 23·6 V. Close agreement was obtained with his results between $V = 0$ and $V = 26$, above which slightly lower values were found by us. For the other values of X/p, the constants of Smit's Hankel function formula for his highest voltage range were calculated from his graphs, and used to find values of $f(V)$ for higher values of V than were given by him, using his formula. For X/p between 4 and 10, the calculations were carried out up to values of V given by the ends of the full lines. The broken lines are graphical extrapolations of the full lines. For $X/p = 3$, $f(V)$ was also computed to be 10^{-8} for $V = 30$. The line for $X/p = 4$ extrapolates to $f(V) = 10^{-8}$ at $V = 34$.



Electron energy distribution. The numbers against the curves are X/p values

Kerr[4] has called attention to the fact that the complicated geometry of Maier-Leibnitz's diffusion apparatus may not have been adequately allowed for in his calculations of collision cross-sections from his experimental data. If this is so, the values of $f(V)$ given by Smit and in the present note may need revision. At present Maier-Leibnitz's data seem to be the best available.

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The time dependence of the sensitivity of photomultiplier tubes

By M. HILLERT, Swedish Institute for Metal Research, Stockholm.

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A study has been made of the regular changes in sensitivity occurring in photomultiplier tubes under typical operating conditions. The particular phenomenon which takes place in the first minutes after a sudden alteration has been made in the illumination, was investigated in detail. An expression is given, capable of describing this change in the case of a sudden increase in the illumination to which the tube is submitted. Certain experimental results indicate that the dynodes are responsible for this effect. Finally, an electrical model of the behaviour of the dynodes is outlined, which functions according to the expression found for the time dependence of the sensitivity.

The so-called photomultiplier tube is a vacuum valve, which in principle consists of a photoelectric cell coupled to an electron multiplier unit, which acts as an amplifier for the primary photoelectric current. This instrument constitutes a most valuable tool for a number of research problems. Its chief advantages are associated with its high sensitivity, its good linearity of response and simplicity of use. To use a photomultiplier tube one has need of a voltage-supply, capable of giving a constant d.c. voltage, and an instrument for measuring the current from the anode, such as a galvanometer. An alternative method is to let the anode current charge a capacitor and then to measure the voltage built up on this capacitor after the lapse of a fixed time. This corresponds to an integration of the anode current. The general characteristics of photomultiplier tubes has been ably reported in several papers, among others by R. W. Engstrom[1] and G. H. Dieke.[2] Fig. 1, taken

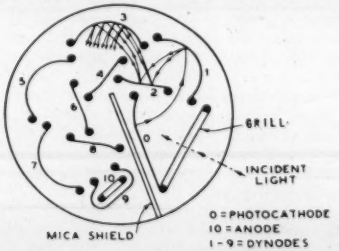

Fig. 1. Cross-section of electrode arrangement of type IP21 multiplier phototube by the Radio Corporation of America

from the R.C.A. pamphlet No. 931–A–3–46, shows a cross-section of the electrode arrangement. The present investigation is limited to the variation with time of the sensitivity of photomultiplier tubes. The sensitivity, S, is defined as the anode current/unit illumination, since the tube response shows good linearity.

FATIGUE AND RECOVERY

When using a photomultiplier for accurate intensity measurements, it is important to know the decrease of sensitivity in use, which is a characteristic feature of such a tube. With a photomultiplier illuminated from a constant light source the anode current, I, is found to decrease, rapidly at first, and then at a slower rate. This decrease can continue for several days, without the attainment of a steady state. When the light is shut off, the tube recovers its sensitivity in the dark, to begin with rapidly, then more slowly. The reduction in sensitivity upon illumination is called fatigue, the reverse effect recovery. The fast decrease in sensitivity at the beginning of an illumination period is very dependent

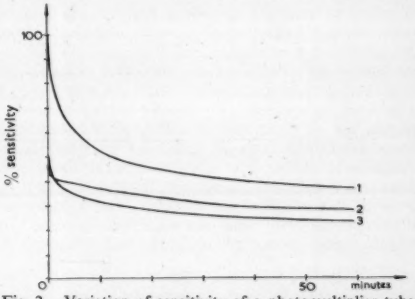

Fig. 2. Variation of sensitivity of a photomultiplier tube

on the extent to which the tube has been "rested" before use. Fig. 2 shows the sensitivity as a percentage of the sensitivity at the start, for a tube which had been kept in darkness for a long time. Curve 1 was measured immediately after such a rest period. After the tube had been subjected to a fatiguing illumination in this way for 1 h and then had been given 1 h in darkness, curve 2 was measured. After further fatiguing illumination, followed by a rest of several days, curve 3 was measured. It is to be noted that curves 2 and 3 cross. This illustrates that the state of a photomultiplier tube and its future behaviour cannot be uniquely ascertained from a knowledge of the present sensitivity alone, but are more complicated functions of the tube's previous history. In practical use it has generally been found convenient always to maintain the photomultiplier in a rather fatigued state. In this way the rapid decrease in sensitivity at the beginning of an illumination period is avoided. In direct reading spectrographs the tubes are generally kept illuminated between readings by a small incandescent lamp, which is only darkened just before

the measurements begin. Individual tubes of the same type and make frequently exhibit rather large differences as regards their fatigue properties.

DELAY

Sometimes the sensitivity of a photomultiplier tube upon illumination is found to increase at first, contrary to expectation. Only after a while does the decrease due to fatigue show up. This peculiar effect has been subjected to a closer examination by the present author. It will here be called delayed response or "delay."

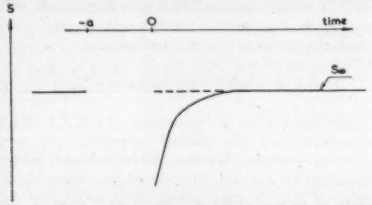

Fig. 3. Delayed response after long periodic fatiguing by illumination

Figs. 3–6 show the effect of delay at different stages of fatigue. In Figs. 3–5 the illumination is relatively weak. In Fig. 6 a much stronger illumination has been used. The anode current was measured with a galvanometer having a response time of about 3 sec. The procedure in taking these measurements was to illuminate the photomultiplier tube from a constant light source and screen off the light periodically for a time interval of length a. Thus in all the diagrams the cell was darkened during the interval $-a < t < 0$, but the voltage was kept constant.

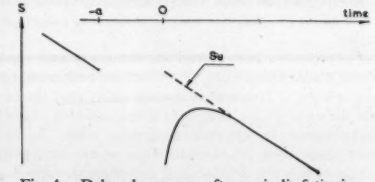

Fig. 4. Delayed response after periodic fatiguing insufficient to reach a steady state sensitivity

The simplest case is the one illustrated by Fig. 3. In this experiment the tube had been subjected to a periodic fatiguing illumination for such a long time before measurement, that its sensitivity had reached an almost steady, low value S_∞. In Fig. .7 the same data as in Fig. 3 are plotted as $\log \dfrac{S_\infty - S}{S_\infty}$ against time t for the interval $t > 0$ during which the illumination was on. This plot shows that it is reasonably accurate to write

$$\frac{S_\infty - S}{S_\infty} = Be^{-t/\tau}$$

Such measurements have been repeated for a number of different light intensities and thus different values of anode current. The constants B and τ were found to

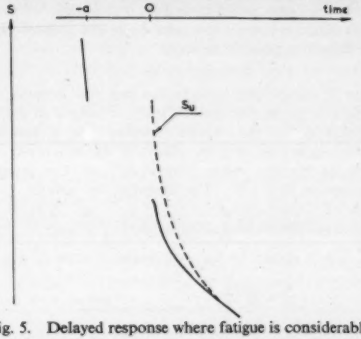

Fig. 5. Delayed response where fatigue is considerable

be essentially independent of the intensity of illumination. Fig. 4 shows what happens when the previous periodic illumination has not been kept up long enough for an approximately constant steady state sensitivity to develop,

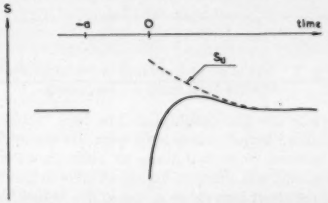

Fig. 6. Delayed response after very strong illumination

due to the usual fatigue effect, before the exposure is interrupted at $t = -a$. If it were assumed, in agreement with previous results, that the recovery during the short dark interval $-a < t < 0$ is negligible, one would

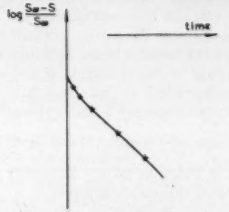

Fig. 7. Logarithmic relation of sensitivity change with time

expect the sensitivity to follow the dotted curve S_u in the absence of delay ("u" meaning "undelayed"). As an extrapolated quantity S_u cannot be measured directly; its

probable course is inferred from other experiments. When $\log \frac{S_u - S}{S_u}$ is plotted against t from the data in Fig. 4, the same result as in Fig. 7 is found. The constants B and τ remain the same as in the case of Fig. 3. It is therefore possible to write

$$S = S_u(1 - Be^{-t/\tau})$$

where S_u varies with time in the way one expects from the fatigue properties and the state of fatigue of the tube in question. If the change expected due to fatigue is very considerable, it is possible for the sensitivity S to reach its highest value immediately at the onset of illumination at $t = 0$. The condition for this is

$$(\text{for } t = 0)\ \frac{d}{dt}(\log S_u) < -\frac{B}{1-B}\frac{1}{\tau}$$

This case is shown by the experimental curve of Fig. 5.

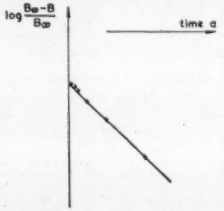

Fig. 8. Variation of constants of photomultiplier with the time interval of screening

In case the photomultiplier has been subjected to strong illumination for a long time, its sensitivity has been reduced through fatigue to such an extent that recovery will take place to an appreciable extent already during the short interval $-a < t < 0$. When illumination is resumed, however, the sensitivity rapidly adjusts itself through fatigue to its former value. This is illustrated by the curve for S_u in Fig. 6, which gives the sensitivity one would expect if there had been no delay. The general formula for I with the same values for the two constants B and τ as determined for the earlier cases were found to fit also this measurement.

Special experiments have been performed in order to determine how the length a of the time interval $-a<t<0$, affects the values of the constants B and τ. The result is that τ is independent of this interval. The constant B increases with increasing a and approaches a limiting value B_∞ when $a > 2$ min. In Fig. 8 $\log\left(\frac{B_\infty - B}{B_\infty}\right)$ is plotted as a function of a. Evidently an analytical representation of this relationship is

$$\frac{B_\infty - B}{B_\infty} = e^{-a/\tau'}$$

In this representation τ' is, within the accuracy of measurement, equal to τ, the time constant in the expression for the variation of S due to delay, using a fixed value for a. In practice it is often possible to arrange to have a dark interval of not less than 2 min, so that no correction for variation in B is needed.

In the cases so far considered, the delay effect appeared as a lag in the response of the anode current to a sudden change of illumination, when, at time $t = 0$, the illumination was suddenly switched on from darkness to a constant value F. A corresponding delay effect has been found more generally when, at time $t = 0$, the illumination of a photomultiplier is suddenly increased from the constant value F_1 to a new constant value $F_2 > F_1$. The manner in which the anode current I responds to this change can be described by the following general formula of which the previous one is a special case:

$$S = S_u(1 - nBe^{-t/\tau}) \text{ where } n = \frac{F_2 - F_1}{F_2}$$

If the illumination is decreased ($F_2 < F_1$), there is again a delay in the current response. No simple equation to represent this case has been found, however.

The values for B and τ, determined from measurements on tubes of R.C.A. type 931 A, 1P21 and 1P28, range from 0·04 to 0·10 for B and from 15 to 35 sec for τ. These measurements were made using anode currents from 1 to 30 μA. The potential difference applied between the cathode and the first dynode, as well as between consecutive dynodes, was 90 V; the anode was kept 65 V positive with respect to the last dynode—the ninth. From these values for B and τ it should be evident that the delay effect can be neglected for practical purposes, if one measures the current directly with an ordinary instrument of rapid response and waits a few minutes before the deflexion is read. In case an integrating instrument is used, the instrument should not be allowed to start integrating until after a few minutes' lapse of time, if it is desired to avoid a correction for delay.

The expressions here found for the delay and for the constant B are exponential, and the same time constant τ occurs in both. This makes it seem likely that the cause of the delay is to be found in some kind of charging up phenomenon in the photomultiplier tube. In order to shed some light on the question, where in the tube the cause of this effect is located, experiments were made in which the delay was measured for an increase in the illumination by a factor of 2, which increase was effected in three different ways. (1) The intensity of the light source was doubled. (2) The illuminated area on the photocathode was increased from the lower half of the cathode to the entire cathode, keeping the light source unchanged. (3) The illuminated area on the photocathode was increased from the left half to the entire cathode.

If the cause of the delay is in the photocathode itself, the results from 2 and 3 ought to agree but could be expected to deviate from the results of experiment 1. If, on the other hand, the cause of this effect rests in some property of the dynodes, one might expect a

difference between experiment 2 and experiment 3 for the following reason. When only the left half of the cathode is illuminated, photoelectrons proceed only from this half. These then give rise to secondary electrons at the dynodes, which secondary electrons may be expected to

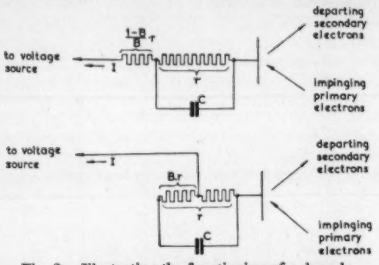

Fig. 9. Illustrating the functioning of a dynode.

show a fairly even distribution over the entire width of the dynodes, resulting from differences in velocity and direction of emission and from non-uniformity of the electric field. When only the lower half of the cathode is illuminated the spread over the surface of the dynodes would be expected to be less, because there are no electric field components in the vertical direction and because the height of the dynodes is considerably greater than their width.

The measurements according to 1 and 3 showed good agreement, but experiment 2 gave a different result, B being appreciably higher in this case. Thus it appears likely that the cause of the relaxation is to be found in the dynodes.

It seems possible to represent the way a dynode functions as regards its delayed response by means of an electric model like the one in Fig. 9a or 9b. The sensitive layer of the dynode is here assumed to represent electrically a capacitor C, shunted by a bleeder resistor r, with the voltage source connected either through a separate series resistor $\frac{1-B}{B}r$ or to a tap on the bleeder at a distance Br. If it is assumed that, within the range of the delay phenomenon, the anode current is proportional to the potential difference across the sensitive layer of the dynode, the empirically found formula for the delay effect is reproduced by these circuits, provided that $Cr = \tau$.

ACKNOWLEDGMENT

The author wishes to express his thanks to Prof. E. Rudberg for his interest in this work and for valuable assistance.

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The variation of the dynamic elasticity of polythene with strain

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The variation in the dynamic elastic modulus of polythene when filaments of the material were subjected to increasing tensile strain is described. The dynamic modulus was obtained from observed values of the velocity of propagation of longitudinal oscillations using several frequencies and at four temperatures. The results are discussed with reference to the structure of polythene filaments and the probable changes that occur on stretching.

In recent papers[1, 2] a method of measuring the dynamic elasticity of high polymer filaments by an investigation of the velocity of propagation and the attenuation of longitudinal oscillations with frequencies between 0·5 kc/sec and 30 kc/sec has been described. The method has been shown to be effective and a technique has been described for determining the velocity of propagation whilst the filament is being strained at a constant rate of elongation. The present paper describes experiments carried out over a range of temperatures to determine the variation of the dynamic modulus with increasing strain. The measurements were carried out on polythene with temperatures between 0° C and 50° C and frequencies between 2 kc/sec and 20 kc/sec.

APPARATUS

The apparatus has been fully described elsewhere and the method and the principle of the measurements explained previously.[2, 3] Oscillations produced electrically were fed into the filament under test by an electromechanical transducer of moving coil or magnetostriction type. A modified gramophone pick-up was used to detect the signal in the filament and the variation of the amplified pick-up signal with distance from the source in both phase and amplitude was observed by means of a double beam oscilloscope. From the experimental observations, values of the velocity of propagation, V, of longitudinal oscillations along the filaments were obtained. Since the wavelength of the oscillations in the material is very considerably greater than the diameter of

the filament the relevant modulus is Young's modulus.[4] For a purely elastic material showing no elastic wave dispersion the modulus of elasticity is equal to $V^2\rho$, where ρ is the density of the material; this simple modulus was used throughout these experiments. Elsewhere[5] calculations have been made giving values of the dynamic modulus of polythene corrected for dissipation. Since, however, these values are within 5% of the simple values because of the low attenuation in this range of frequencies and temperatures, these corrections were omitted. The "static" values of the stress were obtained by a sliding pulley arrangement and calibrated spring balance.[6] The strain was obtained directly from the elapsed time as the end of the filament was drawn at a constant velocity. The rate of strain was varied by altering the gear ratio of the drive and the length of filament, but no appreciable difference in behaviour was noted with strain rate variations of as much as 30 : 1. Most of the measurements were carried out at rates of 0·0015, 0·003 or 0·007/sec. The temperature control was sufficient to maintain the set temperature to within 1° C.

EXPERIMENTAL RESULTS

The results are given for specimens of polythene, "Alkathene" grade 20, of the special batch made by Imperial Chemical Industries, Alkali Division, which has been used throughout the mechanical behaviour investigations being carried out at these laboratories.

Values of the "static" stress and dynamic modulus at three frequencies and 10° C for polythene

		Dynamic modulus dyne/cm^{-2} Frequency		
Strain	Stress dyne/cm^{-2}	2 kc/sec	6 kc/sec	16 kc/sec
0	0·432 × 10^7	7·01 × 10^9	7·80 × 10^9	8·50 × 10^9
0·04	4·175	6·79	7·55	8·30
0·07	7·40	6·65	7·38	8·18
0·11	9·37	6·65	7·30	8·12
0·15	10·65	6·75	7·40	8·12
0·18	11·52	7·01	7·52	8·18
0·26	12·79	7·56	7·98	8·72
0·33	14·30	8·20	8·55	9·48
0·41	16·02	8·88	9·32	10·36
0·48	17·98	9·55	10·20	11·22
0·55	20·25	10·20	11·10	12·02
0·63	22·10	11·05	12·00	13·05
0·71	24·15	11·80	13·00	14·14
0·77	25·90	12·75	14·00	14·98
0·85	28·20	13·70	15·10	16·22
0·93	29·90	14·75	16·25	17·60
1·00	30·95	15·80	17·40	18·70

The filaments, 0·030 in in diameter, were carefully annealed as described previously[1] and the results of several experiments used in obtaining the final values. These results are given for one temperature and three frequencies in the table and Fig. 1, and the variation with temperature shown in Figs. 2 and 3. The stress in the filament was calculated on the actual cross-section, and is not the nominal stress calculated on the original cross-sectional area. The results show that the material drew

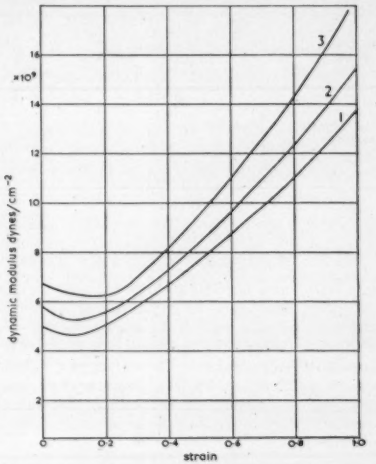

Fig. 1. Dynamic modulus against strain $T = 10°$ C
(1) Frequency 2 kc/s. (2) Frequency 6 kc/s.
(3) Frequency 16kc/s

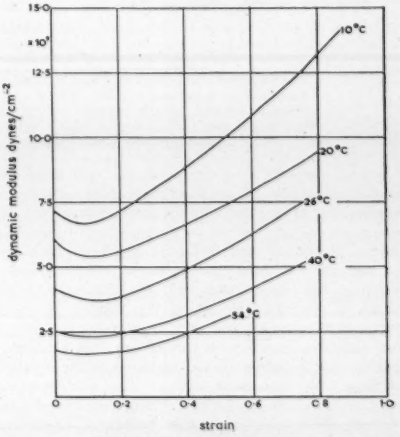

Fig. 2. Dynamic modulus against strain $f = 2$ kc/s

smoothly without "necking"; this was confirmed by visual examination. The results have the form previously reported;[1] the dynamic modulus after a small decrease with strain rapidly increases, until at a strain of 1·0 the value of $V^2\rho$ is between two and three times its initial value. At higher temperatures the minimum in the dynamic elasticity-strain curve is very much less pronounced. The gradient of the "static" stress-strain curve—the "static" modulus—decreases, in the main, throughout the range and is nowhere as large as the dynamic value, though there is evidence of a point of inflexion in the stress-strain curve which would correspond to a minimum in a "static" modulus-strain curve. The qualitatively equivalent effect of increase in frequency and decrease in temperature on the dynamic modulus can be seen from the results.

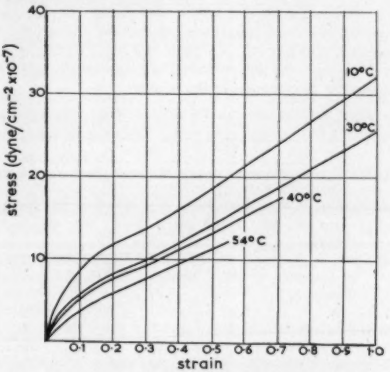

Fig. 3. "Static" stress against strain

DISCUSSION OF RESULTS

The results have two outstanding features, (*a*) the large increase in the dynamic modulus with strain compared with the small decrease in the "static" modulus; (*b*) the minimum value in the dynamic modulus-strain curve and the appearance of the point of inflexion in the "static" stress-strain diagram.

(*a*) The structure of a filament of long chain molecules, such as polythene, has been discussed by Bunn,[7] Meyer[8] and others. The molecules are supposed to exist in a curled-up form similar to that described by Treloar[9] for rubber. In addition however, polythene by its simple chemical nature readily crystallizes, and at room temperature contains about 60% of crystalline material whose structure has been described by Bunn.[10] The actual amount of crystalline material present is still in dispute, agreement between the different methods of analysis, heat capacity, specific volume and X-ray diffraction being poor.[11] Values between 54% and 76% at room temperature have been obtained. It is generally accepted that in annealed material the crystallites are randomly oriented and consist of parts of several molecules, i.e. each long chain molecule will participate in several crystallites. The material not in the crystallites is of an amorphous nature, completely randomly oriented and can be considered as acting as "ties" between the crystallites. The cohesive forces between the molecular segments in the crystallites are due to Van der Waals type linkages, so that the binding energy is relatively low perpendicular to the main carbon chain. The effect of strain on the material is to extend the chains and orient them in the direction of stretch so that in the fully drawn material the distance between the ends of the chains, the chain displacement length,[12] approaches the length of the chain, Flory's chain contour length.[12] The crystallites present initially become oriented in the direction of stretch and may become much larger, the effect observed in rubber.[13] The large increase in the dynamic modulus with strain is consistent with this description of the drawing process. Although the specimens broke before becoming completely oriented, measurements of commercially drawn material confirmed that the modulus approached a steady value when this orientation was complete. The variation in the "static" modulus, i.e. the general decrease with strain, is difficult to account for. It has been shown by photoelastic measurements on polythene[14] that no slip of one molecule past another takes place and that the optical anistropy produced in polythene on straining can be accounted for by the orientation of the crystallites. The difference between "static" and dynamic measurements cannot therefore be due to a supposed slip of one molecule past another. It is suggested that the variation of the "static" modulus is due to the behaviour of the crystallites or even aggregates of crystallites (spherulites) and that there is no appreciable contribution to the "static" modulus by the kinetic elasticity of the molecular chains. As a rough approximation, the crystallites can be regarded as rigid bodies moving in a viscous fluid formed by the amorphous material between them. The macroscopic shear viscosity of such amorphous long chain material is of the order 10^{10} poise. The applied stress is of the order 10^7 dynes/cm^2; then from Stokes' Law the velocity of the crystallites, assumed to be spheres, is of the order 10^{-5} mm/sec. Thus in 1 msec the distance moved is less than 10^{-8} cm. The amplitude of the dynamic signal was approximately 10^{-3} cm, so that the crystal movement would not be a major factor in the dynamic experiments, whereas in the "static" measurements enough time would elapse for the movements of the crystallites to be completed.

(*b*) It is still necessary to account for the minimum value in the dynamic modulus-strain curve and the point of inflexion in the "static" stress-strain curve both below a value of about 0·25 strain. Similar minima in the dynamic modulus when plotted against increase in stress

for filaments of nylon have been reported by Hamburger[15] and the smaller effect occurring in the "static" experiments has been observed by Woods[16] and Chevenard and Champetier[17] in protein fibres. Measurements of the stress-strain properties in connection with the investigation of the photoelastic properties of polythene[14] have shown irreversible effects have occurred before a strain of 0·1 is reached. It was also found that if filaments were removed from the annealing oven too soon, or the temperature of the oven was not high enough, then the dynamic modulus increased continuously with strain or the minimum value occurred almost immediately the specimen was strained. It is therefore probable that the minimum observed in the modulus is due to a change that can be "frozen in" the material by previous treatment.

Woods[16] ascribed the effect he observed in "static" experiments with various natural protein fibres to a change in internal energy resulting from a local disorientation occurring during the initial straining. If the load on the material is K and the extended length L, then $K = (\partial U/\partial L)_T - T(\partial S/\partial L)_T$ where U = internal energy, S = entropy, T = temperature and hence $K = K_U + K_S$. If K_U is negative then there will be a small range of strain over which $\partial K/\partial L$ will be negative, i.e. a minimum in the modulus-strain curve. Hoff[18] has put forward a similar theory where the hindered rotation of the bonds causes the variation of entropy with strain to pass through a maximum value at some small finite strain. In the earlier work of this laboratory[1] the effect was observed with both polythene and Nylon filaments and attributed to a decrease in the proportion of crystalline material, caused by the crystals perpendicular to the direction of stretch being partially broken up during the initial strain. The evidence for this is slight and until improved methods of determining the proportion of crystalline material present are available, this decrease cannot be verified since it would amount to only a few per cent. The French workers Chevenard and Champetier were working with hair filaments and also attributed the point of inflexion in their results to a crystalline change, in this case from α to β keratin. Indirect evidence is also furnished by some experiments carried out with a Neoprene compound loaded with carbon black (36%) in which a minimum in the velocity-strain curve was obtained.[3] The carbon black is known to form a definite structure which is destroyed by subsequent strain.[19]

CONCLUSIONS

The large increase in the dynamic modulus with strain can be satisfactorily explained in terms of the extension of the long chain molecules, whilst the fact that the "static" modulus shows no such increase can be explained by the hindered mobility of the crystallites. The minimum observed in the dynamic modulus-strain curve and a similar inflexion in the "static" stress-strain diagram are attributed to some change that can be "frozen in" the filaments by stressing. Two possible explanations are offered and further work would be necessary to decide between them. The variation in modulus may be due to a decrease in the amount of crystalline material present or to a change in the internal energy of the system due to local disorientation of the molecular chains during initial straining.

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A technique for arc initiation

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A method for initiating an arc discharge between fixed electrodes by an auxiliary spark discharge is discussed. Theoretical limiting values for the parameters of the initiation circuit are obtained, and these are compared with the experimental values found to be satisfactory in a circuit used for initiating a hydrogen arc.

For arc discharge investigations the initiation of the arc is normally accomplished by touching the electrodes together and then drawing them apart. This method has several disadvantages in controlled experimentation, since during the initiation process deterioration of the electrodes can take place, the final gap length cannot easily be accurately controlled, and the observation of a newly formed arc column is not possible. All these difficulties are removed if the arc is initiated between electrodes fixed at the required separation. Two main methods are available for carrying this out. Thus in one method the arc can be initiated by reducing the gas pressure in a sealed arc chamber to a value at which a low-pressure glow discharge can be started, and the current and pressure then increased until the required arc is obtained. The other method, which does not require a sealed chamber, employs a high-voltage impulse generator connected directly across the experimental gap. When the impulse generator is discharged, a low-voltage spark discharge is produced in the test gap which then has a high conductivity for sufficient time to allow the build up of the arc current from its source. This method is useful for experiments concerned with arc pulses. The idea of initiating an arc by means of an initial spark breakdown is by no means new. As early as 1839 Daniell[1] reports this type of initiation, whilst Neef[2] also used the method. Many experimenters since have used the method in various forms, and reference may be made, for example, to the heavy current spark investigations of Norinder and Karsten,[3] whilst the use of auxiliary discharges in practical equipment is well known. It is of interest to note that this type of arc initiation often occurs subsequent to surge breakdown on transmission systems. There does not, however, appear to be any published works on the details of an arc initiation circuit of this type and this is due to the difficulty of accounting for the conducting gap, which during the initiation process is carrying current from both the impulse generator and arc source.

THE DESIGN OF A CIRCUIT FOR ARC INITIATION BY A HIGH VOLTAGE IMPULSE

As indicated, the simplest circuit for arc initiation by a high voltage impulse is obtained when a high-voltage impulse generator and low-voltage arc source are connected in parallel across the test gap in which the arc is to be initiated. In most practical cases, however, the arc source must be adequately protected against the impulse in the event of the test gap resisting breakdown. The required protection can be achieved by connecting a filter network in the low-voltage circuit so that the complete main circuit is given by Fig. 1a. In this Figure the impulse generator is indicated by its equivalent capacitance C_1, discharge resistor R_1 and impulse gap *IG*. The filter circuit comprises the inductor L and capacitance C_2, whilst R_2 is a resistor limiting the arc current to be given by the low voltage arc source E. It

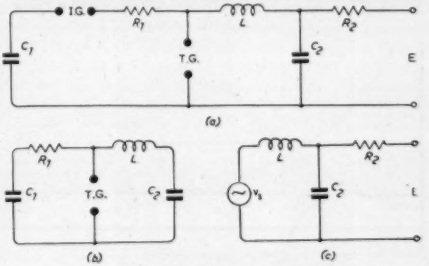

Fig. 1. Arc initiation equivalent circuits

is the purpose of this section to determine the values of the parameters of this circuit which will ensure satisfactory arc initiation. These values are, however, greatly dependent upon the discharge conditions in the test gap. If we consider first the discharge of the impulse generator alone through the test gap, then it is known[4, 5, 6] that after the initial spark breakdown the spark column rapidly expands and the discharge properties become with time closely approximate to those of a transient arc. The current during this time decreases exponentially, and ceases when the voltage available is insufficient to maintain the discharge. This may not occur until the normal arc to glow transition current has been reached.[7] If we now connect the remainder of the circuit across the test gap, then a successful arc initiation will take place if sufficient current can be fed into the gap from the low-voltage arc source, to maintain the arc state produced in the gap by the discharge of the impulse generator. The conditions for a successful initiation can thus be stated as (*a*) The impulse generator discharge voltage v_s should be of the same polarity and less than the arc source voltage E. (*b*) The inequality $v_s < E$ should be maintained for sufficient time to allow the current from the arc source to build up in the test gap to an arc value.

The time available for initiation as defined by this latter condition will depend, for a given circuit, entirely upon the way in which v_s varies with time. If v_s remains constant then the time available is a maximum and equal to the normal impulse generator discharge time t_1 (see Fig. 2a). If, however, as the spark current i_s decreases, v_s increases and becomes greater than E, then

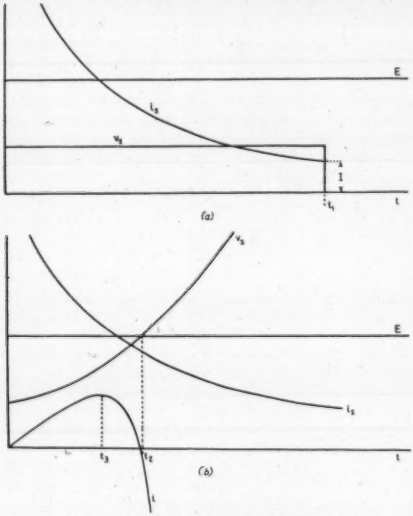

Fig. 2. Assumed limiting spark conditions

a current will be fed into the arc source from the impulse generator. Under these conditions the arc source current i will exhibit a maximum as indicated by Fig 2b, and the time available for initiation can be assumed to be a lower limit and given by the time t_3 for the arc current to reach its peak value.

It is now possible, utilizing the above considerations, to obtain the relative values of the initiation circuit parameters (Fig. 1a). The impulse generator size is determined solely by conditions (*a*) and (*b*) above, and hence by the test gap gas and electrode separation, and by the arc source voltage available. An increase in the energy content of the generator will increase both the current (reducing v_s) and the time available for initiation independent of the form of v_s, whilst an increase in R_1 increases both v_s and the initiation time available. It will be appreciated therefore that in order to ensure satisfactory initiation for a range of electrode separations and gases the largest possible generator capacity and voltage should be used, with a discharge resistance sufficiently low to give a discharge of high current and low voltage. Now with a given arc source voltage E, and a desired final arc current value, the limiting resistor R_2 is fixed, so that only the values of the filter circuit parameters remain to be determined. The basic considerations affecting the filter network components are that the inductor must present an impedance to the impulse voltage considerably greater than that of the discharge resistor, thus allowing the impulse voltage to build up across the test gap to a sufficient value for breakdown, whilst the impedance of the capacitance must be small so that only a fraction of the impulse voltage is impressed upon the low voltage source. The inductor must also be capable of carrying the final continuous arc current. The actual values depend upon the values chosen for the impulse generator and upon the assumed discharge and current conditions in the gap. If we consider first the value of the filter capacitance, then its lower limit is set by the voltage produced across it when the impulse generator discharges and the test gap resists breakdown. Under these conditions since the impedance of C_2 can be considered small compared to R_2, the circuit becomes that given by Fig. 1*b*. If C_1 is charged to a voltage V and then discharged, the voltage across C_2 after the oscillatory components have decayed is

$$v = VC_1/C_1 + C_2 \qquad (1)$$

If the maximum voltage allowable on C_2 is v_m then from (1)

$$C_2 > C_1[V/v_m - 1] \qquad (2)$$

and it will be appreciated that the maximum protection of the arc source is obtained when the filter capacitance tends to an infinite value. The impressed voltage will eventually be reduced to zero by the components of the complete circuit which have been neglected in the analysis.

If we now take the worst condition for initiation, assumed to be obtained when the spark voltage v_s rises as rapidly as its current decays (Fig. 2b), and put v_s in the form $(Al/i_s + B)$ where A and B are constants and l is the electrode separation, then substitution for i_s gives

$$v_s = F \exp(\alpha t) + B \qquad (3)$$

where $\alpha = 1/R_1C_1$ and $F = AlR_1/V$. Assuming now that the impulse generator discharge controls the gap conditions, then current will be fed into the gap by an equivalent voltage

$$v = E - B - F \exp(\alpha t) = G - F \exp(\alpha t) \qquad (4)$$

The circuit under these conditions is given by Fig. 1c and analysis gives a complex general expression for the current fed from the arc source into the test gap. However as C_2 tends to zero the expression becomes that for the simple (LR_2) circuit, giving a time to current peak of

$$t_3 = \frac{1}{\alpha + \beta} ln\left[\frac{G}{F}\left(1 + \frac{\beta}{\alpha}\right) - \frac{\beta}{\alpha}\right] \qquad (5)$$

where $\beta = R_2/L$. It can be shown from the general expression for the current that this time is increased if C_2 is made greater than zero, and becomes a maximum

when the capacitance is infinite. This maximum value is given by

$$t_3 = \frac{1}{\alpha} ln(G/F) \tag{6}$$

It is of interest to note here that t_3 increases with G/F and $1/\alpha$, and thus corresponds to increases in the size of the impulse generator and arc source voltage, thus confirming the statement previously made. It would thus appear that the maximum possible value of C_2 should be used, but an upper limit to this value is fixed by a further consideration. If we assume now that the arc is initiated instantaneously and represents a sudden short circuiting of the test gap, then the current fed into the gap by the arc source is

$$i = \frac{E}{R_2}\left\{1 - \exp(-t/2R_2C_2)\left[\cos \omega t + \left(\frac{1}{2R_2C_2} - \frac{R_2}{L}\right)\frac{\sin \omega t}{\omega}\right]\right\} \tag{7}$$

where $\omega = [4R_2^2C_2/L - 1]^{\frac{1}{2}}$. For large values of C_2 the sinusoidal term predominates and the current can pass through a zero. If this happens it is possible that the already initiated arc will be interrupted at a current zero, owing to the insufficiency of the available restriking voltage. In order to ensure that oscillations do not take place, a maximum value of the capacitance is given by

$$C_2 < L/4R_2^2 \tag{8}$$

Thus equations (2) and (8) determine the limits of the filter capacity.

Considering now the filter inductance, the maximum value is that which allows the arc source to build up a current in the test gap to a minimum arc value in the maximum time available. This condition is obtained when v_s is constant and C_2 infinite, see Fig. 2a. If now this minimum arc current I is the value at which glow to arc transition takes place, it can be assumed that the impulse generator discharge will cease, and the arc source will then just maintain the discharge. With this condition the same time is taken to reach this current by both the arc source and impulse generator, so that

$$t_1 = \frac{IL}{E - v_s} = R_1C_1 ln[(V - v_s)/IR_1] \tag{9}$$

and this gives a maximum limiting value of the inductance

$$L < \frac{(E - v_s)}{I} R_1C_1 ln[(V - v_s)/IR_1] \tag{10}$$

The minimum value of the inductor is determined by consideration of the voltage impressed across the test gap by the impulse generator discharge. This voltage (Fig. 1b), when the filter capacitance is large, is given over the first few microseconds by

$$v = V \exp(-R_1t/2L) \tag{11}$$

If the generator is charged to p times the value required for breakdown, and if a value of $t = \tau$ is assumed in which breakdown must occur, then the minimum value of the inductor which can be used is given by equation (11) as

$$L > R_1\tau/2ln(p) \tag{12}$$

Thus equations (10) and (12) define the limiting values of the filter inductor, and with equations (2) and (8) the filter network is determined, so that with the consideration given for the impulse generator, the complete initiation circuit can be constructed.

A CIRCUIT USED FOR ARC INITIATION IN HYDROGEN AND AIR

A description of an initiation circuit used for certain experiments on a hydrogen arc will be given.[8] The impulse generator was of the Marx type consisting of six stages each with a capacity of 0·5 μF, and having a discharge resistance of 150 Ω. A charging unit utilizing a Cockcroft doubling circuit was used to charge the generator to 4 kV, whilst a simple thyratron triggering circuit was connected to the generator through a three-ball triggering gap. An arc source of 440 V was used with a 40 Ω variable arc limiting resistor. The actual

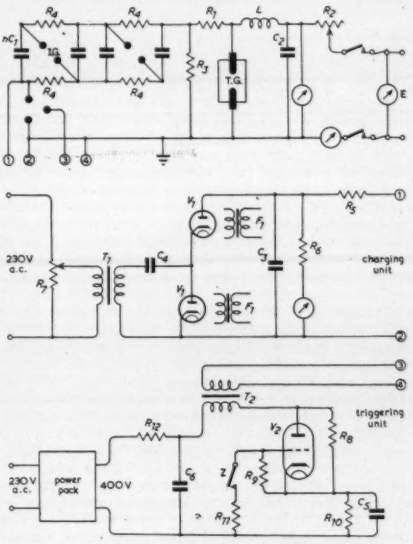

Fig. 3. Arc initiation circuit

R_1 = 150 Ω
R_2 = 40 Ω 10 A
R_3 = 10 MΩ
R_4 = 100 kΩ
R_5 = 3 MΩ
R_6 = 10 MΩ
R_7 = 1 kΩ 50 W
R_8 = 1 MΩ
R_9 = 1 MΩ
R_{10} = 100 kΩ
R_{11} = 10 kΩ
R_{12} = 100 kΩ
nC_1 = 0·5 μF 8 kV
C_2 = 2 μF 5 kV
C_3 = 0·25 μF 8 kV
C_4 = 0·25 μF 8 kV
C_5 = 50 μF 50 V
C_6 = 4 μF 500 V
L = 9mH 20A
V_1 = VUIII rectifiers
V_2 = GTC1 thyratron
T_1 = 230/2 500 V transformer
T_2 = Induction coil
F_1 = 230/4 V 12 kV filament transformer
Z = Triggering switch

circuit used is shown on Fig. 3. It only remains now to compare the theoretical limiting values of the filter network parameters with those found satisfactory in practice. Since the gap length during the experiments was 4·7 mm the value of p (equation 12) is approximately 1·5 and τ can be taken as 5 μsec, so that from equation (12) $L > 1{\cdot}2$ mH. The spark voltage in hydrogen given by the generator described is approximately 100 V, and assuming a transition current of 1 A, equation (10) gives $L < 40$ mH. In the actual circuit an inductance of 9 mH was found to be satisfactory. For an arc current of 10 A an arc limiting resistance of 36 Ω is required and equation (8) then shows that $C_2 < 2\mu F$. With a value of 2 μF an impulse of 1 000 V will be impressed upon the arc source and this was taken to be satisfactory.

In conclusion it is of interest to note some preliminary experiments which were made on the initiating circuit. It was found that a circuit satisfactory for use with arcs in air could be obtained with an arc source of 220 V and a filter capacitance of 100 μF. When the circuit was applied to initiation in hydrogen no satisfactory results were obtainable until the arc source voltage had been raised to 440 and the filter capacity reduced to 2 μF. This effect was presumably due to the higher spark and arc voltages obtained in hydrogen, and due to the presence of oscillat orycurrents and the known difficulty of restriking in hydrogen.

ACKNOWLEDGMENTS

The author wishes to record his appreciation of the interest shown by Professors J. M. Meek and F. J. Teago in the work reported here. The discussions he has had with Dr. J. D. Craggs on the problems associated with the spark channel have been most helpful. The author is grateful for being enabled to do this and other experimental work by the grant to him of a Fellowship by the University of Liverpool, and by the financial aid of the Electrical Research Association.

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Correspondence

The effect of pressure and friction on photographic emulsions

In connexion with the paper by Dr. A. C. Moore in your January issue, the results of some tests carried out at the Road Research Laboratory on the effects of direct loading on photographic emulsions may be of interest. These tests arose from a study of the skidding resistance of roads, and were made in search of a pressure-sensitive material which could be used for investigating pressures between vehicle tyres and the peaks of projections in road surfaces. The work consisted of applying direct loads to various types of emulsion, without exposure to light; this is in contrast to the method of Bäckström.* The loads were applied through steel balls mounted on an arm 2 ft long, by means of which the balls could be lowered slowly on to the surface. Balls were used since the pressure distribution between a sphere and a plane is known.† Emulsions on both plates and films were tested, the emulsions being selected with the co-operation of the Kodak Research Laboratories, as materials most likely to be sensitive to the action of mechanical forces. The emulsions on films were of the X-ray type, namely, Kodirex, Crystallex, Blue Brand and Super Speed (code Nos. 1, 2, 3 and 8 respectively), and those on plates were of the highly concentrated type used for recording nuclear tracks. The plates were tested bare, while the films, which were supported on bases of varying rigidity, were tested either bare or wrapped in light-tight envelopes of paper or rubber. The loads applied ranged from 2–50 lb and were applied through balls of $\frac{1}{4}$ in and $\frac{3}{32}$ in diameter. I.D. 2 developer was used and in every case it was found that when sufficiently high loads were applied (all loads on plates and loads above 10 lb on films), a circular area of the emulsion was blackened on development. The density of this blackened area was measured with a simple spot photometer and was uniform except for a slight falling off in intensity at the edges. The relations between load, "average pressure" (as estimated from the applied loads and area of blackening) and density were investigated. "Pressures" ranging from 3 000 to 40 000 lb/in^2 and densities ranging from 0·2–1·8 were obtained. Using film there appeared to be a direct proportionality between load, "average pressure," and density although the slope of the lines was influenced by the rigidity of the support; this dependence of the results upon the type of support for the emulsion is presumably due to the way in which the forces are distributed throughout the emulsion. With plates, however, while "average pressure" and density increased with load, no linear relations existed as for film. "Pressure" and density increased with load, tending to maximum values at which the plate broke. The results depended for both plates and film on the size of ball used, the smaller ball giving higher densities and pressures, as would be expected. The results also depended on the type of emulsion used. It was found that the pressure sensitivity of the emulsions decreased with decreasing grain size, although grain size was not the primary factor in determining pressure sensitivity. Increased gelatin content decreased the sensitivity to a much larger extent.

Of the emulsions on films, Crystallex was found to be the most sensitive to pressure. With this film in paper or rubber envelopes, impressions were obtained on roads with sharp projections, indicating pressures between tyre and road up to 8 000 lb/in^2. On many surfaces pressures are not high enough to be recorded by this method, and the possibility of obtaining a more sensitive emulsion is being examined.

Road Research Laboratory, Harmondsworth, Middlesex. C. G. Giles Barbara E. Sabey

* Mitchell, J. W. *Photographic Sensitivity* (London: Butterworth's Scientific Publications Ltd., 1951), p. 203.
† Timoshenko, S. *Theory of Elasticity* (New York: McGraw Hill Book Co., Inc.,) p. 339.

New Books

Semi-Conductors. By D. A. WRIGHT. (London: Methuen and Co. Ltd.) Pp. viii + 130. Price 7*s*. 6*d*. net.

About half this book is devoted to a discussion of the basic theory of semi-conductors. The remainder deals with those aspects which are of particular practical interest: secondary electron emission, metal-semi-conductor contacts, thermionic and photoelectric cathodes. Unless the derivation is very simple, only the results of mathematical discussions are given. The presentation is, however, so lucid that the interested student will find no difficulty in filling in the details from the larger treatises to which reference is made. The book can be recommended both to the honours student and the research worker as a useful introduction to more specialized works.

T. B. RYMER

Sourcebook on Atomic Energy. By S. GLASSTONE, Ph.D., D.Sc. (London: Messrs. Macmillan and Co. Ltd.) Pp. 546. Price 24*s*. net.

This book has been prepared at the behest of the United States Atomic Energy Commission by a consultant, Dr. Samuel Glasstone, who was specially appointed for this purpose. The book contains a very readable semi-popular narrative of the past, present and future possibilities of atomic science. It can be read with profit by first year undergraduates in honours schools in physics and for general interest by persons with a sixth form knowledge of science. Like so many U.S. textbooks, this volume is magnificently comprehensive in its treatment of the subject and the use of two columns per page materially assists the readability. Sixteen chapters in the book are devoted to the fundamental concepts of the atom, nucleus and nuclear radiations, the various types of particle accelerating machines, the production and use of isotopes, nuclear fission and nuclear energy piles. Finally, there are two chapters on cosmic radiation and health physics.

It is of considerable interest to note that on page 409 it is stated that "ultimately the cost of electricity obtained from fission energy will not differ outstandingly in one direction or the other from that derived from other sources of power." From such a source the remark may be considered fairly authoritative.

J. V. DUNWORTH

Selected Topics in X-ray Crystallography. Edited by J. BOUMAN. (Amsterdam: North-Holland Publishing Co.) Pp. xv + 375. Price 76*s*. net.

This is the third of a series of monographs on different aspects of theoretical and applied physics to be published under the general editorship of the Dutch physicists, H. B. G. Casimir, H. Brinkman, and J. de Boer. Monographs on nuclear forces and thermodynamics are the first two of the series.

There are already a number of textbooks that provide a general introductory account of X-ray crystallography, so that it is refreshing to come across a volume that is written frankly for specialists. As the title implies, the book consists of a series of partly inter-related accounts of advanced researches conducted at the three X-ray laboratories of the "Technische Hogeschool" at Delft. The researches have a common theme, that of the deviations observed in real crystals from the ideal atomic arrangements of classical X-ray crystallography. Such deviations may be small or large, and the investigations described reveal their importance and their bearing on practical problems.

The book is divided into eight parts, each part containing one or more chapters and covering a different aspect of the X-ray crystallographic research carried out at Delft. The chapters are contributed by individuals who participated directly in the investigations, and there are many names, such as those of W. G. Burgers, P. M. de Wolff, and J. A. Prins, that will be well known to X-ray crystallographers in this country. The editor, J. Bouman, contributes several chapters concerned with theory and basic principles. Part A contains five chapters devoted to special aspects of structure analysis. The reciprocal lattice is introduced on the first page and this section includes a treatment of the X-ray line broadening consequent upon small crystal size. The two sections that follow deal on the one hand with lattice distortions of various kinds and on the other with recrystallization effects in aluminium. Although the investigations originated largely in the metallographic field, the physical meaning of diffuse scattering and of asterisms in Laue patterns are discussed at some length. Those concerned with practical applications of X-ray diffraction technique will be particularly interested in part F. De Wolff treats here the theory of X-ray diffraction by mixed powders, and discusses the limitations imposed by finite absorption and size of the individual particles, and so points towards a proper appreciation of the difficulties of quantitative determinations. In the class of feebly crystalline or "amorphous" substances, rubber, starch, waxes, glasses, and liquids have to be considered. The value of Prins' work on liquid metals is widely recognized, and the chapter in part E on mercury, antimony, selenium, and sulphur provides an able summary of present views on the structure of "amorphous" solids and liquids. The degree of crystallinity in natural rubber is dealt with in part G, and the five chapters of part H contributed by D. R. Kreger provide an authoritative survey of X-ray investigations in the biological field. Some interesting equipment design problems are referred to in the course of the book. Attention may be directed, for example, to the discussion of fine collimators and micro-focus tubes in chapter 4 of part H. Although developed in the first place for the study of biological specimens, the method is applicable to textural problems in metals. The description of curved crystal monochromator techniques in chapter 3 of part F is also of considerable interest to X-ray physicists.

The Dutch have an almost unique talent for producing admirable monographs of this type. Though there are inevitably certain turns of phrase, unusual in an English text, and some further editing would be advantageous, the book is nevertheless thoroughly readable. It is well illustrated and contains many useful references, and the paper and binding are adequate for normal service on the laboratory or library bookshelf.

H. P. ROOKSBY

Notes and Comments

Preparation and mounting of polycrystalline specimens for X-ray analysis

The Equipment sub-committee of the X-ray Analysis Group of The Institute of Physics announces that a memorandum on the preparation and mounting of polycrystalline specimens for X-ray analysis has been drawn up by the powder camera and Geiger–Müller spectrometer panel. Copies may be obtained (free of charge) on application to The Institute of Physics, 47 Belgrave Square, London, S.W.1.

Elections to The Institute of Physics

The following elections have been made by the Board of The Institute of Physics:

Fellows: S. L. Anderson, N. Clarke, M. M. Das, H. L. D. Pugh, R. W. Sillars, E. H. S. van Someren, J. R. S. Waring.

Associates: M. Barbanel, I. J. W. Bisset, N. Brown, G. C. Cato, D. E. C. Corbridge, J. A. F. Cornick, R. C. De Silva, W. K. Dyer, K. E. A. Effat, R. D. Malcomson, M. D. McNicholl, R. N. Thomson, K. R. Vale, J. F. Youd.

Forty-two Graduates, two Subscribers and twenty-two Students were also elected.

Safety in the laboratory. We have received a small manual giving advice on how the effect of accidents can be minimized or even prevented altogether. It is published by The Dunlop Rubber Company Ltd. for use by their employees, and in twenty pages summarizes a very large number of causes of accidents, and emphasizes the need of forethought and reasonable care.

Although this booklet includes details relevant only to The Dunlop Rubber Company Ltd. it is thought that it might be of use in guiding other firms in the preparation of similar booklets.

The new Patents Act. On the 1st January, 1950, the new Patents Act came into force. This Act alters the rights of manufacturers and inventors in a number of important and different ways and will affect application for patents, assignments, action in the Courts, the compulsory grant of licences and the rights of the Crown.

Readers of this Journal will find a pamphlet entitled *The New Patents Act* by Robert Lochner, a useful guide to their rights under the new Act. This thirty-page pamphlet deals with applications for patents, the rights of the patentee, the rights of the public, the rights of the Crown, appeals and a number of miscellaneous points. Copies are obtainable from the National Union of Manufacturers, 6 Holborn Viaduct, London, E.C.1, price 2s. 6d. net.

Errata. In the article *Fibre formation in synthetic polymers: methods and features*, by F. Happey, published in May issue, Figs. 6(c) and (d) were inadvertently interchanged.

In the table, Terylene monomer *should be* $OH.(CH_2)_2 . OH$ *not* $OH.(CH_2) . OH$ as printed.

Journal of Scientific Instruments

Contents of the June issue

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Double beam recording infra-red spectrometer. By K. S. Tetlow, J. McAuslan, K. J. Brimley and W. C. Price.

The stability of mica standards of capacitance. By G. H. Rayner and L. H. Ford.

A membrane manometer with secondary air transmission. By C. P. Luck.

The design of thermistor thermometers with linear calibration. By W. R. Beakley.

A new form of *f*/0·71 lens for 35 mm cine-radiography. By C. G. Wynne.

A thermal switch for use at low temperatures. By G. O. Jones.

A method of keeping the luminance of a lamp constant without the use of an electric meter. By R. W. G. Hunt.

Variations in extinction voltages of glow-discharge voltage-regulator tubes. By F. A. Benson.

Effects of geometry on scattering distributions in the Wilson cloud chamber. By A. V. Crewe and A. E. Litherland.

LABORATORY NOTES

A compact sliding vacuum seal. By R. I. Garrod.

A divided light guide for coincidence counting of scintillations due to alpha particles. By B. Makiej.

Storage cabinet for radio assay preparation. By O. Kantorowicz.

A simple apparatus for recording time intervals. By E. F. Maillard.

Capsules for neutron irradiation of small quantities of materials. By J. V. P. Long.

MANUFACTURERS' PUBLICATIONS

NOTES AND COMMENTS

British Journal of Applied Physics

Special Articles and Original Contributions accepted for publication in future issues of this Journal

SPECIAL ARTICLE

Procedure in industrial physics and its implications for education. By R. S. Silver.

ORIGINAL CONTRIBUTIONS

Aerial prospecting for radioactive minerals. By H. P. Peirson and E. Franklin.

Solution of certain unsteady heat flow problems by relaxation methods. By A. Gilmour.

The scattered light method of exploration of stresses in two and three dimensional models. By H. T. Jessop.

Application of an oblique incidence method to the determination of principal stresses on the free boundaries of three dimensional "frozen stress" photoelastic models. By V. M. Hickson.

The photoelectric measurement of coal dust stains on filter paper. By C. N. Davies and Mary Aylward.

The preparation of high melting single crystals and bicrystals with predetermined crystallographic orientation. By K. V. Gow and B. Chalmers.

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SPECIAL ARTICLE

An introduction to the rationalized M.K.S. system of units*

By G. F. NICHOLSON, B.Sc., A.K.C., M.I.E.E., Royal Naval College, Greenwich

It is shown that the rationalized M.K.S. system of units when adopted in presenting the subject of electromagnetism, supplied the need for a single system of units with the least possible disturbance to current practice. The informative natures of the formulae and dimensional expressions arising from the use of the system are stressed and the case for the general adoption of the system in physics is examined. Some units and conversion factors are tabulated. Difficulties arising from the historic development of magnetic units are discussed.

THE CASE FOR A SINGLE SUFFICIENT SYSTEM OF UNITS

The object of this article is to introduce the rationalized M.K.S. system of units to readers who have not yet had occasion to examine it very closely.

For many years students of electromagnetism have had their attention distracted from their main job, the understanding of the physical meaning of the phenomenon which they happen to be studying or measuring, by having to grapple with two C.G.S. systems of units, sometimes called absolute, inter-connected by the velocity of electromagnetic waves in free space, and a third, practical, system connected to the other two by a series of entirely uneducative powers of ten. From the new student's point of view at least, there is a case for the invention of a single sufficient system of units.

The specification for such a system would certainly contain a clause demanding the least possible mental disturbance for those in the habit of using the older systems. Of all the arrangements which have been tried the rationalized M.K.S. system is the only one which can be said to cover this clause and to do it admirably. By a device, which will be explained, it avoids introducing any new unit names with the exception of one for a new, and very convenient, mechanical unit. All units can be expressed in terms of mechanical units and of the practical electrical units, the ampere, the coulomb, the volt, the ohm, the henry and the farad. The system can indeed claim to be a single sufficient system which requires no learning by older scientists.

As the International Electrotechnical Commission at its meeting in Paris last July formally adopted the totally rationalized M.K.S. system, the unrationalized system will not be discussed. One outstanding advantage of the rationalized system is that, when it is used, the mathematical expressions for the electromagnetic relationships assume shapes which are helpfully informative instead of cryptic. Literal formulae do not depend upon any system of units for their shapes. But there are limited formulae into which numbers can be substituted and which, in the study of electromagnetism, come to be regarded as the most familiar expressions of the relationships in everyday use. It is well known that these include such coefficients as 4π, 8π, 3×10^{10}, 9×10^{20} and the powers of ten already mentioned. As these demand an effort of memory and contain no information, they can only be a further distraction to the student. Examples will be considered later to show that the M.K.S. formulae not only tell the student unequivocally what he is dealing with, but they even suggest to him how to think about it.

Further examples will show that dimensional equations in the M.K.S. system are equally informative. At every stage of a problem they have a physical meaning and help to keep the student in touch with what is going on, a faculty which can be justly denied to such dimensional terms as $M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} k^{\frac{1}{2}}$.

A PRESENTATION OF ELECTROMAGNETISM USING THE SYSTEM

A method of presenting the subject of electromagnetism to a hypothetical new student using the rationalized M.K.S. system of units will now be examined. It is not suggested that this method would be the best for a real student, although, with slight modifications, it might do very well. In this context it is merely a vehicle for carrying familiarity with the system of units to the reader.

MECHANICAL UNITS

Before a start is made, however, a few words must be said about the mechanical units with which the student is assumed to be already familiar. The metre, the kilogram and the second have been chosen as basic because, while they have the advantage of being already in general use, the unit of work based upon them turns out to be the joule, and that of power the watt, giving a one to one relationship with the volt-ampere of electrical power. The new unit, which has unfortunately to be introduced, is the newton of force. It is the force which imparts to a mass of 1 kilogram an acceleration of 1 m/sec/sec, and is equivalent to the gravitational pull on a mass of 102 g (1 000/9·81). The joule is, of course, the newton-metre. At the I.E.C. meeting referred to above the newton was formally adopted as the unit of force.

ELECTROSTATIC UNITS

It must now be decided from what fourth quantity the student shall start his studies of electricity. At the July meeting of the I.E.C. the ampere was chosen as the fourth unit, but the author understands that it was explicitly

* Based on a lecture to the Electronics Group of The Institute of Physics delivered in London on 20th February, 1951.

stated that this was merely for purposes of legal definition. Cornelius and Hamaker[1] state with great emphasis that it is immaterial what electrical quantity is regarded as fundamental and what as derived.

For teaching purposes, then, the choice is free. As it seems easier for a student to pass from quantity to flow than from flow to its integral with respect to time, and as it is believed that very many modern teachers start teaching from the electron, electric charge will be taken as the fourth quantity. At first the coulomb will be defined merely as a multiple of the charge on the electron. The actual multiple ($6{\cdot}29 \times 10^{18}$) is an historical accident, as the practical system of units carried no reference to the electron. The coulomb, then, is arbitrary. But so, as it turns out, are the metre, the kilogram and the second, although they were not intended to be. A start is made, then, from four arbitrary units; and this is quite consistent.

Fundamental observations upon electric charge must now be made. As the reader will very soon appreciate, it is convenient to start one's investigations upon uniform fields. So, instead of considering point charges, as in the classical treatment, we will consider charges spread over flat plates. Attention will be turned to a part of an infinite flat plate carrying a uniform charge of electrons, which are assumed to have been removed from a second plate parallel to the first, so that the plates carry equal and opposite charges. In the classical case of point charges three observations are made, which may be called the inverse square law observation, the charge observation and the permittivity observation. Analogous observations must be made on the parallel plate system.

The student, having observed that there is a mutual attraction between the plates, is tempted to investigate the region between them with a small exploratory charge. His first quantitative observation is that, if the charge density on the plates is fixed, the force on the exploratory charge is independent of its position between the plates. He may invent the term, "uniform field." His second observation is that the force on the exploratory charge is proportional to the charge density on the plates. And his final observation is that the constant of proportionality between this force and the charge density is changed if the homogeneous medium between the plates is changed. No further observations are required. The remaining phenomena of electrostatics follow by deductive reasoning.

To help in his reasoning the student will find it useful to define difference of potential in terms of the work done in moving charge. His next unit, then, will be the volt, defined as the joule/coulomb. He may also at this point define potential gradient and assign to it the unit, volt/metre. As the volt is the joule/coulomb, and the joule is the newton-metre, the volt/metre can also be written newton/coulomb. He will realize that this unit measures the quantity which is uniform between his parallel plates.

His first deduction is that, since for a given charge density on the plates, the charge on a given area is fixed and the potential gradient between the plates is proportional to it, then, for a given plate separation, the difference of potential between corresponding opposite areas and the charge on them are proportional. He will define capacitance, and his next unit will be the farad or coulomb/volt. In carrying out this deduction he will notice that the capacitance of a parallel plate capacitor is proportional to the area of the plates and inversely proportional to the plate separation. He may write

$$C = Q/V = kA/d \tag{1}$$

The constant of proportionality, k, is a measure of the effect of the medium, and is clearly numerically equal to the capacitance of unit parallel plate capacitor having this medium as a dielectric. This is his definition of permittivity. It has dimensions, farads/square metre/metre, or, more shortly, farads/metre.

To obtain a mental picture of what is going on between the plates he now invents the concept of electric flux. He should allow one unit of flux to pass from unit charge on one plate to unit charge on the other. Then no new unit is required. The unit of electric flux is the coulomb. He should at this point define charge density on the plates and flux density between them and measure them in coulombs/square metre.

If he divides equations (1) by A/d he finds that

$$k = \frac{Q/A}{V/d} \tag{2}$$

But Q/A is the flux density, D, and V/d is the potential gradient, E, so that

$$k = D/E \tag{3}$$

Before leaving electrostatics a word must be said about the inverse square law. Suppose that the finite plates of a charged parallel plate capacitor are distorted mechanically until one of them is a small sphere and the other a larger sphere concentric with the first, it is easy to show the student that the charges on each sphere will distribute themselves uniformly. Then the charge density on the inside of the outer sphere is $Q/4\pi R^2$, where R is the radius of that sphere. This is also the value of the flux density near to the surface of the sphere. Using equation (3) he can write

$$E = Q/4\pi R^2 k$$

This is the inverse square law. It is noticed that a coefficient, 4π, has crept in. But in this case it is a coefficient full of information. It exposes the mechanism of the inverse square law. It states that the influence of a charge, Q, uniformly distributed about a point, is spread over a larger and larger sphere as the distance from the point increases. 4π will occur in all expressions dealing with spherical symmetry. 2π will appear when there is cylindrical symmetry. The word "rationalized" in the title of our units simply means that 4π makes its appearances with very good reasons instead of with no reason at all.

ELECTRODYNAMIC UNITS AND A DIGRESSION

The student is now ready to study electrodynamics. His next unit will be the ampere, or coulomb/sec, and as he will probably meet Ohm's law at this point he will also define the ohm as the volt/ampere.

He may now return to his parallel plates. This time he will consider a current flowing, say, from left to right in one plate and a return current from right to left in the other, the current density across the plates being uniform. Having observed that, due to the currents in the plates, they experience a mutual repulsion, he will decide on the analogy of his electrostatic experiments to investigate the region between the plates with an exploratory current. He will find that a small turn of wire carrying a current and placed between the plates with its plane at right angles to the direction of the plate current requires a torque to keep it in this plane, and that for a fixed current density in the plates this torque is independent of the position of the turn. His second analogous observation is that the torque is proportional to the current density in the plates. His third observation is that the torque depends upon the medium between the plates.

Here a teacher may find it convenient to suggest to his student a change of method. This raises a point of very great importance. The student may be left for the moment while it is discussed with the reader.

Up to this point electrodynamics and electrostatics have been treated analogously, and current density with respect to the width of a plate measured in amperes/metre, has been seen to be the analogue of charge density with respect to area, measured in coulombs/square metre. Later it will be shown that the ampere/metre is the unit of magnetic potential gradient. So that we have H analogous to D. Many theoretical physicists consider this analogy eminently reasonable. However, for symmetry in our units we would like H to be analogous to E and so current to voltage. It should obviously be investigated whether this divergence of requirements implies some wrong thinking somewhere, and whether any system of units can be blamed for it. Much has been written on the matter. Our only concern here is to find out whether it could be taken as grounds for condemning the M.K.S. system.

The reason for the convenience of taking current and voltage as analogues is that e.m.f. is defined in terms of work per unit electric charge, and current or m.m.f. was originally defined in terms of work per unit magnetic charge. On this basis all systems of units have been built. Although the isolated magnetic pole is now discarded by most people, and m.m.f. is treated as a line integral of magnetic field strength, the fact remains that the structure of the magnetic units is founded on the magnetic pole. Magnetism is in fact a highly artificial science. It was developed and formulated before certain gaps in our knowledge had been filled. The early workers in magnetism were geniuses of the first order, and the very brilliance of their work may dazzle us from seeing the light of new knowledge when it comes. After all magnetism has now become an attempt to represent what we believe to be a dynamic phenomenon by a static field. It suggests that certain analogies should be drawn between the magnetostatic and the electrostatic fields. There is a temptation to push these too far, and anomalies are met which are inherent in the method of approach. These anomalies are, quite rightly, called difficulties. The difficulty under discussion is a case in point.

A very short examination of the M.K.S. units arrayed in Table 1 will be sufficient for finding a very pleasing symmetry in them, a symmetry which is altogether advantageous both to the new student and to the practising scientist. It might, however, be advanced as a valid criticism of the system that the symmetry was too beautiful, and that nothing in nature really corresponded to it. But it must be remembered that the system is designed to cause the least mental disturbance to busy scientists hardened to the older systems, and to that end it is based on the practical electrical units. These in turn are derived from the magnetic C.G.S. system and so from the unit pole. The primary blame is on the artificiality of magnetism. The symmetry, whether it be good or evil, is implicit in all systems of electrical units. Only the M.K.S. system is so clear and so easy to look at as a whole that in it the symmetry stands out as a challenge. As has already been suggested, the M.K.S. system is above all things informative. It is not only informative about physical facts and relationships, but it even throws light upon the ways in which men have thought about them, and may go so far as to suggest that these ways are open to criticism. This is a virtue, not a vice.

The conclusion seems to be either that this artificial symmetry is so evil that we must scrap all existing systems of units and start again, or that we admit it and that wherever we put the blame, it must not be upon the M.K.S. system.

We may now return to our student. We shall suggest that instead of making deductions from his fundamental electrodynamic observations he continues to make observations. He should try to maintain a constant rate of change of current density in his plates. That means a constant rate of change of current in two opposite strips of finite dimensions. He will find that an e.m.f. over and above that demanded by Ohm's law has to act round the circuit and that it is proportional to the rate of change of current in the strips. The strips have an inertia with respect to change of current as mass has with respect to change of velocity. His next unit will be a measure of the inertia effect and will be the henry, defined as the volt/ampere/sec, or as it is usually written, the volt-second/ampere, the unit of inductance.

His next observations are that, if he increases the length of the strips and their separation, the inductance will rise proportionally to their product, that is to the area of a plane joining corresponding longitudinal edges of the strips, and that if he reduces the width of the strips so as to increase the current density for a given current,

the inductance will increase inversely as the width. He may write

$$L = \frac{V}{\mathrm{d}I/\mathrm{d}t} = \mu\frac{A}{d} \tag{4}$$

where A and d are perpendicular to the corresponding quantities in the electrostatic case. μ is a measure of the effect of the medium. It is called its permeability and is measured in henries/metre.

It should be pointed out to him that it is not necessary to use both these approaches to electro-dynamics at once. Equation 4, for instance, may be regarded as a summary of his fundamental observations, and energy considerations may be applied to establish the laws of forces between electric currents.

He will now find it convenient to invent a magnetic flux within a system having inductance, analogous to the electric flux in a system having capacitance, that is at right angles to the area, A, and along the distance, d, in the expression $\mu A/d$. As the torque on his exploratory loop was constant all over the inter-plate region he will naturally make the flux density uniform between the plates. If with a constant current in the strips he increases the area, A, it follows that the flux will increase proportionally. But so does the inductance. He may, then, extend equations (4) to read

$$L = \frac{V}{\mathrm{d}I/\mathrm{d}t} = \mu\frac{A}{d} = \frac{\phi}{I} \tag{5}$$

In order that ϕ/I may be not only proportional to L but equal to it he must choose an appropriate unit of magnetic flux. It is seen to be the volt-second (sometimes called the weber).

If now he divides equation (5) by A/d he obtains

$$\mu = \frac{\phi/A}{I/d} \tag{6}$$

But ϕ/A is the magnetic flux density, B. By analogy with the electrostatic case he will call I/d the magnetic potential gradient, H, and will measure it in amperes/metre. This automatically makes the ampere the unit of magnetic difference of potential. He has established that

$$\mu = B/H \tag{7}$$

If such an artificial invention of flux irks him, the historical association of flux and pole strength can be explained to him after a discussion of the electrodynamic nature of permanent magnets. The value of the convention may be illustrated by using the expression ϕ/I for inductance to show that the inductance of a long tube with respect to peripheral current is also $\mu A/d$. From this, progress can be made to the N-turn solenoid (in which, owing either to its length or to its being bent into a toroid, the end effects can be neglected). Here, as the current, I, only refers to an Nth of the peripheral current and as the e.m.f. acting in the whole coil is N times that in one turn,

$$L = N^2\mu A/d \tag{8}$$

SUMMARY OF THE MAIN UNITS

Reluctance, a very important quantity to designers of magnetic circuits, is discussed below. The units for all the other commonly used quantities have already appeared, and the reader will have noticed that with the exception of the newton no new names have been used. As was promised, a single sufficient system has been evolved by a very slight modification of approach to units already in common use. It was stated that this had been achieved by some special device. This device is simply to gather all the irrationalities in past modes of thought on the subject and to pack them into the only two numbers which the student is asked to remember by rote. These are the numbers attached to the permeability and permittivity of free space. μ_0 is $4\pi/10^7$ henries/metre and k_0 is $(1/36\pi) \times 10^{-9}$ farads/metre. Relative permeabilities and permittivities, being ratios, will remain unchanged. For instance, if a certain specimen of mica has a relative permittivity of 8, its absolute permittivity which must enter into all relevant relationships is $(8/36\,\pi) \times 10^{-9}$ farads/metre. In general

$$\mu = \mu'\mu_0 \quad \text{and} \quad k = k'k_0 \tag{9}$$

where μ' and k' are relative permeabilities and permittivities respectively.

Table 1. *Summary of M.K.S. units*

Mechanical Quantities			
length	*metre*	force	*newton* (kg-m/sec²)
mass	*kilogram*	work	*joule* (n-m)
time	*second*	power	*watt* (j/sec)
Electrical Quantities			
charge	*coulomb*	current	*ampere* (c/sec)
		e.d.p.	*volt* (j/c)
Circuit Parameters		**Specific Quantities**	
resistance	*ohm* (volt/amp)	resistivity	*ohm-metre*
conductance	*mho* (amp/volt)	conductivity	*mho/metre*
inductance	*henry* (volt-sec/amp)	permeability	*henry/metre*
capacitance	*farad* (amp-sec/volt)	permittivity	*farad/metre*
Electric Field Quantities		**Magnetic Field Quantities**	
flux	*ampere-sec*	flux	*volt-sec* (or *weber*)
flux-density	*ampere-sec/metre²*	flux-density	*volt-sec/metre²*
field strength	*volt/metre*	field strength	*ampere/metre*
e m.f.	*volt*	m.m.f.	*ampere*

The subject as far as it has been taken is summarized in Table 1. In this table some of the mechanical units have been tabulated in the top two panels, the basic units being on the left. Under these is the coulomb because it has been used as a basic unit for the purposes of this discussion. If this were a legal document the ampere would occupy this position without causing any trouble, and the coulomb would appear with the volt in the right hand panel as the ampere-second. The next panel on the left contains the units of the circuit parameters. Their dimensional symmetry needs no emphasis. In addition to the information which the dimensions immediately give about the nature of the quantities, they show at a glance such relationships as that CR, LG and

$\sqrt{LC}$ all have the dimensions of time, that ωL has the dimension, ohm, and that ωC has the dimension, mho. The opposite panel on the right contains the corresponding specific quantities. As resistivity and conductivity are mutually reciprocal, the last three symmetrical quantities will cover all requirements. From them it is seen that a right prism of cross-sectional area, A square metres, and length, d metres has a conductance.

$$G = \sigma A/d$$

d being measured in the direction of current flow, an inductance

$$L = \mu A/d$$

d being measured in the direction of the magnetic flux and a capacitance

$$C = kA/d$$

d being measured in the direction of the electric flux.

The corresponding mixed practical and C.G.S. expression for capacitance, $C = Ak/(4\pi d \times 9 \times 10^{11})$, contains the same information, but it is shrouded in a distracting veil of memory-taxing numbers.

The bottom two panels display the electric and magnetic field quantities in a particularly striking dimensional symmetry.

RELUCTANCE AND SOME OTHER QUANTITIES

The unit of reluctance has been omitted from the table as it is not essential to the development of the subject of electromagnetism. It is, however, a quantity of the greatest value to the engineer. Without it electrical machine design would be much more cumbersome than it is. It is a very well justified artificiality within an artificiality. The outer artificiality is magnetism itself, while the inner one is the forced analogy between the magnetic and the electric circuit. Its unit is the ampere-turn/volt-second. If we neglect the turn as having no dimensions this becomes the ampere/volt-second, which is seen to be the reciprocal of the henry as defined above. The henry written backwards, the yrneh, has actually been suggested. It is presumably pronounced as an affectionate diminutive of Ernest. This is aesthetically deplorable. Henry and yrneh conjure up a revolting picture of symmetrical Siamese twins. It would seem that the right thing to do would be to confess manfully one's double artificiality and to call the unit the magnetic ohm. This would have the advantage of supplying information about the way in which the quantity is used. It would further imply that the unit of permeance was the magnetic mho, and this would differentiate it from inductance which seems to have the same dimensions. In fact the number of turns appears differently in the expressions for the two quantities. Inductance is expressed as $N\phi/I$, whereas the expression for permeance is ϕ/NI.

Other quantities which it was not considered necessary to tabulate are electric polarization measured in ampere-seconds/square metre, magnetic induction measured in volt-seconds/square metre and electric and magnetic susceptibility which are numerics, being one less than the relative permittivity and permeability respectively.

EXAMPLES OF THE INFORMATIVE NATURE OF THE SYSTEM

A few random examples will now be taken to illustrate further the informative nature of the M.K.S. formulae and of their associated dimensions. First consider the magnetic potential gradient at a distance, r, from a long, thin wire carrying a current, I. In the magnetic C.G.S. system this appears as $2I/10r$ oersteds. The I/r carries the physical information. The $2/10$ demands a slight effort of memory and is devoid of information. It is doubtful if the units tell one very much. In the rationalized M.K.S. system the expression is $I/2\pi r$ amperes/metre. I amperes is the m.m.f. acting round $2\pi r$ metres. Both the formula and the dimensions display the required information with the greatest clarity, This is a case of cylindrical symmetry as indicated by the 2π.

Next let us turn to the energy stored in a magnetic field. The energy stored in the field of an inductance appears both in the practical and in the M.K.S. systems as $\frac{1}{2}LI^2$ joules. But when we come to consider energy density the practical system goes into partnership with the C.G.S. and we meet $\mu H^2/8\pi$ ergs/cubic centimetre. Here the 8π supplies the distraction. The dimensions of μ and H do not suggest any immediate connexion with L and I. In the M.K.S. system the expression is $\frac{1}{2}\mu H^2$ joules/cubic metre. As μ is measured in henries/metre and H in amperes/metre the relationship with the expression, $\frac{1}{2}LI^2$, is carefully underlined again both by the formula and by the dimensions.

The positions of 4π and of c in Maxwell's equations constitute a perpetual difficulty for students, the mastery of which in no way increases their supply of useful knowledge. In the M.K.S. system Maxwell's equations become

$$\begin{aligned} \text{curl } H &= \dot{D} + J & D &= kE \\ \text{curl } E &= -\dot{B} & B &= \mu H \\ \text{div } D &= \rho & &= \sigma E \\ \text{div } B &= 0 \end{aligned}$$

The dimensions of these equations can be checked by merely glancing at them.

The fruitful parallelism between transmission line and radiation relationships which only the rationalized M.K.S. system makes possible has been exhaustively treated by Booker[2] and will not be further discussed here.

THE CASE FOR THE GENERAL ADOPTION OF THE SYSTEM

There remains one point to discuss, a point which may be of the greatest importance to future scientists. Several times in this article the M.K.S. system has been

referred to as a single sufficient system of units. It follows that unless it supplants other systems it loses a great deal of its value. There seems therefore a case for investigating the implications of applying the system (to the exclusion of others) to all branches of physics. If this could be done without undue interference with current scientific work, there is little doubt that the benefit to the future scientist would be very great indeed. In his student days especially he would be free to concentrate upon essential things. The rate of acquisition of knowledge and the progress of science should be accelerated.

If the views of the advantages of the M.K.S. system as applied to electromagnetism, which this article is intended to support, are generally accepted, it must be agreed that work would be eased by its introduction into any branch of physics into which electromagnetism intrudes. The effect of its introduction upon any other branch of physics would be merely to change certain familiar numbers by powers of ten. This effect can be reviewed under four heads.

(1) The number to be remembered suffers no change.

(2) The number to be remembered is slightly less convenient than before. The author has found only one example of this class.

(3) The number to be remembered is more convenient than before, especially if prefixes such as milli- and micro- are admitted.

(4) A few noughts have to be added to a number which is associated with so many noughts already, that, as far as the new student is concerned, it really does not matter.

As an example of class (1) the gram-molecule occupies 22·4 litres. The kilogram-molecule occupies 22·4 cubic metres. This applies whenever grams and litres appear, one in the numerator and the other in the denominator. For instance, the densities of gases are usually expressed in grams/litre. The same number will be attached to them in kilograms/cubic metre.

The only example of class (2) which the author can quote is the density of water. In the M.K.S. system it is 1 000 kilograms/cubic metre. This is unfortunate, although probably not catastrophic. All densities of liquids and solids as usually expressed will have to be multiplied by 1 000. Specific gravities, being ratios, will remain unaltered.

As an example of Class (3), the millinewton/square metre might be a very convenient unit of pressure for the vacuum engineer. It is equivalent to 0·0000075 mm of mercury, which is in the region in which he is interested. The unit has the advantage of looking as if it were a measure of pressure. A further example is supplied by piezo-electric constants. That of quartz is about 2 $\mu\mu$C/newton (in place of 6×10^4 e.s.u./dyne). If the inverse piezo-electric effect is being considered this quantity may more aptly be written 2 micro-microns/volt. Both forms are immediately seen to be dimensionally the reciprocal of electric potential gradient, and so, as usual, throw light upon the quantity being measured. Whether the micron as a substitute for the Angström unit comes into this class the author is not competent to state.

The fourth class includes such obvious cases as the mass and charge of the electron, Planck's constant and Boltzmann's constant.

A fifth class might have been included in which the number can be said neither to gain nor to lose in convenience by the change. Young's modulus, for instance, in newtons/square metre would have a tenth of its value in dynes/square centimetre.

Table 2. *Conversion factors*

Length	1 metre = 3.28 feet	1 foot = 0.3048 metre
Mass	1 kilogram 2.205 pounds	1 pound = 0.4536 kilogram
	= 0.0685 slug	1 slug = 14.6 kilograms
Force	1 newton = 7.23 poundals	1 poundal = 0.1383 newton
	= 0.2247 pound weight	1 pound weight = 4.45 newtons
Torque	1 joule per radian = 0.738 pound-foot	1 pound-foot = 1.356 joules per radian
Work	1 joule = 0.738 foot-pound	1 foot-pound = 1.356 joules
Moment of inertia	1 kilogram-metre^2 = 23.76 pound-ft^2	1 pound-foot^2 = 0.0421 kilogram-metre^2
	= 0.738 slug-feet^2	1 slug-foot^2 = 1.356 kilogram-metre^2

All these effects are seen to be trivial. A very much more serious obstacle to the immediate exclusive adoption of the M.K.S. system is the fact that a great body of literature exists in which the C.G.S. systems have been used. It must, however, be recognized that an important body of literature using the M.K.S. system is growing, gradually in this country and very fast in America. It appears inevitable that during the next few years all the systems will have to be mastered by at least the more advanced students. It is probable that as a generation grows up which has been grounded in the M.K.S. system, the other systems will gradually fall into desuetude. Many will draw the conclusion that this process ought to be accelerated.

CONVERSION BETWEEN ENGLISH AND M.K.S. UNITS

As can be seen from Table 2 the M.K.S. system makes the conversion to and from the mechanical engineer's English units neither more nor less comfortable than it was in the case of the C.G.S. systems, except that the conversions are between two systems instead of between three systems.

It is only essential to remember, say, the three numbers underlined in the table, together with the acceleration due to gravity in one system or the other. It will probably be well known in both.

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Availability and irreversibility in thermodynamics

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Connexion is made between Gibbs's "available energy of the system and medium" and the criterion of stability. The available energy concept is developed for systems which communicate only with the uniform medium or atmosphere and for systems which communicate with an additional reservoir of specified temperature. The treatment is extended to problems of transient and steady flow. A measure of departure from the ideal, the *irreversibility*, is defined and examined for its significance. Performance coefficients are devised for several classes of processes. Finally, the generality of the method is exhibited by analysis of a variety of thermodynamic phenomena.

INTRODUCTION

The concept of an idealized process with which an actual process may be compared is common to all branches of thermodynamics. In view of the second law the idealized process is usually so selected as to be one of maximum production of work. The obtaining of maximum work involves the establishment of some restricting conditions regarding the possible behaviour of the system. For example, the physicist and the chemist have sometimes prescribed an environment of fixed temperature with which the otherwise isolated system must be in temperature equilibrium initially and finally—the maximum work then being the decrease in the so-called Helmholtz free energy; or they have sometimes prescribed an environment of fixed pressure and temperature with which the system must be in both pressure and temperature equilibrium initially and finally—the maximum useful work being the decrease in the so-called Gibbs free energy; the engineer has sometimes prescribed adiabatic conditions between some specified initial state and a specified final volume or pressure—the maximum work being the decrease in energy or enthalpy (depending upon the nature of the process) at constant entropy. In this latter instance it has often been pointed out that the final state of the idealized process may be quite different from that of the actual process.

None of these idealizations has generality—each is an *ad hoc* device of limited utility. The more general approach to the statement of the thermodynamically most beneficial result and to the evaluation of departures from it has been given limited attention.

The foundations of the general approach were laid by J. W. Gibbs in his second paper on thermodynamics.[1] Gibbs imposed on the behaviour of the system the condition that it should be isolated except for communication with a stable environment of uniform pressure and temperature. He put no restrictions, other than those imposed by the nature of thermodynamics, upon the selection of the system or of its initial and final states.

Maxwell[2] adopted a modified version of Gibbs's approach which omitted from consideration the pressure of the environment. Darrieus,[3] starting from Maxwell's method, developed a treatment of engineering problems in steady flow which was expanded upon by Keenan[4] and others. Published quantitative studies of irreversible processes that stem less directly from the work of Gibbs are less general in application. For example, the recent work of Tolman and Fine[5] is restricted to cyclic and steady-flow processes.

The purpose of the present paper is to develop the method of Gibbs more generally than has hitherto been done. Connexion is made between Gibbs's "available energy of the system and medium" and the criterion of stability. The available energy concept is developed for systems which communicate only with the uniform medium or atmosphere and for systems which communicate with an additional reservoir of specified temperature. The treatment is extended to problems of transient and steady flow. A measure of departure from the ideal, the *irreversibility*, is defined and examined for its significance. Performance coefficients are devised for several classes of processes. Finally, the generality of the method is exhibited by analysis of a variety of thermodynamic phenomena.

In this generality may be found the justification of the availability concept. It is through this concept that processes as widely different as the decay of motion in a viscous fluid, the rectification of a binary mixture, and the dissociation of hydrogen peroxide can be examined from a common basis of comparison and their thermodynamic quality compared quantitatively by means of the *irreversibility* or a *coefficient of performance* as here defined.

STABILITY, MAXIMUM WORK, AND AVAILABILITY

System exposed to an infinite atmosphere only.—Virtually all problems which can be treated adequately by the methods of thermodynamics are terrestrial: that is, they relate to the behaviour of systems which are surrounded by an essentially infinite atmosphere. A major exception to this latter generalization is found in the subject of meteorology wherein the system under consideration is the atmosphere itself. For other terrestrial problems, however, the system considered is small in mass and extent compared with the surrounding atmosphere which, for purposes of analysis, may be thought

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of as an environment whose temperature and pressure are unaltered by any process experienced by the system.

By including within the system as much material or machinery as is affected by the process (except for the atmosphere) one may consider any process which occurs as one in which the system interacts with the atmosphere only. Gibbs[6] has shown that for any process which can occur under these circumstances

$$\Delta\Phi < 0 \tag{1}$$

where $\Delta\Phi$ denotes the increase in the quantity Φ which is defined as follows:

$$\Phi \equiv E + p_0V - T_0S \tag{2}$$

where E denotes the energy of the system, V its volume, S its entropy, p_0 the pressure of the atmosphere, and T_0 its temperature on the Kelvin scale.

Since it is evident that processes or changes can occur until the pressure of the system is uniformly p_0 and its temperature uniformly T_0 and perhaps even thereafter, then the state from which no spontaneous changes can occur is the state for which the system has the pressure p_0 and the temperature T_0 and for which Φ has the minimum of all possible values. This is the state of stable equilibrium if there is only one such state, and of neutral equilibrium and maximum stability if there is more than one.

Gibbs referred to the difference

$$\Phi - \Phi_{min}$$

where Φ refers to a state in question and Φ_{min} to the most stable state, as the "available energy of the body and medium" for the state in question[7] (Gibbs uses the term *body* for what is called the *system* here and the term *medium* for what is called the *atmosphere* here). By this he meant the maximum *useful* work—that is, work in excess of that done against the atmosphere—which could be obtained from the system and atmosphere, without aid from other things, by any possible processes.

The point of view of the last paragraph differs from that of those which preceded it in that if useful work is delivered then something other than the system or the atmosphere must be affected by the process through the reception of work. The only other thing considered to be affected, however, is a work reservoir, such as a coiled spring or a flywheel, which operates adiabatically in the course of the process.

The proof of Gibbs's statement, which he does not give explicitly, may be given as follows. If a system which is surrounded by an atmosphere at p_0 and T_0 experiences a change from state 1 to state 2 while it receives net heat (positive or negative) from the atmosphere only, the useful work which will be delivered to things other than the system and atmosphere cannot exceed that of a reversible process between states 1 and 2. For if this were not true then it would be possible to execute the hypothetical process from 1 to 2 which produces work in excess of that of the reversible process and to complete a cyclic change for the system by means of the reverse of the reversible process from 2 to 1. This cycle would have a net production of positive work and, since the atmosphere at T_0 would be the only heat reservoir engaged in the process, the cycle would constitute a perpetual-motion machine of the second kind. The hypothetical process is therefore impossible and no process can produce useful work in excess of that of the reversible process.

By similar reasoning it may be shown that all reversible processes between states 1 and 2 having heat transfer between system and atmosphere only must produce identical quantities of useful work.

In order to evaluate the useful work of an infinitesimal reversible process one may set up a reversible means of heat transfer between the atmosphere at T_0 and the system at T, say, which consists of a reversible cyclic engine of small enough dimensions so that one cyclic operation will be required to absorb or deliver an infinitesimal amount of heat (Fig. 1). Let dE, dV, and dS denote respectively the energy, volume, and entropy changes experienced by the system in going from the first prescribed state to the second. These quantities would obviously have the same values for this change of state regardless of the nature of the process or the amount of useful work produced. Let δQ_0 denote the heat received by the reversible engine at T_0 from the atmosphere. The quantity δQ_0 (unlike dV, dE, and dS) will have different values for the same change of state in the system, depending upon the nature of the process and the amount of useful work produced. In what follows the symbols d and δ will be used to differentiate between quantities like dV on the one hand, which are fixed by the end states of the system, and δQ_0 on the other, which are not so fixed.

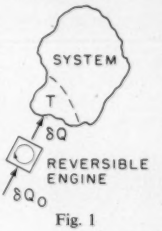

Fig. 1

The magnitude of δQ_0 may be, of course, either greater or less than zero. A magnitude less than zero would denote heat flow away from the engine to the atmosphere.

Now the work done by the system and the cyclic engine in combination is given by

$$-dE + \delta Q_0$$

where dE, the increase in energy of the system, is also the increase in energy of the system and the cyclic engine combined. Of this work the amount p_0dV must be expended in displacing the atmosphere. Therefore the useful work of the reversible process which is the maximum useful work $(\delta W_u)_{max}$ of all possible processes is given by

$$(\delta W_u)_{max} = -dE - p_0dV + \delta Q_0$$

By the definition of the temperature scale, however,

$$\delta Q_0/T_0 = \delta Q/T$$

where δQ denotes the heat received by the system from

the reversible engine. Moreover, the change of entropy dS of the system in the course of this reversible process is given by

$$dS = \delta Q/T$$

It follows upon substitution into the previous expression for $(\delta W_u)_{max}$ that

$$\begin{aligned}(\delta W_u)_{max} &= -dE - p_0 dV + T_0 dS \\ &= -d\Phi \end{aligned} \tag{3}$$

and, in general,

$$\delta W_u \leqslant -d\Phi \tag{4}$$

An alternative method of deriving (4), which is suggested by the method of Gibbs in proving (1), is as follows. Since the atmosphere is a fluid which is never in any but stable equilibrium states, one may write

$$T_0 \delta S_a = \delta E_a + p_0 \delta V_a \tag{5}$$

where subscript a refers to the atmosphere. From the second law of thermodynamics it may be said that the entropy of the system-atmosphere combination cannot decrease, or that

$$\delta S_a + dS \geqslant 0 \tag{6}$$

Gibbs considered the large atmosphere as one whose outer bounds were fixed in position, so that

$$\delta V_a + dV = 0 \tag{7}$$

Then the useful work of any process between the prescribed end states of the system is given by

$$\delta W_u = -\delta E_a - dE \tag{8}$$

since the system-atmosphere combination receives net heat from nothing else.

Upon substituting for δE_a in (8) from (5), eliminating δS_a and δV_a by means of (7) and (8) respectively, and noting that T_0 can never be less than zero, one gets

$$\delta W_u \leqslant -dE - p_0 dV + T_0 dS$$

which is (4).

It might be noted that this proof may be modified by considering the atmosphere to be a finite one which is restrained by an appropriate envelope to pressure p_0. Then instead of (7) and (8) one has

$$\delta W_u = -\delta E_a - dE - p_0(\delta V_a + dV) \tag{9}$$

Substitution from (5) and (6) into (9) again yields (4).

Under the restrictions imposed—namely, that heat can be transferred only between system and atmosphere—useful work can be obtained only from changes of state for which Φ decreases. Moreover, since the useful work may be in any degree less than the maximum value, it may be zero for any change of state for which Φ decreases. Thus spontaneous changes can occur only to states of less or equal values of Φ.

Changes of state for which Φ increases can be accomplished only with the aid of useful work supplied from outside the system-atmosphere combination. Such changes cannot occur spontaneously.

For a finite change from the system in state 1 to the system in state 2

$$W_u = \int_1^2 \delta W_u$$

$$W_u \leqslant -\int_1^2 d\Phi$$

or

$$W_u \leqslant \Phi_1 - \Phi_2 \tag{10}$$

It follows that for a given state 1 of the system there is a maximum value of the useful work which can be obtained from the system-atmosphere combination for all possible changes of state of the system. This is, obviously,

$$\Phi_1 - \Phi_{min}$$

where Φ_{min} is the minimum value of Φ for all possible states of the system. This minimum value corresponds to the most stable state of the system in the presence of the atmosphere. Necessary, but not sufficient, requirements for this state are a pressure of p_0 and a temperature of T_0.

It is proposed to call this maximum value of the useful work corresponding to a state of the system within the atmosphere the *availability* instead of the longer term used by Gibbs and to denote it by the symbol Λ. Thus

$$\text{Availability} \equiv \Lambda = \Phi - \Phi_{min} \tag{11}$$

It may be said that for any state of any system in a stable atmosphere

$$\Lambda \geqslant 0 \tag{12}$$

and that for the most stable state of the system

$$\Lambda = 0 \tag{13}$$

Moreover, for any one system-atmosphere combination, Φ differs from Λ by a constant. A geometrical representation of Φ or Λ as the vertical distance above a plane in which two independent properties of the system (such as V and S) were the other two co-ordinates, would be a surface with its lowest point representing the most stable state. If two or more points are equally low and lower than all others, then these are equally stable states and equilibrium is neutral as between them. For the Λ surface, as distinguished from the Φ surface, the lowest point or points will lie at the zero or datum level.

It follows from (11) that the increase in availability for any process is given by

$$\Delta\Lambda = \Delta\Phi \tag{14}$$

and from (10) that

$$W_u \leqslant -\Delta\Lambda = -\Delta\Phi \tag{15}$$

System exposed to a heat reservoir in addition to an infinite atmosphere.—Often one is interested in processes which involve heat transfer between a system and some reservoir at a temperature different from that of the atmosphere. As an example of the latter one might cite the hot reservoir of a power plant.

Of course, the reservoir may be included within the system for purposes of analysis, and this device yields a

satisfactory answer in terms of change in Φ of the reservoir and the system. For convenience, however, it would be better to put the answer in terms of the quantity of heat withdrawn from the reservoir. This may be done as follows:—

For a change of state in the system while it receives net heat from the atmosphere and from a reservoir R, one may, from the point of view of the preceding paragraph, write from (10)

$$W_u \leqslant -\Delta\Phi - \Delta\Phi_R \tag{16}$$

where the last term refers to the reservoir R and the next-to-the-last to the system as usual. This may be rewritten in the form

$$(W_u)_{max} = -\Delta\Phi - \Delta E_R - p_0\Delta V_R + T_0\Delta S_R \tag{17}$$

The term reservoir generally implies a system which passes only through stable states and which if it expands or contracts does so slowly in the presence of the atmosphere. Letting Q_R denote the heat flow *from the reservoir* to the system, one may therefore write

$$Q_R = -\Delta E_R - p_0\Delta V_R = -T_R\Delta S_R \tag{18}$$

Substituting from (18) into (17), one gets

$$W_u \leqslant -\Delta\Phi + Q_R\left(\frac{T_R - T_0}{T_R}\right) \tag{19}$$

or for an infinitesimal process

$$\delta W_u \leqslant -\mathrm{d}\Phi + \mathrm{d}Q_R\left(\frac{T_R - T_0}{T_R}\right) \tag{20}$$

An alternative proof of (19) and (20) may be devised in which the work of a reversible process $(W_u)_{max}$ is evaluated for a process involving reversible cyclic engines to transfer heat across finite temperature differences. It will not suffice, however, merely to insert an engine between the reservoir and the system because then the heat leaving the reservoir would not be of the same quantity as that reaching the system. Instead, one may use two engines (Fig. 2): one to take heat $\mathrm{d}Q_R$ from the

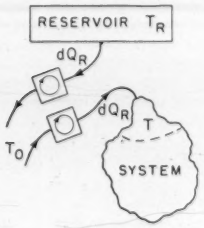

Fig. 2

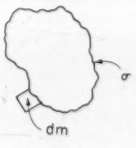

Fig. 3

reservoir and reject heat to the atmosphere and one to take heat from the atmosphere and reject $\mathrm{d}Q_R$ to the system. Now, since the system draws on the atmosphere only, the maximum useful work for it is simply $-\delta\Phi$, and the maximum useful work from the remainder of the operation is the work of the first reversible engine which is $\mathrm{d}Q_R(T_R - T_0)/T_R$. The sum is as indicated in (20).

It should be noted that the heat from the reservoir is treated differently here as compared with the heat from the atmosphere. For example, the former appears explicitly in (19) and (20), whereas the latter does not. The implication is that Q_R, like the change of state of the system, is prescribed. The heat from the atmosphere, on the other hand, is not prescribed and varies in fact with the magnitude of W_u. For this reason the symbol $\mathrm{d}Q_R$ is used for the prescribed infinitesimal quantity of heat from R to correspond with the prescribed changes in properties of the system $\mathrm{d}E$, $\mathrm{d}V$, and $\mathrm{d}S$.

FLOW PROCESSES AND MAXIMUM SHAFT WORK

Flow across a control surface.—Consider a closed control surface σ (Fig. 3) in a field of fluid flow which exchanges heat only with the atmosphere. When the mass element $\mathrm{d}m$ crosses the surface σ from outside to inside, the useful work of the process executed by a system consisting of all the fluid finally inside σ is given by

$$\delta W_u \leqslant -\mathrm{d}\Phi$$

or

$$\delta W_u \leqslant -\Phi_\sigma'' + (\Phi_\sigma' + \phi_x\mathrm{d}m) \tag{21}$$

where subscript σ refers to fluid inside surface σ, superscripts $'$ and $''$ refer to initial and final states, and ϕ_x denotes the value of Φ per unit mass for fluid outside σ which is about to enter.

Alternatively

$$\delta W_u \leqslant -\mathrm{d}\Phi_\sigma + \phi_x\mathrm{d}m \tag{22}$$

where $\mathrm{d}\Phi_\sigma$ denotes the increase in the magnitude of Φ summed up for all fluid found within σ instantaneously.

If σ lies within an extensive fluid field, then some of the useful work represented by (22) is done on or by fluid which follows the element $\mathrm{d}m$. The remainder must be delivered outside of σ by a shaft, a piston rod, electrical conductors, or the like. Calling this part of the useful work *shaft work* and denoting it by the symbol W_y, one may write

$$\delta W_y = \delta W_u + (p_x - p_0)v_x\mathrm{d}m \tag{23}$$

The $+$ sign in the right-hand member of (23) arises from the fact that when $(p_x - p_0)$ is positive and $\mathrm{d}m$ is positive (flowing in, by the implied convention) then the useful work done in pushing $\mathrm{d}m$ across σ is negative. Implied in (23) is the simplifying assumption that only normal stresses occur at the boundary of $\mathrm{d}m$—that is, the control surface is so placed that no shear exists where fluid flows through the surface. It follows from (22) and (23) that

$$\delta W_y \leqslant -\mathrm{d}\Phi_\sigma + [\phi_x + (p_x - p_0)v_x]\mathrm{d}m$$

or

$$\delta W_y \leqslant -\mathrm{d}\Phi_\sigma + [e_x + p_x v_x - T_0 S_x]\mathrm{d}m \tag{24}$$

where e_x, v_x and s_x denote the energy, volume, and entropy each per unit mass of the fluid which is about to cross σ.

If the fluid has appreciable velocity and it flows in a gravitational field of strength g, then one has

$$e = u + \frac{c^2}{2g_0} + z\frac{g}{g_0} \tag{25}$$

where u denotes the energy per unit mass of fluid at rest at the datum level, c the velocity of the fluid, z its height above the datum plane, g the acceleration of gravity, and g_0 the acceleration given to unit mass by unit force. Substituting (25) into (24) one gets

$$\delta W_y \leqslant -\,\mathrm{d}\Phi_\sigma + \left[h_x - T_0 s_x + \frac{c_x^2}{2g_0} + z_x\frac{g}{g_0}\right]\mathrm{d}m$$

or

$$\delta W_y \leqslant -\,\mathrm{d}\Phi_\sigma + \left[b_x + \frac{c_x^2}{2g_0} + z_x\frac{g}{g_0}\right]\mathrm{d}m \tag{26}$$

where

$$b \equiv h - T_0 s$$

and

$$h \equiv u + pv \tag{27}$$

the latter being the enthalpy.

It may readily be seen that if a reservoir R is considered to be outside σ and to supply heat $\mathrm{d}Q_R$ to the fluid inside σ while $\mathrm{d}m$ flows in, then

$$\delta W_y \leqslant -\,\mathrm{d}\Phi_\sigma + \left[b_x + \frac{c_x^2}{2g_0} + z_x\frac{g}{g_0}\right]\mathrm{d}m + \frac{T_R - T_0}{T_R}\mathrm{d}Q_R \tag{28}$$

Steady flow.—Application to steady flow results in two important modifications of the more general equations which were developed above. The first is that

$$\mathrm{d}\Pi_\sigma = 0$$

where Π_σ denotes the total value of any property, such as V, S, or Φ, of the mass of fluid found within surface σ at any instant. The second is that the mass flow across σ can be subdivided into two equal mass flows, one in and one out. The most general of the flow equations (28) then becomes

$$\frac{\delta W_y}{\mathrm{d}t} = P_y \leqslant \Sigma_{in}\left(b + \frac{c^2}{2g_0} + z\frac{g}{g_0}\right)w - \Sigma_{out}\left(b + \frac{c^2}{2g_0} + z\frac{g}{g_0}\right)w + \frac{T_R - T_0}{T_R}\frac{\mathrm{d}Q_R}{\mathrm{d}t} \tag{29}$$

where P_y denotes the shaft power flowing out of σ, Σ_{in} a summation over all the streams flowing into σ, Σ_{out} a summation over all the streams flowing out of σ, and $\mathrm{d}Q_R/\mathrm{d}t$ the rate of heat flow from reservoir R to material inside σ. This last equation may be compared with the so-called energy equation of steady flow in the form

$$P_y = \Sigma_{in}\left(h + \frac{c^2}{2g_0} + z\frac{g}{g_0}\right)w - \Sigma_{out}\left(h + \frac{c^2}{2g_0} + z\frac{g}{g_0}\right)w + \frac{\delta Q_a}{\mathrm{d}t} + \frac{\mathrm{d}Q_R}{\mathrm{d}t} \tag{30}$$

where $\delta Q_a/\mathrm{d}t$ denotes the rate of heat flow from the atmosphere to material inside σ.

IRREVERSIBILITY

A quantitative definition.—From (15) a quantitative definition of irreversibility can be devised. Letting I denote the irreversibility of a process which the system-atmosphere combination executes, one may write

$$I = (W_u)_{max} - W_u \tag{31}$$

$$= -\,\Delta\Lambda - W_u \tag{32}$$

$$= -\,\Delta\Phi - W_u \tag{33}$$

Substituting into (33) the definition (2) of Φ and for W_u the integrated form of (8), and noting from (7) that

$$\Delta V = -\,\Delta V_a$$

one gets

$$I = T_0\Delta S + \Delta E_a + p_0\Delta V_a$$

which, in accordance with (5), may be reduced to

$$I = T_0\Delta(S_a + S) \tag{34}$$

A similar result may be had by using (9) without (7). Thus, the irreversibility becomes equal to the increase in entropy of everything involved in the process multiplied by the temperature of the atmosphere. It is evident from both (31) and (34) that

$$I \geqslant 0 \tag{35}$$

If heat is received from a reservoir R then the irreversibility is given, in accordance with the definition (31), by

$$I = -\,\Delta\Phi + Q_R\frac{T_R - T_0}{T_R} - W_u \tag{36}$$

Considering the system and atmosphere combined as a system, one may write

$$W_u = -\,\Delta E - \Delta E_a + Q_R$$

Upon substituting this into (36) and proceeding as in the previous paragraph, one gets

$$I = T_0\Delta(S_a + S + S_R)$$

Irreversibility in flow across a control surface.—According to (31) one gets for the flow of $\mathrm{d}m$ across σ irreversibility of the magnitude

$$\delta I = (\delta W_u)_{max} - \delta W_u$$

Since $(\delta W_u)_{max}$ differs from $(\delta W_y)_{max}$ by the amount $(p_x - p_0)v_x\mathrm{d}m$ and δW_u differs from δW_y by the same amount, it follows that

$$\delta I = (\delta W_y)_{max} - \delta W_y \tag{37}$$

This is a consequence, of course, of eliminating by hypothesis shear stress at the boundary of the mass $\mathrm{d}m$. It is not implied, however, that the process of flow across σ is reversible. For example, σ may coincide with a thin porous wall through which the fluid flows irreversibly. This would involve shear between the fluid and the wall, but not between the fluid $\mathrm{d}m$ and the surrounding fluid which pushes it across σ.

It has been shown[8] from the first law of thermodynamics that

$$\delta W_y = -\mathrm{d}E_\sigma + \left(h_x + \frac{c_x^2}{2g_0} + z_x\frac{g}{g_0}\right)\mathrm{d}m + \mathrm{d}Q_R + \delta Q_a \tag{38}$$

for a control surface which is of invariable volume. Substituting into (37) the expression for $(\delta W_y)_{max}$ given by (28) and that for δW_y in (38), one gets

$$\delta I = -p_0\mathrm{d}V_\sigma + T_0\mathrm{d}S_\sigma + (b_x - h_x)\mathrm{d}m - \frac{T_0}{T_R}\mathrm{d}Q_R - \delta Q_a \tag{39}$$

Since $\mathrm{d}V_\sigma$ is zero by hypothesis, the last equation becomes

$$\delta I = T_0\left(\mathrm{d}S_\sigma - s_x\mathrm{d}m - \frac{\mathrm{d}Q_R}{T_R} - \frac{\delta Q_a}{T_0}\right) \tag{40}$$

or, since the increase in entropy $\mathrm{d}S$ of all the fluid finally found within σ is

$$\mathrm{d}S = \mathrm{d}S_\sigma - s_x\mathrm{d}m$$

$$\delta I = T_0(\mathrm{d}S + \mathrm{d}S_R + \delta S_a) \tag{41}$$

If the control surface is permitted to change in volume by amount $\mathrm{d}V\sigma$ during this process and δW_y is made to include the useful part of the work done at the moving boundary, or $(p_\sigma - p_0)\mathrm{d}V_\sigma$, then the right-hand member of (38) is altered by adding a term $-p_0\mathrm{d}V_\sigma$, the right-hand member of (39) by cancelling out the term $-p_0\mathrm{d}V_\sigma$, and (40) and (41) remain unchanged. Examples of this case would be the intake and discharge processes in a reciprocating machine.

The equations developed above for flow processes are applicable to processes involving flow in or out. The convention adopted requires that for flow in the quantity $\mathrm{d}m$ should be a positive number, and for flow out a negative number. In both instances the subscript x refers to states just outside the control surface. For flow in, the state outside is essentially independent of the state inside. For flow out, on the other hand, the state outside may be a direct consequence of a state immediately on the other side of the surface σ. For example, if no heat flows across the surface, then the enthalpy h_x will be identical with the enthalpy h immediately on the other side of σ.

Irreversibility in steady flow.—For steady flow (40) reduces to

$$\left.\begin{aligned}\delta I &= T_0\left(-\Sigma s_x\mathrm{d}m - \frac{\mathrm{d}Q_R}{T_R} - \frac{\delta Q_a}{T_0}\right)\\ &= T_0(-\Sigma s_x\mathrm{d}m + \mathrm{d}S_R + \delta S_a)\end{aligned}\right\} \tag{42}$$

Thus the irreversibility becomes the product of T_0 and the sum of the net flux of entropy *out* of the control surface, the increase in entropy for the reservoir R, and that for the atmosphere. For example, the irreversibility per unit mass of a single stream passing in steady flow across a control surface would be

$$I = T_0(s_2 - s_1 + \Delta S_R + \Delta S_a)$$

where subscripts 1 and 2 refer to entrance and exit respectively.

An alternative, though entirely equivalent, development of (42) consists of getting the rate of production of irreversibility

$$\frac{\delta I}{\mathrm{d}t} = (P_y)_{max} - P_y$$

by subtracting the right-hand member of (30) from that of (29) and then multiplying through by $\mathrm{d}t$.

COEFFICIENTS OF PERFORMANCE

Various coefficients of performance in the nature of efficiencies can be devised in view of the first and second laws of thermodynamics. It would seem to be desirable to define these coefficients in such a way that they would not exceed unity for any processes to which they may properly be applied.

Starting from the relation

$$W_u \leqslant (W_u)_{max}$$

which is implicit in the definitions of its two members, one can devise the coefficient

$$C_1 = W_u/(W_u)_{max} \tag{43}$$

This coefficient cannot exceed unity, provided that $(W_u)_{max}$ is greater than zero.

If, however, $(W_u)_{max}$ is less than zero, then W_u may also be less than zero; and since no lower limit can be set for W_u the value of the coefficient may not only exceed unity, but may actually extend toward infinity. The coefficient then would be greater in magnitude the greater the degree of irreversibility. A more rational definition of a coefficient where $(W_u)_{max}$ is less than zero would be

$$C_2 = (W_u)_{max}/W_u \tag{44}$$

a quantity which under these circumstances increases in magnitude with decrease in irreversibility and has the value unity for the reversible process.

By the definition of the irreversibility I, (31), one may write (43) in the form

$$C_1 = 1 - I/(W_u)_{max} \tag{45}$$

which in combination with (35) indicates as noted above that

$$C_1 \leqslant 1$$

provided that $(W_u)_{max}$ is greater than zero. The analogous statement for C_2 is

$$C_2 = 1 - I/(-W_u)$$

which indicates that

$$C_2 \leqslant 1$$

provided that $(W_u)_{max}$ (and, therefore, W_u) is less than zero.

The coefficient C_1 is appropriate for processes which are intended to produce useful work: for example, the expansion process in a heat engine or the combustion process in a Diesel engine. It compares the useful work produced with the sum of the expenditures of Φ and of the work-producing capacity of heat from a reservoir; thus,

$$C_1 = \frac{W_u}{-\Delta\Phi + Q_R\left(\frac{T_R - T_0}{T_R}\right)} \tag{46}$$

The coefficient C_2 is appropriate for processes which consume useful work: for example, compression combined with heat rejection in a heat pump. It compares the sum of the gain of availability for the system and reservoir with the useful work consumed; thus,

$$C_2 = \frac{\Delta\Phi - Q_R\left(\frac{T_R - T_0}{T_R}\right)}{(-W_u)} \tag{47}$$

By writing (19) in two other ways two more pairs of coefficients may be devised. Thus, from

$$W_u + \Delta\Phi \leq Q_R\left(\frac{T_R - T_0}{T_R}\right) \tag{48}$$

one may define

$$C_3 = \frac{W_u + \Delta\Phi}{Q_R\left(\frac{T_R - T_0}{T_R}\right)} \tag{49}$$

and

$$C_4 = 1/C_3 = \frac{Q_R\left(\frac{T_R - T_0}{T_R}\right)}{W_u + \Delta\Phi} \tag{50}$$

It is clear that

$$C_3 < 1$$

when $$Q_R\left(\frac{T_R - T_0}{T_R}\right) > 0$$

and that $$C_4 < 1$$

when $$Q_R\left(\frac{T_R - T_0}{T_R}\right) < 0$$

Thus C_3 might be used when the principal expenditure was heat from a hot reservoir or heat to a cold reservoir, and C_4 when the principal product was the heating of a hot reservoir or the cooling of a cold one.

From

$$W_u - Q_R\left(\frac{T_R - T_0}{T_R}\right) \leq -\Delta\Phi \tag{51}$$

one may define

$$C_5 = \frac{W_u - Q_R\left(\frac{T_R - T_0}{T_R}\right)}{-\Delta\Phi} \tag{52}$$

and

$$C_6 = \frac{1}{C_5} = \frac{\Delta\Phi}{-W_u + Q_R\left(\frac{T_R - T_0}{T_R}\right)} \tag{53}$$

Again $$C_5 < 1$$

when $$\Delta\Phi < 0$$

and $$C_6 < 1$$

when $$\Delta\Phi > 0$$

Thus C_5 might be used when the principal expenditure was a loss in availability of a system, and C_6 when the principal product was an increase in availability of a system.

The coefficients C_1, C_2, C_3, and C_4 (but not C_5 and C_6) are applicable to cyclic processes in a system. Since for these, however, $\Delta\Phi$ is zero, it follows that

$$C_1 = C_3$$

and $$C_2 = C_4$$

Coefficient C_1 may be used for work-producing cycles, coefficient C_2 for work-consuming cycles. The use of C_1 in this sense was implied by Kelvin in 1848.[9] When C_2 is applied to a refrigeration cycle Q_R will be positive and $(T_R - T_0)$ will be negative. For a heat pump Q_R will be negative and $(T_R - T_0)$ will be positive. For both, W_u will be negative.

Coefficients can as readily be devised for flow processes by the method just outlined. One special case has been proposed and discussed at some length[10] for steady-flow processes in turbines in the absence of a reservoir R. It derives from (29) and may be stated in the form

$$\epsilon = \frac{W_y}{m(b_1 - b_2)} \tag{54}$$

where W_y here denotes the shaft work produced when m units of mass enter the device at the condition corresponding to subscript 1 and an equal mass leaves at a condition corresponding to subscript 2. If terms other than b in (29) are significant, (54) would have to be modified accordingly.

APPLICATIONS

Pure substance.—A simple application of some of the equations developed above can be made to a system consisting of a pure substance under conditions such that two independent properties are sufficient to determine the state. More specifically, this is a system homogeneous in state in the absence of capillarity, motion, gravity, electricity, magnetism, and chemical change.

For an infinitesimal change of state of unit mass of such a system

$$\delta W_u \leq -\mathrm{d}\Lambda = -\mathrm{d}\Phi = -\mathrm{d}\phi = -\mathrm{d}\omega \tag{55}$$

where $$\omega \equiv u + p_0 v - T_0 s$$

as distinguished from the more general quantity

$$\phi \equiv \epsilon + p_0 v - T_0 s$$

The property ω is, for any fixed atmospheric conditions, a function of two independent properties such as v and s. Expanding (55), one gets

$$\delta W_u \leqslant -\mathrm{d}u - p_0 \mathrm{d}v + T_0 \mathrm{d}s$$

For infinitesimal changes between equilibrium states the first and second laws of thermodynamics give

$$-\mathrm{d}u = -T\mathrm{d}s + p\mathrm{d}v \qquad (56)$$

so that

$$W_u \leqslant (p - p_0)\mathrm{d}v - (T - T_0)\mathrm{d}s \qquad (57)$$

Upon integration of (57) over a path passing through equilibrium states, one gets for the maximum useful work the difference between an area A on the pressure-volume diagram and an area B on the temperature-entropy diagram (Fig. 4).

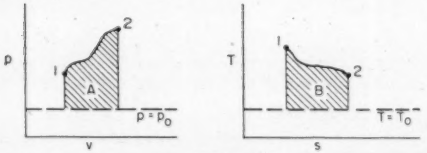

Fig. 4

In particular, for cooling at constant-volume ($\mathrm{d}s < 0$) the maximum useful work is the work which would be obtained from a reversible cyclic engine which receives heat from the system at the variable temperature T and rejects heat to the atmosphere at the constant temperature T_0. This is the area B' of Fig. 5. For expansion at constant entropy the maximum useful work is the expansion work in excess of that done against the atmosphere in a reversible adiabatic change. This is the area A' in Fig. 6.

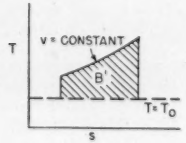

Fig. 5

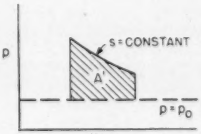

Fig. 6

Between two states having the same pressure and temperature (57) may be integrated to give

$$W_u \leqslant (p - p_0)\Delta v - (T - T_0)\Delta s \qquad (58)$$

provided that a continuous series of intermediate equilibrium states exist at the same pressure and temperature. This would be true for a change of phase between two saturation states. If, on the other hand, no such series of intermediate equilibrium states exists at that pressure and temperature, then (58) is not valid, because the integration of (58) implies the integration of (56) which can be valid only for a path through equilibrium states. Thus, (58) will not hold for a change from a supersaturated vapour state to a liquid state at the same pressure and temperature. For such a change (57) may be integrated over any path which consists entirely of equilibrium states, such as the constant-temperature path from the supersaturated-vapour state to saturated vapour to saturated liquid to the liquid state ($vgfl$, Fig. 7).

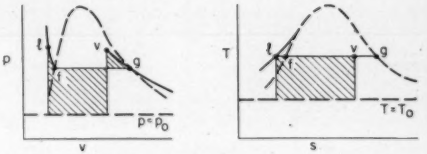

Fig. 7

The pure substance has been treated by Gibbs at some length.[11] His discussion omits capillarity, gravity, electricity, magnetism and chemical change, but includes motion and non-homogeneity of state. He represents states by points in a co-ordinate space in which the vertical axis is the energy axis and the two horizontal axes are those of entropy and volume. All equilibrium homogeneous states in this space lie on a "primitive surface" in this space and all non-homogeneous equilibrium states on "derived surfaces." The "availability," as defined here, is the vertical distance the state point lies above a plane which is tangent to the primitive surface at the point representing the most stable state. Other representations of availability can be devised, but few observations could be made from them that are not expressed or implied by Gibbs.

Combustion.—Equations (15), (31), (32), and (33) may be applied to any thermodynamic system that may be considered to be enclosed within a stable and uniform atmosphere. They may, therefore, be applied to processes involving, mixing, solution, or chemical reaction. In order to determine the change in Φ or Λ and the value of $(W_u)_{max}$ it is necessary only to know the change in energy, volume, and entropy of all the material which is involved in the process.

A highly simplified case of a chemical reaction will serve as an illustration.

Consider a reaction like the formation of nitric oxide,

$$\tfrac{1}{2}\mathrm{N_2} + \tfrac{1}{2}\mathrm{O_2} = \mathrm{NO}$$

for which the number of moles of products is the same as the number of moles of reactants and for which both

reactants and products are, say, diatomic. The problem may be further simplified by the assumption that reactants and products are perfect gases having identical molal specific heats which are independent of temperature.

For an adiabatic reaction at constant volume the energy of the system is unaltered. Letting subscripts P and R refer to products and reactants respectively, one may write

$$U_P - U_R = 0 \tag{59}$$

In terms of the energy of reaction

$$(U_{RP})_0 \equiv U_{P0} - U_{R0}$$

at temperature T_0, (59) becomes

$$(U_P - U_{P0}) - (U_R - U_{R0}) + (U_{RP})_0 = 0 \tag{60}$$

which is also

$$C_v(T_P - T_0) - C_v(T_R - T_0) + (U_{RP})_0 = 0$$

or

$$C_v(T_P - T_R) + (U_{RP})_0 = 0 \tag{61}$$

where C_v is the specific heat at constant volume of the system either as products or as reactants, its value being by hypothesis the same constant for both. From (61) it is clear that the temperature rise in combustion $(T_P - T_R)$ is constant and, therefore, independent of the temperature T_R at which the reaction begins.

The maximum useful work for this process—or the decrease in availability—is given by

$$\begin{aligned}(W_u)_{max} &= \Phi_R - \Phi_P \\ &= T_0(S_P - S_R)\end{aligned} \tag{62}$$

because both $(U_P - U_R)$ and $(V_P - V_R)$ are zero. But

$$\begin{aligned}(S_P - S_R) &= (S_P - S_{P0}) - (S_R - S_{R0}) + (S_{RP})_0 \\ &= C_v\left(\ln\frac{T_P}{T_0} - \ln\frac{T_R}{T_0}\right) + (S_{RP})_0 \\ &= C_v \ln\frac{T_P}{T_R} + (S_{RP})_0\end{aligned} \tag{63}$$

where $(S_{RP})_0$ is the entropy of reaction at T_0. Substituting (63) into (62), one gets

$$(W_u)_{max} = T_0\left[C_v \ln\frac{T_P}{T_R} + (S_{RP})_0\right] \tag{64}$$

Since, from (61),

$$\frac{T_P}{T_R} = 1 - \frac{(U_{RP})_0}{T_R C_v}$$

the temperature ratio T_P/T_R decreases with increase in temperature (for U_{RP} less than zero—that is, for a temperature rise in combustion). It follows from (64) that the maximum useful work decreases with increase in the temperature of the reactants.

The irreversibility is defined by

$$I = (W_u)_{max} - W_u$$

and W_u is in this case zero; therefore,

$$I = (W_u)_{max}$$

and the irreversibility is seen to decrease with increase in the temperature at which the reaction begins. This is a major element in the justification for compression of a charge in an internal-combustion engine before ignition.

If the combustion occurs adiabatically at constant pressure p, instead of at constant volume, the result is similar. Once more the temperature rise is independent of the temperature of the reactants. The maximum useful work becomes

$$(W_u)_{max} = (p - p_0)(V_P - V_R) + T_0(S_P - S_R)$$

This time, however, the useful work is not zero but

$$W_u = (p - p_0)(V_P - V_R)$$

and the irreversibility is, as before,

$$I = T_0(S_P - S_R)$$

which again is a quantity which decreases with increase in the temperature at which reaction begins.

Flow into a chamber.—The adiabatic flow of a perfect gas from a region of constant conditions into an evacuated chamber may be used to illustrate a problem in nonsteady flow. Referring to conditions in the outside region by subscript x one may write for (38)

$$\delta W_y = 0 = -\mathrm{d}E_\sigma + h_x \mathrm{d}m \tag{65}$$

and for (28)

$$(\delta W_y)_{max} = -\mathrm{d}\Phi_\sigma + b_x \mathrm{d}m$$

Upon integrating each of these between the initial condition of no fluid inside the chamber to the final condition, for which subscript f will be used to denote the condition in the chamber, one gets

$$-mu_f + mh_x = 0 \tag{67}$$

and

$$(\delta W_y)_{max} = -m\phi_f + \Phi_i + mb_x \tag{68}$$

where m denotes the mass which flows into the chamber. It is necessary to introduce Φ_i, the initial value of Φ in the chamber, into (68) because, unlike U_i, it is not zero. Specifically,

$$\begin{aligned}\Phi_i &= U_i + p_0V_i - T_0S_i \\ &= p_0V_i \\ &= p_0mv_f\end{aligned} \tag{69}$$

From (67) one gets

$$u_f = h_x \tag{70}$$

or

$$T_f = kT_x \tag{71}$$

where

$$k = c_p/c_v$$

from (68) and (69)

$$(W_y)_{max}/m = (h_x - u_f) + T_0(s_f - s_x)$$

which, in view of (70), reduces to

$$(W_f)_{max}/m = T_0(s_f - s_x) = T_0(c_p \ln k - R \ln p_f/p_x) \quad (72)$$

This result could have been anticipated by noting that the maximum shaft work per unit of mass entering is also the irreversibility of this process and is therefore the product of T_0 and the increase in entropy of the fluid.

It may be noted from (72) that the maximum shaft work and the irreversibility per unit mass are infinite for the first elementary mass which enters the chamber ($p_f = 0$) and that just before flow ceases ($p_f = p_x$) these quantities are reduced to $c_p T_o \ln k$.

The steam power-plant cycle.—A steady-flow steam power-plant cycle may be used to illustrate several types of application of the availability, maximum work, and irreversibility concepts. The various processes involved may be classified according to whether they are (*a*) adiabatic—that is, employing no heat reservoirs external to the water passing through the cycle, (*b*) exchanging heat with the atmosphere only, and (*c*) exchanging heat with a hot reservoir.

Examples of (*a*) are expansion in a turbine, compression in a feed pump, and heating feed water with steam bled from the turbine. An example of (*b*) is the condensation process. Examples of (*c*) are the steam-generator processes: namely, heating of feed water, evaporation, superheating, and reheating.

The change in availability and the maximum useful work (W_u) of a unit mass of fluid may be evaluated for each process in the cycle. These are, however, of less engineering interest than the maximum shaft work $(W_f)_{max}$ for each process. For the cycle as a whole the summations of $(W_u)_{max}$ and $(W_f)_{max}$ are identical because their difference, the useful work done on adjacent fluid, sums up to zero. The irreversibility summed up for the cycle will be equal to the difference between the summation of the maximum shaft work (or maximum useful work) and the net work of the cycle.

In dealing with the (*c*) type of process—namely, that involving heat exchange with a hot reservoir—several choices are open to the analyst. He may, for example, choose to employ as a reservoir a stream of products of combustion such as might be found in an actual power plant. In that event (29), with the last term equal to zero, must be applied to the stream of water and to the stream of products of combustion as well in order to find the maximum shaft work. This method brings into consideration the characteristics and behaviour of the stream of products of combustion. A logical extension of it would be to include in the application of (29) the entire process of the air-fuel stream from its entrance to the power plant in the form of reactants to its exit in the form of cooled products of combustion. Any analysis which includes the behaviour of the stream of hot gases is, of course, concerned with a non-cyclic process.

It might be desired, however, to study a cyclic process as such in which heat exchange is with conventional "reservoirs." In that event, the temperature of the hot reservoir is arbitrary. For a given cycle the irreversibility will be greater the higher the temperature selected for the hot reservoir, and the lowest magnitude of the irreversibility will correspond to the lowest possible temperature of the reservoir. For example, in a steam cycle the lowest possible temperature for a single hot reservoir which supplies heat to the cycle without the aid of intermediate cyclic machinery would be the highest temperature attained by the steam in the course of heating.

If a reservoir temperature is selected higher than this minimum, the increase in the value of the irreversibility will be equal to the maximum useful work which could be obtained by virtue of heat transfer from the higher temperature and to the lower one—the magnitude of the heat being that taken from either reservoir by the cycle. Thus the increase in irreversibility is attributable to heat flow across the finite difference between the temperatures of the two reservoirs.

CONCLUSIONS

Quantitative concepts of maximum useful work, availability, irreversibility, and quality of performance of a thermodynamic task may be defined from consideration of the first and second laws of thermodynamics for all processes between equilibrium states of a system operating within an infinite stable atmosphere. These concepts may be extended to cover flow across a control surface and, as a more special case, to steady flow through a control surface. They may be applied to as wide a range of processes and as great a variety of systems as the science of thermodynamics itself.

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The boundary conditions at a mist-water interface

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The boundary conditions are formulated mathematically for two extreme cases, which may be described as the case of high turbulence and small droplets and the case of low turbulence and large droplets. The type of experiment needed to give a more precise significance to these descriptions is indicated.

INTRODUCTION

The calculation of diffusional processes in a turbulent medium is essentially a problem in differential equations and boundary conditions, and is one of considerable interest. In the case of an atmosphere free from mist the diffusional equations and boundary conditions may be regarded as known. We shall find, however, that when mist is present the problem is more complex. Though its presence does not essentially complicate the differential equations in the regions where the turbulent diffusion dominates the molecular diffusion, we shall find that it is not possible to specify mathematical boundary conditions, for the region of the water surface, likely to be valid under all physical conditions. The difficulty will arise from the great difference in the diffusional properties of water vapour and water droplets in a non-turbulent layer. We shall find that by making two different assumptions about the properties of the boundary layer we shall obtain two different mathematical boundary conditions, which, from their derivation, may be called the case of high turbulence and small droplets and the case of low turbulence and large droplets. They will only be realized exactly in extreme cases. All cases should lie between them, but it is impossible to predict theoretically over what range of physical conditions each will give a good approximation. The calculations will show that the application of these two boundary conditions to a given problem will often lead to widely different solutions; it is consequently important to investigate experimentally whether either or both of them occur under practical conditions (with a view to determining ultimately the range of approximate validity of each). A preliminary investigation of this kind will be described in a paper by D. J. Evans (see page 196).

As will be seen later the point of difficulty may be investigated experimentally by considering cases in which the pressure at a point is not changing progressively with time, and in which, owing to the interface employed being vertical, the gravitational settling of the mist (superposed on the diffusional processes) plays a negligible part. Thus in our discussion we shall omit effects due to these causes.

The quantities to which the equations of turbulent diffusion apply may be taken as

(1) the total mass q_2 of water (vapour and liquid) per unit mass of atmosphere; and

(2) a quantity, say Y, which for an atmosphere which is unsaturated, or supersaturated, or contains droplets, may be defined by

$$Y = \phi + c_p T - q_1 L(T) \qquad (1)$$

where $c_p = c_{pd} + q_2(c_{pw} - c_{pd})$

c_{pd} = Specific heat of dry air at constant pressure

c_{pw} = Specific heat of water vapour at constant pressure.

q_1 = Mass of liquid water/unit mass of atmosphere.

$L(T)$ = Latent heat of vaporization of water at temperature T.

T = Temperature which would be recorded by a small stationary thermometer or thermocouple.

ϕ = The gravitational potential gh where h is the height above a standard level.

q_1 and $L(T)$ are to be given values calculated from q_2 and T by using tables of latent heat and saturation vapour pressure.

It may be mentioned that in any part of the atmosphere the formation of droplets from a supersaturated state does not alter the value of Y.

Considering now a portion of a horizontal water surface, we take a parallel plane P in the atmosphere, near the surface but not actually in the boundary layer, so that at P turbulent diffusion dominates molecular. We consider also a parallel plane S (moving if necessary) which lies a little below the water surface and across which there is no mean flow of matter. We will select for the zero of our thermometer scale some temperature in the small region between P and S, and we will choose the water surface as the zero level of ϕ. By writing down the equations for the flow (in the region between P and S) of water and energy, we obtain two equations which we will write in an approximate form obtained by omitting factors of the type $1 - q_2$ and $1 - (Y/L)$ where q_2 and Y have values in the range between P and S. These equations are

$$\left.\begin{aligned} K\rho(\partial Y/\partial z) &= Q - L_0 r \\ K_1\rho(\partial q_2/\partial z) &= r \end{aligned}\right\} \qquad (2)$$

and thus, by division $$\frac{Q}{r} = L_0 + \left(K\frac{\partial Y}{\partial z}\Big/ K_1\frac{\partial q_2}{dz}\right) \qquad (3)$$

The quantities on the left-hand sides of (2) have their values in the plane P, and r is the rate at which the

water surface is gaining mass, per unit area, from the atmosphere. Q is the rate of flow of heat (away from the surface) through unit area of the plane S. L_0 is the latent heat at the temperature of the surface. The z direction is into the atmosphere and normal to the surface. ρ is the density of the atmosphere.

There is usually no serious error in assuming the turbulent diffusion coefficients K and K_1 (which vary greatly in space) to be identical throughout the whole turbulent region; and in that case the differential equations expressing, in that region, the rates of change of Y and q_2 at a point moving with the local mean wind velocity are of identical form, containing the same K. In the absence of mist near the surface, no serious error will usually result from assuming that this identity persists right up to the water surface (that is, throughout the region where turbulent diffusion does not dominate molecular).

THEORY OF CASE 1 AND CASE 2

To discuss the boundary problem when there is mist in the region of the surface we will consider two cases:—

(1) If the droplets are very small, and if we assume that droplets which in consequence of their Brownian and other motions come into contact with the surface do not leave it again, then (by an argument familiar in the boundary problems which arise in other branches of physics*), the concentration of droplets at the water surface may be regarded as negligible compared with their concentration elsewhere; that is, q_1 is negligible there. This state of affairs is assisted by distillation from the curved surfaces of the droplets on to the plane water surface. Thus by (1) we may say that at the water surface the value of Y/c_p is simply the temperature of the water surface, and q_2 has the corresponding saturation value. If, now, the wind is highly turbulent and hence the boundary layer very thin, the differential equations, with identical K's, may be regarded as holding right up to a surface at which the conditions are practically those, just stated, which prevail at the surface itself. When these mathematical conditions are satisfied we shall say that case 1 boundary conditions are holding.

(2) In non-turbulent air the diffusional properties of water droplets and water vapour differ greatly. Case 1, in which the physical conditions were such that this difference played a negligible part, may be regarded as the extreme case of a mist in the form of very small droplets contained in a highly turbulent wind. We now consider an opposite case, in which we have larger droplets, and in which, owing to a low turbulence of the wind, the boundary conditions approximate to those which would be imposed by the presence at the water surface of an appreciable layer of air in laminar flow. To investigate the laws of diffusion in such a layer, we now write K_1 for the diffusivity of water vapour; ρ and K for the atmospheric density and thermal diffusivity (that is, $k/c_{ps}\rho$ where k is the thermal conductivity and c_{ps} the specific heat at constant pressure), all measured in absence of liquid water. Then if we assume that the mass flow of droplets is, owing to their small diffusivity, negligible compared with that due to vapour, we have

$$K\rho c_{ps}\frac{\partial T}{\partial z} = Q - L_0 r; \quad K_1\rho\frac{\partial q_s}{\partial z} = r$$

and hence, on dividing $\frac{Q}{r} = L_0 + \frac{c_{ps}}{f\alpha}$ (4)

Here q_s is the mass of water vapour in unit mass of an atmosphere just saturated; f is K_1/K; and α, which is $(\partial q_s/\partial T)_p$, is, of course, well known as a function of temperature (and pressure).

The expression (4) may seem slightly unsatisfactory, since, even though we may think of T as only changing by about a degree in passing through the non-turbulent layer, the right-hand side of (4) can, owing to the rapid variation of α with temperature, depend appreciably on whether we insert the value of α at the one side or the other. A more detailed calculation, taking account of the convection of the droplets by the vapour, and the variation of q_1 in the layer, indicates that the value, say α_1, at the atmospheric side of the layer should be inserted.

When the relation (4) is approximately satisfied, we shall say that case 2 boundary conditions are holding.

GENERAL PRINCIPLES OF THE TYPE OF EXPERIMENT NEEDED

In illustration, and as an indication of the type of experiment needed to show the range of validity of the two cases, we consider warm air, laden with droplets, flowing down the inside of a vertical pipe (long compared with its diameter) cooled externally. Water will be deposited on the inside of the pipe and will run down it. We shall take the external cooling to be such that the temperature of the inner surface of this water is constant along the pipe (for though this is hardly a practicable case it affords a good illustration). As the zero of the thermometer scale we will choose this temperature.

If now the case 1 boundary conditions are holding, the values of q_2 and Y at the water surface are q_{s0} and zero, q_{s0} being the value of q_s at temperature zero. In these laboratory experiments ϕ may be neglected. Remembering the formal identity of the diffusion equations of Y and q_2, we see that at any point in the pipe Y and q_2 are (when a steady state has been attained) related by

$$\frac{Y}{q_2 - q_{s0}} = \frac{Y^\circ}{q_2^\circ - q_{s0}} \qquad (5)$$

where q_2° and Y° characterize the atmosphere entering at the top of the pipe. Thus the conditions at any point in the pipe are specified by some point lying on the

* As we approach the surface diffusion changes its character, ceasing to be the difference of two opposing flows; the flow becomes unidirectional, and if the concentration had not greatly diminished the net flow would have greatly increased; which it has not done.

straight line PQ in the figure. (In a given section of the pipe the representative point will range from Q, attained at the wall, to some point on PQ, attained at the centre.) P may lie to the left or right of the vertical axis, according as the initial air is more or less heavily laden with droplets. If at the lower end of the pipe the air emerging from the whole cross-section, or from any part of it, is brought to a state of uniformity by a suitable mixing process, its state is represented by a point on PQ. Instead of analysing the emergent air we might, as an equivalent test of the boundary conditions, measure the quantity of heat, say $\bar{Q}$, which has passed into the pipe per unit mass of water deposited on it. For case 1 boundary conditions we have

$$\bar{Q} = L_0 + [Y^\circ/(q_2^\circ - q_{s0})] \qquad (6)$$

as is seen from (3) and (5), noting that (3) though stated for a horizontal surface may be applied also to a vertical surface, and remembering that we are assuming the K's identical.

If on the other hand case 2 boundary conditions are holding, we have by (4)

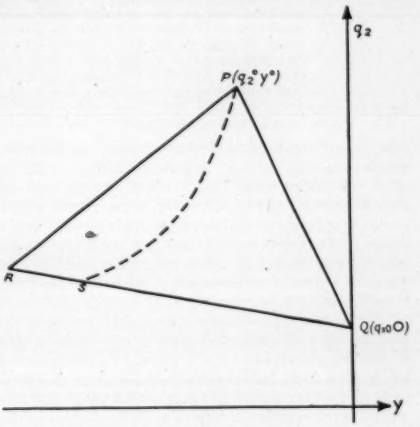
$$\bar{Q} = L_0 + (c_{ps}/f\alpha_1) \qquad (7)$$

and, as regards the mist at any point in the pipe, it is readily seen that (except perhaps in a region which lies in the boundary layer and contributes negligibly to the total atmospheric flow) its representative point lies on a straight line, PR in Fig. 1, given by

$$\frac{Y - Y^\circ}{q_2 - q_2^\circ} = \frac{c_{ps}}{f\alpha_1}$$

The end R of this line is its intersection with the isothermal QR drawn through Q (the slope $\partial Y/\partial q_2$ of this isothermal is approximately $-L_0$). The mist emerging from the lower end of the pipe, after suitable mixing, will be represented by a point on PR, the point R being attained at the end of a long pipe. (To be strictly accurate we should say that if the length of the pipe is increased sufficiently the representative point of the emergent mist will, after reaching the vicinity of R, move slowly to Q. Through this latter range the mist is at temperature zero and the droplets are slowly diffusing out of it; for however small their diffusivity in the non-turbulent layer may be they will eventually reach the wall.)

To assist in visualizing the matter let us suppose that the original mist was some 35° C warmer than the water surface. Through P we will draw the line PS of constant q_1. Its slope $\partial Y/\partial q_2$ is approximately c_p/α_T, which, owing to the (roughly exponential) increase of α_T with T, increases as the line passes down from P through diminishing values of T. At P, that is at the temperature of the original mist, α_T is, say, ten times α_1. On the other hand, the quantities f and c_{ps}/c_p are of order unity. Thus in the region of P the slope of PS is much less than that of PR, and, in fact, S will be well to the right of R. Thus throughout the whole length of



(q_2Y) diagram illustrating the theory of the experiments

PR the air has a greater liquid water content than the original mist; whereas on the case 1 line PQ the air on reaching Q is free from mist.

An alternative type of experiment would consist of measuring $\bar{Q}$ for a vertical pipe, cooled internally, situated in a descending mist-laden wind whose representative point in regions remote from the pipe is (q_2°, Y°). Regarding the outer surface-temperature of the water condensed on the pipe as a constant, taken as the zero of the temperature scale, it is seen that in this experiment the two expressions given above for $\bar{Q}$ still hold in case 1 and case 2 respectively.

Experiments on the boundary conditions at a mist-water interface

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Experiments with mist-laden air passing down vertical pipes have been performed with the intention of testing the theory, proposed by Davidson[1], in which the boundary conditions at a mist-water interface were formulated for two extreme cases. In the first case, which he calls Case 1, it was assumed in the theory that the equations of turbulent diffusion might be regarded as holding right up to the water surface, and that the concentration of liquid water in the atmosphere there would be negligible; this case would be expected to arise under conditions of high turbulence and small droplet size. In the opposite extreme case, which he calls Case 2, the conditions were regarded as approximating to what would occur if the droplets were larger and there were a completely non-turbulent layer of appreciable thickness at the interface. It is important to investigate whether the formulae derived for these two extreme cases, between which all cases should lie, do in fact arise under practical conditions. The experiments here described were carried out over a wide range of physical conditions (of temperature, water content, and wind velocity), and it was found that the Case 1 formulae held over a large range; with diminishing turbulence the results tended towards, and finally attained, the Case 2 formulae.

1. Introduction

Davidson[1] has made a theoretical study of the differential equations and boundary conditions in the passage of a mist-laden wind over a water surface, and the theory, though mainly of interest in large-scale phenomena, may be tested by laboratory experiments. It was found in the theory that the boundary conditions do not admit of a mathematical formulation which may be expected to hold over all ranges of wind turbulence and droplet size, but that their formulation is comparatively simple in two extreme cases, which may be called the case of high turbulence and small droplets and the case of low turbulence and large droplets. All cases should lie between them, but it is impossible to predict theoretically over what range of physical conditions each will be a good approximation. Since the application of the two boundary conditions to a given problem will often lead to widely different solutions, it is important to investigate experimentally whether either or both of them occur under practical conditions (with a view to determining ultimately the range of approximate validity of each). Two experiments, suitable in principle for the purpose, were described at the end of the paper, and these are the experiments which have been carried out in the present investigation. The simple apparatus employed, though not capable of high accuracy, is adequate for our purpose.

2. Experiments with Mist Passing Down the Outside of a Vertical Pipe Cooled Internally

As explained at the end of Davidson's paper, the procedure in such an experiment is to measure the quantity

$$\bar{Q} = \frac{\text{quantity of heat passing into the pipe in a given time}}{\text{mass of water deposited on the pipe in that time}}$$

and to compare the measured value with the two values predicted for $\bar{Q}$ by the case 1 and case 2 boundary conditions. These two values are given by the expressions (6) and (7) in his paper.

2.1. Apparatus

Fig. 1 is a schematic diagram of the apparatus used. *B* was a long glass tube with very thin walls, down the outside of which the mist-laden air flowed. Inside the tube flowed a rapid stream of water whose temperature was measured both on entering and leaving the mist by means of the thermocouples *a* and *b*. CC_1 was a very wide outer jacket enclosing the mist, and was lagged on the outside so that as little heat as possible escaped to

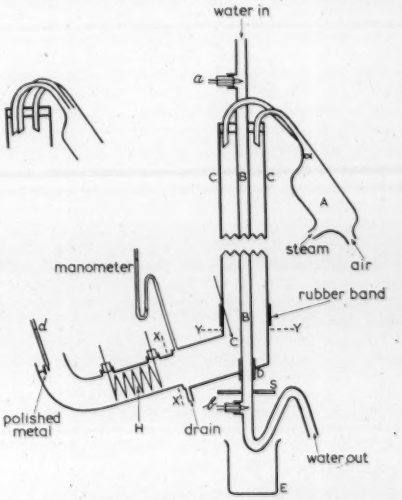

Fig. 1. Schematic cross-section of the apparatus used in the passage of mist-laden air outside a vertical pipe.

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the surroundings. To make the mist, air from a blower and steam from a boiler were mixed in the large glass vessel, shown in the inset diagram, Fig. 1. The mist entered the top of CC_1 by three wide tubes (of which only two are shown in the main diagram) so as to distribute it over the cross-section of CC_1. To vary the velocity of the wind in CC_1, the air from the blower, before entering the mixing chamber, passed through a vessel with side taps through which any desired amount of air could be allowed to escape into the atmosphere. The mist, after flowing down over the tube B, passed over a heating coil H, disposed so as to produce both heating and stirring, and then to a thermocouple d with a piece of polished metal beside it. The portion of the apparatus between the lines X and Y was made of copper. D was a short copper tube through which passed the glass tube B. The portion XY was joined to CC_1 by a rubber band through which was inserted a thermocouple c. Any water deposited on the walls of the wide jacket CC_1, or falling as drops from the entry pipes at the top of it, was drained off through a small pipe Z. The mist, flowing down in the jacket, transferred heat and water to the tube B. The water thus deposited on the cold surface of B acted as the water surface of the theory. (In this series of experiments its temperature was always in the region of atmospheric temperature.) The stream of water inside B was sufficiently rapid to make the difference of temperature of the unflowing and outflowing water of the order of 1° C. Such a value satisfies two conditions (required for the application of the theory) that it shall be small, but measurable with fair accuracy.

2.2. EXPERIMENTAL PROCEDURE

The water deposited on the cold surface of the tube B flowed down that surface, passed through the annular space between B and D, and the quantity collected in a measured time was weighed. To prevent any mist which passed through D from condensing on the cold surface of the beaker and falsifying the weight, a screen S was placed across the base of D as shown, leaving sufficient clearance for the water to flow down the outside of B undisturbed. The rate of flow of water through the inside of B was measured similarly, and its rise of temperature between the thermocouples a and b was noted. The temperature of the main body of mist passing down the wide jacket CC_1 was measured by the thermocouple c, located in the main stream, well away from the surfaces. The current in the heating coil H was varied until the air, which is misty at X, became just saturated at d. This saturation point was observed by means of a piece of polished metal placed near d. The location of thermocouple (and adjacent plate) was varied in the tube, and the dewpoint was found to be practically independent of this location. From this temperature and the measured pressure we could then calculate the total mass of water (vapour + droplets) contained in unit mass of mist in the main stream—i.e. the q_2^0 of the theory. The temperature of thermocouple c then enabled us to calculate the value of the quantity Y^0 of the theory. We had then sufficient data to examine whether either of the expressions (6), (7) was holding in these experiments.

2.3. RESULTS OF THESE EXPERIMENTS

The experiments were carried out under a wide range of conditions. From the measurements in any one experiment we calculate the values of the three quantities:—

The observed $\bar{Q}$;

$$L_0 + \frac{Y^0}{q_2^0 - q_{s0}}$$

which we will call B, and which is the theoretical value of $\bar{Q}$ if case 1 boundary conditions are assumed;

$$L_0 + \frac{c_{p2}}{f\alpha_1}$$

which we will call C, and which is the theoretical value of $\bar{Q}$ if case 2 boundary conditions are assumed.

The results can conveniently be exhibited by tabulating (see table) the two ratios $B/\bar{Q}$ and $C/\bar{Q}$. If case 1 is holding, then $B/\bar{Q}$ is unity: on the other hand, if case 2 is holding, then $C/\bar{Q}$ is unity. In making the numerical calculations of B and C, estimates have to be made of the quantities q_{s0}, Y^0 and α_1. They involve the temperature of the water surface, and this has to be estimated from the temperature of the water in the pipe, which is somewhat lower. Fortunately the values of $B/\bar{Q}$ and $C/\bar{Q}$ are not much affected by reasonable changes in this estimate.

From the table the following conclusions may be drawn. At the highest wind velocities and lowest liquid water content (and probably smallest droplet size) the ratio $B/\bar{Q}$ is not very different from unity ($\approx 0{\cdot}9$) whereas the ratio $C/\bar{Q}$ is far from unity ($\approx 1{\cdot}7$). Consequently under these conditions case 1 was being approached. Again, at the other end of the table, with the opposite conditions (low wind velocity, high liquid water content and probably largest droplet size) it is seen that $B/\bar{Q}$ is very much less than unity, whereas $C/\bar{Q}$ is approaching unity: that is, under these conditions case 2 is being approached. These results are in full accordance with the theory; and furthermore it will be noted that throughout the table $B/\bar{Q}$ is less than unity and $C/\bar{Q}$ is greater than unity. It may therefore be concluded that the conditions in the boundary layer throughout the range here investigated were always intermediate between the cases 1 and 2 of the theory, and that both extremes were closely approached.

It has been pointed out to the author that a probable type of intermediate case is one in which the conditions of case 1 hold except that q_1 is not negligible at the surface; and that the way in which this leads to an intermediate value of $\bar{Q}$ may be illustrated by regarding q_1 as

having a constant value, say $\bar{q}_1$, along the pipe. The term $Y^0/(q_2^0 - q_{s0})$ in B would then have to be replaced by $(Y^0 + \bar{q}_1 L_0)/(q_2^0 - q_{s0} - \bar{q}_1)$, and thus $\bar{Q}$ would be increased to an intermediate value.

Results of tabulating the two ratios $B/\bar{Q}$ and $C/\bar{Q}$

	$B/\bar{Q}$	$C/\bar{Q}$	
Highest wind velocity	0·89	1·7	
	0·83	1·6	
	0·82	1·5	
	0·68	1·4	
	0·63	1·3	Increasing droplet content ↓
	0·60	1·3	
Medium wind velocity	0·73	1·1	
	0·68	1·3	
		1·2	
Lowest wind velocity	0·31	1·1	
	0·22	1·2	

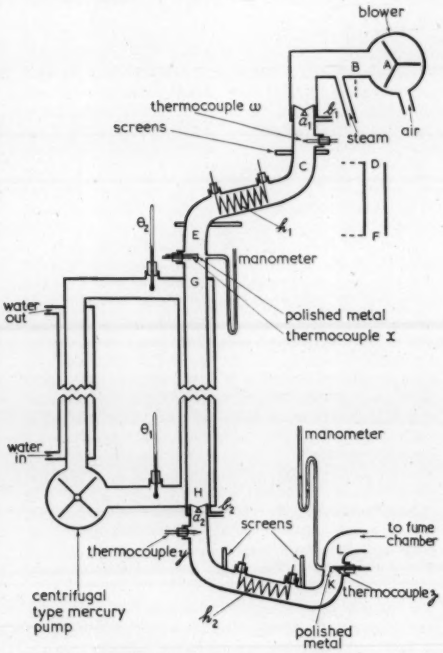

Fig. 2. Cross-section of the apparatus used in the passage of mist-laden air inside a vertical pipe

3. EXPERIMENTS WITH MIST-LADEN AIR FLOWING DOWN INSIDE A VERTICAL PIPE COOLED EXTERNALLY

The theory of such an experiment is given at the end of Davidson's paper and is illustrated by his Fig. 1. It will be seen that we have to measure the temperature of the water surface (to determine the point Q in the figure) and the temperature and composition of the entering mist (to determine the point P). We have also to measure the temperature and composition of the emergent mist and to insert its representative point in the figure. If this point lies on the straight line PQ, case 1 is holding: if on PR, case 2 is holding.

3.1. APPARATUS

The apparatus is illustrated in Fig. 2. The mist, made in the same way as in the preliminary experiments, flowed down the inside of a thing iron pipe the outside of which was cooled by a rapidly flowing stream of mercury. The mercury was pumped through a closed system, part of which was cooled by a water jacket. The temperature of the mercury at the two ends of the iron pipe differed only slightly. The course of the mist through the apparatus is given by *BCEGHKL*.

AB was a *T* piece made of iron tubing;
a_1 and a_2 were small metal cones joined to the tubes by three short lengths of wire;
b_1 and b_2 were narrow outlet tubes for condensed water;
EG and *KL* were two short lengths of glass tubing each having two side arms, one leading to a manometer and the other for the insertion of a thermocouple and a small piece of polished metal;
GH was the main length of thin iron pipe, where the flow of heat and water under investigation took place;
h_1 and h_2 were heating coils contained in straight lengths of glass tubing;
w, *x*, *y*, *z* were thermocouples screened from the heating coils as shown.
The tube *GH* was surrounded by the rapid stream of mercury, the (slightly different) temperatures of which at the bottom and top of the apparatus were measured by thermometers θ_1 and θ_2.

3.2. PROCEDURE

The mist-laden air was brought to a state of uniformity by mixing at *a* and its temperature measured by thermocouple *w*. It then passed over h_1 in which the current was adjusted so that the misty air became just saturated, and its temperature was measured by thermocouple *x*. From these observations the values of q_2^0 and Y^0 were obtained by the use of standard tables. Having thus analysed the mist supplied at the top, it remained to allow the flow of heat and water to the sides of *GH* to take place, and to analyse the emergent mist. To do this, the apparatus was disconnected along the dotted

lines *B*, *CD* and *EF*. *BC* was moved horizontally to the left until *C* was vertically above *E*. *CE* was then joined by means of a straight glass tube, and the gap formed at *B* was closed by an additional piece of pipe. The total resistance which the apparatus opposed to the flow of air was only slightly altered by these changes. The water condensed on the walls of *GH* acted as the water surface of the theory. The rapid stream of mercury and the thinness of the metal pipe served the purpose of making the temperature of the water surface as near as possible to the measured temperature of the mercury.

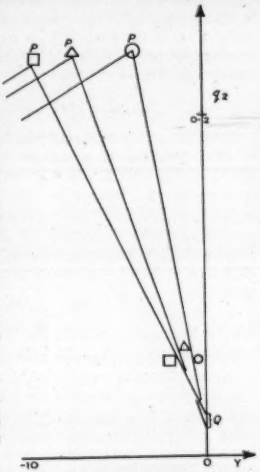

Fig. 3. Typical experimental (q_2Y) diagrams

The emergent mist from *H* was analysed in the same way as the incident mist, so as to determine its representative point (q_2, Y). The point Q in the figure was determined by calculating the saturation value q_{s0} at the temperature of the water surface, taken as the average of the (slightly different) temperatures θ_1 and θ_2.

3.3. RESULTS

The temperature of the mercury was varied and mists of various temperatures, water content and velocities were employed. The velocity of the mist in the pipe was always greater than the velocities in the second and third section of the table, § 2.3. It was found that the representative point of the emergent mist always lay much closer to the case 1 line *PQ* than to the case 2 line *PR*. In fact the departure from the *PQ* line was never greater than could be attributed to experimental inaccuracy. Fig. 3 shows some typical examples.

4. CONCLUSIONS

From the two series of experiments it may be concluded that the case 1 formulae hold over a wide range of physical conditions, outside which the results tend towards (and attain) the case 2 formulae.

ACKNOWLEDGMENTS

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Solution of certain unsteady heat flow problems by relaxation methods

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The use of relaxation methods to solve unsteady heat flow problems in which only one space dimension is involved is demonstrated by application to problems of heat transfer through the walls of highly insulated containers used for transporting frozen produce which requires low storage temperatures. In the problems considered various boundary conditions are encountered and these are readily dealt with by the relaxation technique.

Although the comparatively recent use of the Laplace Transform in solving heat conduction problems[1] has simplified the analytical solution of many such problems, the mathematics and computations involved in applying the analytical solution to a particular problem is usually so heavy, except in the simplest of cases, as to make this method prohibitive. As a result a certain amount of attention has been paid lately to numerical methods for solving problems of heat transfer. Southwell[2] and Emmons[3, 4] have shown how relaxation methods may be applied to steady state problems, and the latter has shown how the method may be extended to solve transient heat flow problems. Dusinberre[5] has also developed a numerical method for solving transient heat flow problems which may be extended to take account of convection and radiation boundary conditions. A special case of his method is the famous Schmidt formula. The purpose of this paper is to show how Southwell's relaxation methods may be applied to certain types of unsteady heat flow problems and the method, although developed without reference to Emmons's work, shows considerable similarity to the one he adopts.

APPLICATION OF THE METHOD TO PRACTICAL PROBLEMS

The problems solved below have occurred in work on refrigerated transport in road/rail insulated containers, but it is evident from the examples themselves that they have a far wider application. The road/rail containers for the transport of perishable produce are essentially an insulated rectangular box. In the years before the war the commonest type was comparatively lightly insulated with sides of approximately 2 in thickness. In making calculations on the heat which the refrigerant

and the load must absorb during the journey, it has been customary to use the overall heat transfer coefficient determined experimentally under steady state conditions of heat flow (and expressed in B.Th.U./h/°F temperature difference between interior and exterior). For practical purposes this has worked reasonably satisfactorily since, even when the container structure starts in a warm condition, the steady state temperature gradient through the walls is established within a comparatively short time, and furthermore the amount of heat abstracted from the structure is not large compared with the ordinary steady state heat flow. However, a striking feature in post-war years has been an increasing demand for the more highly insulated type of container for such things as ice-cream and quick-frozen produce, which call for lower storage temperatures than was customary in the past. A 9 in thickness of insulation is used on these vehicles, but even with this thickness the simple calculation just described no longer gives a reasonable approximation to the truth. In fact, for a short journey the steady state temperature gradient in the walls may not be reached. It was with this in mind that relaxation methods were applied to give a more accurate means of calculating the heat loads to be overcome. It has been found that the methods of relaxation do permit practical solutions provided that the problem may be simplified to one involving one linear dimension, leaving the other dimension of the relaxation network for time. Accordingly, the examples are worked out either for a uniform plain slab or a spherical shell.

Of the three examples described below, the first assumes a plain slab initially at a uniform temperature, with one face altered to, and maintained at, a different temperature and the other face maintained at the original temperature. This corresponds to the practical case in which the warm container is loaded with a load of high thermal conductivity so that the interior walls of the container are suddenly cooled to load temperature. The solution gives a heat flow coefficient for the container, varying with time but including all sources of heat. It represents the extreme case of maximum rate of heat flow from the walls for given temperature drop. The results for different thicknesses can be compared, showing for example that for shorter journeys a thickness is reached beyond which no further reduction in total heat loss is obtained. Observations of this sort are useful in assessing the desirable or economic thickness of insulation required for specified conditions.

In the second example a common practical case is considered in which cooling takes place through the medium of a tank containing frozen eutectic solution and of fixed area and surface temperature, and in this case the problem is complicated by the introduction of assumed heat transfer coefficients at the surfaces. By these means the heat which must be absorbed by the plate during the freezing process can be more accurately calculated than was possible before.

The third example endeavours to show how one may tackle the effect on a load of given thermal properties placed in the container.

The container for which these calculations were made is approximately cubical with a capacity of 193 ft^3. Its walls are 9 in thick made of a material whose thermal conductivity is 0·02 B.Th.U./h/ft^2(°F/ft); density 4·5 lb/ft^3 and specific heat 0·5.

METHOD OF CALCULATION

In all the problems considered here the container wall is either regarded as a slab of infinite length or as a spherical shell and all effects of corners, doors, etc., have been neglected. In cases in which the wall is regarded as a slab of infinite length we are faced with the mathematical problem of finding the temperature θ at any distance x from the inner wall face at any time t in terms of x and t, where θ must satisfy the heat conduction equation—

$$\frac{\partial^2\theta}{\partial x^2} = \frac{c\rho}{K}\frac{\partial\theta}{\partial t}$$

in a two-dimensional region $0 < x < l, 0 < t < \infty$ and any given boundary equations for $x = 0$, $x = l$ and $t = 0$.

If we divide the region into a number of rectangles with sides of length a and b parallel to the axes of x and t, x being plotted horizontally and t vertically, and if the values of the temperature at the points shown in Fig. 1 are θ_0, θ_1, θ_2, θ_3, θ_4, then for the typical point 0 the calculus of finite differences tells us that

$$2b\left(\frac{\partial\theta}{\partial t}\right) \doteqdot \theta_1 - \theta_3$$

$$a^2\left(\frac{\partial^2\theta}{\partial x^2}\right) \doteqdot (\theta_2 + \theta_4 - 2\theta_0)$$

and from the heat conduction equation we have

$$2bK(\theta_2 + \theta_4 - 2\theta_0) = c\rho a^2(\theta_1 - \theta_3)$$

as the equation connecting the value of the temperature at the point 0 with that at the four surrounding points. Thus for each point of the network there is an equation

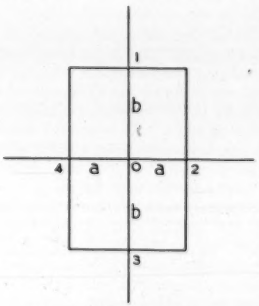

Fig. 1. Relaxation network

of this form although there will be different finite difference equations for boundary points. Our problem is solved when we have found values of θ for each point of the network satisfying these finite difference equations.

If we put $x = a\xi$, $t = c\rho a^2\zeta/K$ where $b = c\rho a^2/K$ then the heat conduction equation becomes

$$\frac{\partial^2\theta}{\partial\xi^2} = \frac{\partial\theta}{\partial\zeta}$$

and its finite difference form $(\theta_2 + \theta_4 - 2\theta_0) = \frac{1}{2}(\theta_1 - \theta_3)$,

i.e. $$-\theta_1 + 2\theta_2 + \theta_3 + 2\theta_4 - 4\theta_0 = 0$$

Putting $R_0 = -\theta_1 + 2\theta_2 + \theta_3 + 2\theta_4 - 4\theta_0$ we call R_0 the residual at the point 0 and after putting in any values of θ we choose at each point of the network and calculating the corresponding residuals the relaxation process consists in a systematic reduction of these residuals. When they are zero or as near zero as possible the process is complete. Although we are free to put in any initial values of θ we wish, knowing that we will obtain a close approximate solution in the end the nearer our initial values of θ are to the actual values the less will be the computation involved. It may be shown that if we alter the value of θ_0 by $+1$, R_0 will be altered by -4 and the residuals at the surrounding points, 1, 2, 3, 4, by $+1$, $+2$, -1 and $+2$ respectively. This gives us what is known as the relaxation pattern and it is by repeated and judicious application of this pattern at all points of the network that the residuals are reduced and the final solution obtained. Since one of the variables used is time, unlike most relaxation problems, not all the boundaries are closed, but later on in this paper it is shown that this need not present much difficulty.

1. *Heat flow through a slab* $0 < x < l$.—This problem, originally worked out by Laplace Transforms, was used to estimate the relative efficiency of various thicknesses for the container wall. The face $x = l$ is permanently at the temperature V_0, the face $x = 0$ is initially at temperature V_0, but for time $t > 0$ the temperature is zero. If K is the conductivity of the slab, c the specific heat, ρ the density of the material, the equation of linear flow is

$$\frac{\partial^2\theta}{\partial x^2} - \frac{c\rho}{K}\frac{\partial\theta}{\partial t} = 0$$

Now if we put $x = a\xi$, $t = c\rho a^2\zeta/K$ where a has the dimension of length, ξ and ζ are dimensionless variables and the equation becomes

$$\frac{\partial^2\theta}{\partial\xi^2} = \frac{\partial\theta}{\partial\zeta}$$

In order to solve this problem by relaxation we take distance as the horizontal axis and time as vertical axis and divide the region $0 < x < l$, $0 < t < \infty$ into a network of squares of side a. Unlike most relaxation problems we have only three closed boundaries, but as in these problems we know there must be a final steady state which within the desired limits of accuracy will occur within a finite time, all we need do is to draw in as many squares as we like of side a on the t axis and continue relaxing until for some value of t the residuals are so small that there is no point in relaxing any further.

The equation

$$\frac{\partial^2\theta}{\partial\xi^2} = \frac{\partial\theta}{\partial\zeta}$$

in finite difference form is

$$(\theta_2 + \theta_4 - 2\theta_0) = \tfrac{1}{2}(\theta_1 - \theta_3)$$

i.e. $$-\theta_1 + 2\theta_2 + \theta_3 + 2\theta_4 - 4\theta_0 = 0$$

The heat flow across the face $x = 0$ after time t is given by

$$H = K\int_0^t \left(\frac{\partial\theta}{\partial x}\right)_{x=0} dt$$

This is obtained by approximate integration and the values obtained for 3 in, 6 in, and 9 in slabs in comparison with those obtained from the Laplace Transform solution are tabulated in the table. It will be noticed that the difference between the two solutions increases with the dimensions of the slab. The reason for this is that the value of a was so chosen that the range $0 < x < l$ was divided into six so that the net used was three times finer for the 3 in slab than it was for the 9 in slab and as in all relaxation problems the finer the net the more accurate is the result.

2. *A slab* $0 < x < 1$ *one face of which is in contact with a well-stirred fluid.*—In the foregoing problem the slab representing the container wall was cooled by assuming the inner face to be maintained at a constant

Heat flow through container wall (Problem 1). Total quantity of heat in B.Th.U./ft²/°F temperature difference which has crossed inner wall after a given time.

thickness	*3 in*		*6 in*		*9 in*	
hours from start	*relaxation solution*	*Laplace transform solution*	*relaxation solution*	*Laplace transform solution*	*relaxation solution*	*Laplace transform solution*
1	0·20	0·24	0·20	0·24	0·13	0·24
2	0·33	0·34	0·30	0·34	0·25	0·34
4	0·51	0·51	0·43	0·48	0·40	0·48
8	0·83	0·83	0·63	0·68	0·60	0·68
12	1·15	1·15	0·80	0·85	0·74	0·83
24	2·11	2·11	1·28	1·34	1·10	1·20
36	3·07	3·07	1·78	1·83	1·45	1·55
48	4·03	4·03	2·27	2·31	1·80	1·87

temperature. We now consider the case in which, in actual fact, the cooling is performed by means of a eutectic plate, a flat box containing a frozen eutectic solution, separated from the wall slab by an air space. The plate is regarded as a surface of given area maintained at a constant temperature and the air is assumed to be "well stirred" so that its temperature is the same at all points in the space at any given time, two surface heat transfer coefficients, namely between the plate surface and air, and between slab surface and air, are also introduced.

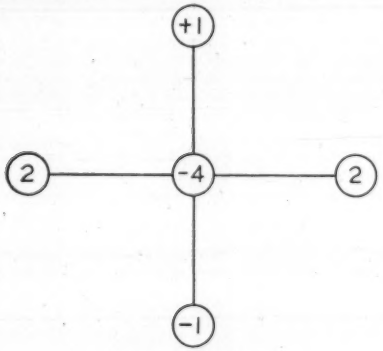

Fig. 2. Relaxation pattern

Thus let A be the area of the plate.
A' be the surface area of the inner wall of the container.
H the heat transfer coefficient between plate and air.
H' the heat transfer coefficient between wall and air.
K the conductivity of the container wall.
c the specific heat of the container wall.
$M'c'$ the thermal capacity of the air.
V_0 the temperature of the air outside the container and initial temperature of the wall and air inside the container.
T_0 the temperature of the eutectic solution.
u, θ the temperature of air and wall respectively.

At $x = 0$ we have the following boundary conditions

$$M'c'\frac{du}{dt} + HA(u - T_0) - A'K\frac{\partial\theta}{\partial x} = 0 \qquad (1)$$

$t > 0$

$$A'K\frac{\partial\theta}{\partial x} - H'A'(\theta - u) = 0 \qquad (2)$$

At $x = l$, $\theta = V_0$ $\qquad t > 0$

In the wall θ must satisfy the equation

$$\frac{\partial^2\theta}{\partial x^2} = \frac{c\rho}{K}\frac{\partial\theta}{\partial t} \qquad (3)$$

Now from (2) $\qquad u = \frac{-K}{H'}\frac{\partial\theta}{\partial x} + \theta$

Therefore substituting in (1) we have

$$M'c'\left[-\frac{K}{H'}\frac{\partial^2\theta}{\partial x\partial t} + \frac{\partial\theta}{\partial t}\right] + HA\left[-\frac{K}{H'}\frac{\partial\theta}{\partial x} + \theta - T_0\right] - A'K\frac{\partial\theta}{\partial x} = 0 \qquad (4)$$

Putting $x = a\xi$; $t = c\rho a^2\zeta/K$ as before this becomes

$$-\frac{M'c'K^2}{H'c\rho a^3}\frac{\partial^2\theta}{\partial\xi\partial\zeta} + \frac{M'c'K}{c\rho a^2}\frac{\partial\theta}{\partial\zeta} - \frac{K}{a}\left(\frac{HA}{H'} + A'\right)\frac{\partial\theta}{\partial\xi} + HA\theta - HAT_0 = 0$$

and (3) is $\qquad \frac{\partial^2\theta}{\partial\xi^2} = \frac{\partial\theta}{\partial\zeta}$

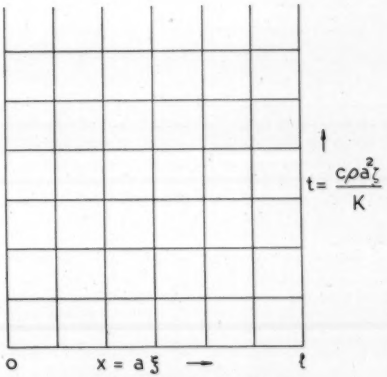

Fig. 3. Network for problem 1

This is of the form

$$-B\frac{\partial^2\theta}{\partial\xi\partial\zeta} + C\frac{\partial\theta}{\partial\zeta} - D\frac{\partial\theta}{\partial\xi} + HA(\theta - T_0) = 0$$

where B, C, D are constants.

In finite difference form this is

$$-\tfrac{1}{4}B[\theta_{II} + \theta_{IV} - \theta_{III} - \theta_1] + \tfrac{1}{2}C[\theta_1 - \theta_3] - \tfrac{1}{2}D[\theta_2 - \theta_4] + HA[\theta_0 - T_0] = 0$$

where the suffixes relate to the position of points as shown in Fig. 4. θ_1, θ_4, θ_{IV} which lie outside the boundary $x = 0$ are called fictitious points and it is our aim to eliminate these points between the finite difference forms of the boundary equations and the heat conduction

equation (the θ governing equation) in order to derive a boundary equation involving actual values of θ only (i.e. values of θ either on the boundary $x = 0$ or within the wall). From the θ governing equation we have

$$\theta_4 = \tfrac{1}{2}\theta_1 - \theta_2 - \tfrac{1}{2}\theta_3 + 2\theta_0$$
$$\theta_1 = \tfrac{1}{2}\theta_5 - \theta_{II} - \tfrac{1}{2}\theta_0 + 2\theta_1$$
$$\theta_{IV} = \tfrac{1}{2}\theta_0 - \theta_{III} - \tfrac{1}{2}\theta_7 + 2\theta_3$$

Therefore substituting in the above equation we have for the boundary equation at $x = 0$

$$-\tfrac{1}{4}\mathrm{B}[2\theta_{II} + \tfrac{1}{2}\theta_0 - 2\theta_{III} - \tfrac{1}{2}\theta_7 + 2\theta_3 - \tfrac{1}{2}\theta_5 + \tfrac{1}{2}\theta_0 - 2\theta_1] + \tfrac{1}{2}\mathrm{C}[\theta_1 - \theta_3] - \tfrac{1}{2}\mathrm{D}[2\theta_2 - \tfrac{1}{2}\theta_1 + \tfrac{1}{2}\theta_3 - 2\theta_0] + HA[\theta_0 - T_0] = 0$$

Therefore

$$[4\mathrm{B} + 4\mathrm{C} + 2\mathrm{D}]\theta_1 - 8\mathrm{D}\theta_2 - [4\mathrm{B} + 4\mathrm{C} + 2\mathrm{D}]\theta_3 + [8\mathrm{D} + 8HA - 2\mathrm{B}]\theta_0 + \mathrm{B}\theta_5 + \mathrm{B}\theta_7 - 4\mathrm{B}\theta_{II} + 4B\theta_{III} - 8HAT_0 = 0$$

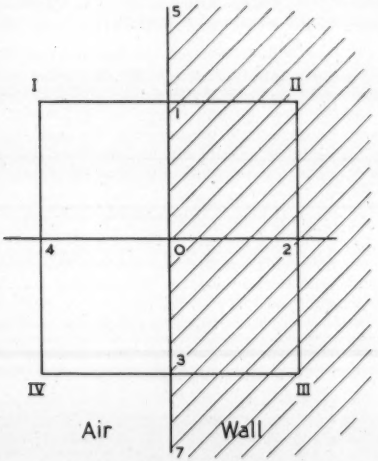

Fig. 4. Boundary points for problem 2

In the problem considered the width of the container wall was 9 in. H and H' were unity. A and A' were 18 and 181 ft^2 respectively, $M'c'$, the thermal capacity of the air, was 3·85 B.Th.U./°F, K was 0·02 B.Th.U./h (ft^2) (°F/ft); $\rho = 4{\cdot}5$ lb/ft^3; $c = 0{\cdot}5$ and $T_0 = -6°$ F. This gave for the boundary equation $x = 0$,

$$22{\cdot}728\theta_0 + 4{\cdot}216\theta_1 - 14{\cdot}52\theta_2 - 4{\cdot}216\theta_3 + 0{\cdot}02\theta_5 + 0{\cdot}02\theta_7 - 0{\cdot}08\theta_{II} + 0{\cdot}08\theta_{III} + 49{\cdot}338 = 0$$

and for relaxing this was taken as

$$23\theta_0 + 4\theta_1 - 15\theta_2 - 4\theta_3 + 49 = 0$$

The heat flow across the inner surface of the container wall is calculated as in problem 1 and the air temperature is calculated from equation (2).

3. *Heat flow across the walls of a spherical container* $a_2 < r < a_3$ *enclosing a spherical load* $0 < r < a_1$, *the intervening space* $a_1 < r < a_2$ *being filled with a well stirred fluid.*—So that the results of this problem should be related as closely as possible to the case of a load inserted in an insulated container, the inner surface area of the spherical shell and the surface of the load were taken to be the same as those of the cubical container and load. With $a_1 = 3$ ft, $a_2 = 3$ ft 6 in, the problem was worked out for a load at temperature 0°F inserted into the container with the outside air temperature remaining constant at 60° F. The temperatures of the outside and centre of the load after 48 h were 8° F and 2° F respectively. Experiments carried out on the transport of ice-cream, in which temperatures were measured by thermocouples inserted at various positions of the load, showed that after 48 h the temperatures of the outside and centre of the load had increased by 8° F and 4° F respectively above the initial temperature of 7° F, showing reasonable agreement between the experimental and the calculated results.

Let θ_1, θ_2, θ_3 be the temperatures of the load, air and container wall at the time t. If K_1 is the conductivity of the load and K_3 is the conductivity of the container walls with $A_1 = 4\pi a_1^2$ the surface area of the load, $A_2 = 4\pi a_2^2$ the inner surface area of the container and other constants as before, we have for boundary equations at $r = a_1$

$$M'c'\frac{d\theta_2}{dt} + H_2A_2(\theta_2 - \theta_3) + A_1K_1\frac{\partial\theta_1}{\partial r} = 0 \qquad (5)$$

and

$$K_1\frac{\partial\theta_1}{\partial r} = H_1(\theta_2 - \theta_1) \qquad (6)$$

Whilst at $r = a_2$ boundary equations are

$$M'c'\frac{d\theta_2}{dt} + A_1H_1(\theta_2 - \theta_1) - A_2K_3\frac{\partial\theta_3}{\partial r} = 0 \qquad (7)$$

$$K_3\frac{\partial\theta_3}{\partial r} = H_2(\theta_3 - \theta_2) \qquad (8)$$

Within the container walls θ_3 must satisfy the equation

$$\frac{c\rho}{K_3}\frac{\partial\theta_3}{\partial t} = \frac{\partial^2\theta_3}{\partial r^2} + \frac{2}{r}\frac{\partial\theta_3}{\partial r} \qquad (9)$$

and within the load θ_1 must satisfy the equation

$$\frac{c_1\rho_1}{K_1}\frac{\partial\theta_1}{\partial t} = \frac{\partial^2\theta_1}{\partial r^2} + \frac{2}{r}\frac{\partial\theta_1}{\partial r} \qquad (10)$$

Put $\theta_1 r = \phi$; $\theta_3 r = \psi$ and equations (5) and (6) become

$$\frac{c\rho}{K_3}\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial r^2} \quad \text{and} \quad \frac{c_1\rho_1}{K_1}\frac{\partial\phi}{\partial t} = \frac{\partial^2\phi}{\partial r^2} \qquad (11)$$

whilst equations (5), (6), (7), (8) become

$$M'c'\frac{d\theta_2}{dt} + A_2H_2\left(\theta_2 - \frac{\psi}{a_2}\right) + A_1K_1\left(-\frac{\phi}{a_1^2} + \frac{1}{a_1}\frac{\partial\phi}{\partial r}\right) = 0 \quad r = a_1 \quad (12)$$

$$K_1\left(-\frac{\phi}{a_1^2} + \frac{1}{a_1}\frac{\partial\phi}{\partial r}\right) = H\left(\theta_2 - \frac{\phi}{a_1}\right) \quad r = a_1 \quad (13)$$

and

$$M'c'\frac{d\theta_2}{dt} + A_1H_1\left(\theta_2 - \frac{\phi}{a_1}\right) - A_2K_2\left(-\frac{\psi}{a_2^2} + \frac{1}{a_2}\frac{\partial\psi}{\partial r}\right) = 0 \quad (14)$$

$$K_3\left[-\frac{\psi}{a_2^2} + \frac{1}{a_2}\frac{\partial\psi}{\partial r}\right] = H_2\left[\frac{\psi}{a_2} - \theta_2\right] \quad r = a_2 \quad (15)$$

Now from (9) $\theta_2 = \frac{\phi}{a_1} + \frac{K_1}{H_1}\left(\frac{1}{a_1}\frac{\partial\phi}{\partial r} - \frac{\phi}{a_1^2}\right)$

Therefore substituting in (12) we have,

$$\frac{M'c'K_1a_1}{H_1}\frac{\partial^2\phi}{\partial r\partial t} + M'c'\left[a_1 - \frac{K_1}{H_1}\right]\frac{\partial\phi}{\partial t} + K_1a_1\left[\frac{H_2A_2}{H_1} + A_1\right]\frac{\partial\phi}{\partial r} + \left[A_2H_2a_1 - \frac{A_2H_2K_1}{H_1} - A_1K_1\right]\phi - \frac{a_1^2}{a_2}A_2H_2\psi = 0 \quad (16)$$

which is the boundary equation for $r = a_1$.

Similarly for $r = a_2$ boundary condition is

$$-\frac{M'c'K_2a_2}{H_2}\frac{\partial^2\psi}{\partial r\partial t} + M'c'\left[\frac{K_2}{H_2} + a_2\right]\frac{\partial\psi}{\partial t} - K_2a_2\left[\frac{H_1A_1}{H_2} + A_2\right]\frac{\partial\psi}{\partial r} + \left[A_1H_1a_2 + \frac{A_1H_1K_2}{H_2} + A_2K_2\right]\psi - A_1H_1\frac{a_2^2}{a_1}\phi = 0 \quad (17)$$

taking a as a unit of length and putting $r = a\xi$; $t = c\rho a^2\zeta/K_2$ the governing equation for ψ becomes

$$\frac{\partial^2\psi}{\partial\xi^2} = \frac{\partial\psi}{\partial\zeta}$$

whilst the governing equation for ϕ is

$$\frac{c_1\rho_1}{K_1}\frac{K_2}{c\rho}\frac{\partial\phi}{\partial\zeta} = \frac{\partial^2\phi}{\partial\xi^2}$$

and boundary equations (16) and (17) reduce to the form

$$B\frac{\partial^2\phi}{\partial\xi\partial\zeta} + C\frac{\partial\phi}{\partial\zeta} + D\frac{\partial\phi}{\partial\xi} + E\phi - F\psi = 0$$

and may be expressed in finite difference form as

$$B[\phi_{II} - \phi_I + \phi_{IV} - \phi_{III}] + 2C[\phi_I - \phi_3] + 2D[\phi_2 - \phi_4] + 4E\phi_0 - 4F\psi_0 = 0$$

Then, as in the previous example, we eliminate the fictitious values of ϕ, ψ between this equation and the appropriate governing equation so obtaining the boundary equations for the load surface and the surface of the inner wall.

RELAXATION TECHNIQUE

In the three problems described here it has been possible to calculate an eventual steady state. In problem 3 it is clear that since we are assuming the air outside to be always at a constant temperature (60° F in the problem worked out) the final steady state occurs when load, container wall and air in the container are all at 60° F. In the other problems the steady state may be obtained by equating the time derivatives to zero. The relaxation process may then be carried out as follows. Insert your initial conditions and at every other point of the network insert the values for the steady condition. The residuals will be zero, except for the first row. If these points are relaxed the residuals will spread and the process may be carried on until residuals *everywhere* are so small that no more relaxation is possible. In practice this work can be considerably reduced by good guesswork. Guess whereabouts the steady state will occur and between this row and the first row ($t = 0$) insert graduated values of temperature calculating the corresponding residuals.

THE TIME INTERVALS

In the problems worked out the time interval has always been taken as $c\rho a^2/K$ where a is the mesh length in inches. If we were to take the time interval as $\alpha c\rho a^2/K$ (α a constant) we could alter the duration of the time interval without altering the space interval. The equation of heat conduction in finite difference form would then become

$$2\alpha(\theta_2 + \theta_4 - 2\theta_0) = (\theta_1 - \theta_3)$$

i.e. $$-\theta_1 + 2\alpha\theta_2 + 2\theta_3 + 2\alpha\theta_4 - 4\theta_0 = 0$$

but for a certain range of values of α oscillatory errors occur. In these cases both the space interval and time interval will need to be altered together. Fowler discusses this point in his paper on analysis of numerical solutions of transient heat flow.(6)

ACKNOWLEDGMENT

I should like to thank Miss P. Burley for the invaluable assistance she has given me in carrying out the necessary computational work and Mr. T. A. Eames for his continued encouragement and helpful discussions. This paper is published by kind permission of Mr. T. M. Herbert, Director of Research, The Railway Executive.

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New Books

A Century of Science. Edited by Dr. H. DINGLE. (London: Hutchinson and Co. Ltd.) Pp. ix + 338. Price 15*s.* net.

The scope of this book is best indicated by the contents list which is: Concept of energy, by W. Wilson; Field physics, by J. L. Synge; Particle physics, by H. T. Flint; Structure of the atom, by W. Wilson; Structure of molecules and the chemical elements, by J. R. Partington; Geology, by W. T. Gordon; The earth's atmosphere, by P. A. Sheppard; Constitution and evolution of stars, by W. H. McCrea; Structure of the universe, by Sir Harold Spencer Jones; Organic evolution, by E. B. Ford; Coming of man and Progress of *Homo sapiens*, by F. E. Zeuner; Genetics and embryology, by G. B. de Beer; Physiology and histology, by K. J. Franklin; Biochemistry, by F. G. Young; Medicine, surgery and their scientific development, by E. Ashworth Underwood; General psychology, by Sir Cyril Burt; Medical psychology, by Aubrey Lewis; Significance of science, by H. Dingle.

The century covered by the articles in this most interesting volume is roughly 1850–1950, and it represents a century in which the advancement of science has been more rapid and spectacular than the advancements which have been chronicled for any other period in our history, and the period is remarkably well illustrated by the fact that the Crystal Palace marks for Britain the beginning of the period and the Festival of Britain the end. Physical science was built on the firm foundation laid by Newton in his *Principia*, and by the concept of a physical universe composed of billiard-ball atoms and an elastic-solid ether as the appropriate medium to carry light waves and radiation. Later in the period Lord Kelvin was to pronounce the opinion that if there was one thing of which he was completely certain and assured, it was that of the reality and substantiality of the luminiferous ether, and Clerk Maxwell suggested that further advances in physical science would accrue as the result of highly accurate experimenting in the valuation of physical quantities, and this indeed was true of certain notable advances, such as the third Lord Rayleigh's experiments on the density of gases which resulted in the discovery of a new elementary gas, argon.

Yet 1879 saw the birth of Albert Einstein whose work was destined to result in formation of field theories which put a wholly new complexion on the concepts of modern physics. So, too, with the concept of the atom, which has evolved from the hard massy sphere contemplated by Newton to the incredibly complex structure which has replaced it.

But there are spectacular advances to be chronicled in all the other sciences; experimental psychology is almost entirely a child of this period, and organic evolution, although its roots were planted in an earlier age, received remarkable impetus from the work of Charles Darwin and A. R. Wallace and, at a different angle, from the epoch-making researches of Gregor Mendel.

So, also, with the advances made in medicine and in surgery. All this, and much more, is to be found in this admirable collection of essays. Professor Dingle is to be heartily congratulated both on his selection of topics and on the brilliance and lucidity of the contributors. The book is well produced, the price is reasonable, and the contents are readable.

ALLAN FERGUSON

An Introduction to Electron Optics. By L. JACOB, Ph.D., F.Inst.P. (London: Methuen and Co. Ltd.) Pp. x + 150. Price 8*s.* 6*d.* net.

As an introduction to the subject this book is exceedingly uneven. Chapter I is a very brief account of its historical development. The "Basic Principles" of Chapter II include the action function and eikonals of Hamilton. As they are hardly referred to again, and make the subject seem unnecessarily forbidding to a newcomer, they might well have been left to the end; the generality of the concept of refractive index in electron optics could be brought out without their aid. The "Useful Optical Concepts" of Chapter III are either too elementary or too briefly explained; it would have been better to assume them to be familiar to an Honours student, as they should be, and discuss more fully their relevance to electron optics. Chapter IV on the electrostatic field again starts forbiddingly, without concessions to those who have no knowledge of vector analysis or Bessels functions, but the ensuing treatment of field mapping and trajectory tracing is well done. The discussion of electrostatic lenses (Chapter VI) and their aberrations (Chapter VII) is very well presented, with the experimental side to the fore, and the same is true of the corresponding Chapters X and XI on magnetic lenses. Phase focusing is very briefly mentioned in Chapter VIII, the properties of the magnetic field in Chapter IX, and the deflexion of beams in Chapter XII. The final Chapter (XIII) on the properties of beams is one of the best in the book, making very clear the problems encountered in making beams follow prescribed paths. It confirms the impression that the author would have produced a much more useful book if he had confined its scope to the experimental side of electron optics. As it is, the enquiring student is likely to be put off by the obscurity and disconnectedness of the opening theoretical discussion, which is needlessly added to by the confused notation, the same symbol being used for several quantities without adequate explanation. If the student persists, he will be rewarded by the presentation of a good survey of the properties and practical applications of electron lenses. Those already well acquainted with the field will also find much valuable practical information here. It must be said that the list of references at the end is a bibliographical horror, nearly all journals being indicated simply by initials. The initiated will know what *Z.f.T.P.* or *J.O.S.A.* mean, but most will be stumped by *P.C.P.S.* Ample space remains for a key to be given, even if the publishers could not print all titles in their accepted abbreviation. This lack of consideration for the student is symptomatic of all that is less satisfactory than it might have been in what could be an exceedingly valuable introductory monograph.

V. E. COSSLETT

Electromagnetic Waves and Radiating Systems. By E. C. JORDAN. (New York: Prentice-Hall Ltd.) Pp. x + 710. Price 32*s.* 6*d.* net.

A study of electromagnetic waves occupies such an important place in electrical engineering and physics that a comprehensive treatise on the subject is continually in wide demand. A number of authors have endeavoured to meet

this need and there are already available works that have become classics, but because of the breadth and complexity of the material these works tend to provide more particularly for the advanced student. Professor Jordan and the publishers have recognized that there is consequently a gap in the literature to be filled by a book of a more introductory character, and that objective is admirably achieved in the present volume. All the arguments start right at the beginning and they are developed with a lucidity that is most gratifying. At the same time the author has been careful to avoid over-simplification and he includes sufficient detail to enable a proper understanding of each facet of the subject to be obtained.

The book opens with a brief survey of the applications of vector analysis and shows clearly the value of this approach, first in dealing with problems of steady fields and subsequently in an examination of time-varying fields. On this background the author builds an excellent treatment of electromagnetic wave propagation both in space and in association with various forms of guide, including the transmission line, the enclosed and the surface waveguide. Reflexion, refraction, power flow, and radiation problems are each dealt with in turn and this is followed by a very thorough investigation of the basic characteristics of various aerial systems. The book is divided into sections with good references linking together different parts of the subject. Rationalized M.K.S. units are employed throughout, and excellent use is made of graphical illustrations and of numerical calculations. The author conforms to the American practice of using the z axis in Cartesian co-ordinates for the principal direction of propagation of a wave and it is arguable whether this is the best procedure in a book offering a closely co-ordinated picture of guided waves because in transmission line work the x axis is almost invariably chosen for that purpose. The section on aerials might well have contained some discussion of artificial dielectrics and micro-wave lenses, since this is a subject that has in recent years become of very considerable importance. The symbols used are clear and unambiguous, but wave impedance represented by η is perhaps a little unusual. There are a few minor errors such as on p. 140, in equation (5–71), on p. 275 in equation (9–40), and on p. 375 where P is written instead of ρ.

Taking this volume as a whole there is very little on which one can offer serious criticism and a great deal deserving of the highest praise. The book should prove immensely valuable to a wide circle of readers and in that difficult achievement both the author and the publishers are to be heartily congratulated.

H. M. BARLOW

Handbook of Experimental Stress Analysis. Editor-in-Chief M. HETENYI, Ph.D. (London: Messrs. Chapman and Hall Ltd.) Pp. xii + 1077. Price 120*s*. net.

This book is a text-book *par excellence* for all those engaged in the direction or solution of stress analysis problems. Each chapter has been subscribed by men experienced in the subject in which they separately deal. Each chapter has a bibliography of anything from 10 to 70 references with a brief description of the scope of each reference. The book is worth buying for the references alone. Although the book is of U.S. origin, a few authors from other countries have also contributed.

The opening chapter on the mechanical properties of materials follows normal text-book lines, but this is followed by a detailed description of various types of testing machines. Tensile, torsion, creep, fatigue, and hardness machines are all described in detail and the chapter is profusely illustrated with photographs. Mechanical extensometers are still often greatly preferred for the measurement of stresses in structures or models where their bulk and low frequency response are not disadvantages. A description of the development of mechanical gauges is given, together with detailed descriptions of magnification motions, accuracy, and sensitivity of each type of instrument dealt with. The accuracy of the engineers' dial gauge and its many applications in extensometer arrangements are discussed and illustrated. Very little is included on mechanical extensometers of the recording type however. There is still a wide field for the use of recording mechanical gauges even in this era of the resistance strain gauge, and some mention could well have been made of the very excellent and well-known strain recorder manufactured by the Cambridge Instrument Co. Ltd. One type of instrument, the Goodyear stress change recorder, is of the statistical type, i.e. it records over a long period of time the number of stress cycles of different ranges. The reviewer feels that there is a need for greater use of this type of instrument in structures such as bridges, ships, etc., to give a more fixed idea of the working stresses. The acoustic gauge is briefly mentioned. From his limited experience of these gauges, they are extremely good, especially for low stress levels and under poor conditions of set up, e.g. on structures in the open air.

It is not proposed to describe the material contained in the chapter on Optics. It does not appear, however, to contain sufficient information for all laboratory set ups. There has been a great amount written about the wire resistance strain gauge and its amplifying and recording equipment, that the author is to be congratulated on his having compressed so much useful information into one chapter. The wire resistance gauge novice or expert will find most of what he needs either in the text or the bibliography. The chapters on inductance and capacitance type gauges are equally informative. Steady state vibration measurements and fatigue stresses are dealt with under one chapter heading. In particular, the problems of bending stresses in a vibrating ship and in a reciprocating engine are explained. Twenty types of vibration measuring instruments are described and a table shows which instruments are suitable for various frequency ranges. Harmonic analysis of records, errors, accuracy, and calibration are also fully explained. This section concludes with a description of transient stresses and their interpretation. The analysis of strain gauge rosettes is exhaustively dealt with in a simple and straightforward manner, special reference being made to solutions using Mohr's circle, vector, and graphical solutions, analytical methods, and the use of mechanical computers.

Other subjects discussed are working stresses, residual, and temperature stress measurements, and methods of crack detection. One of the most interesting subjects dealt with is the interpretation of service fractures. This deals mainly and exhaustively with the various types of fatigue fractures and is copiously supplied with photographs. The use of brittle coatings such as the American "Stress coat" which is similar to the British "Strainlac" is described as is also the use of brittle models manufactured from plaster of paris. The subject of model analysis deals with various techniques,

such as the Beggs Method, the Eney deformeter, the Gottschalk Constinstat, the moment indicator, and the moment deformer, all of which are based on the Müller Breslau principle. The application of these and other methods is described in relation to various types of plane frames. The subject of creep in celluloid is dealt with and methods to overcome it described.

No mention at all is made of the more complicated three dimensional structural models which are in extensive use in ship design and elsewhere. The special problems of scale, material, and validity of results which have to be considered in such complicated models deserve a chapter of their own. (A paper on this subject was read by J. A. H. Paffett, R.C.N.C., to the North East Coast Institution of Engineers and Ship-Builders on 14th October, 1949.) The chapter dedicated to "analogies" deals firstly at great length with the Saint Venant theory for bars in torsion using a membrane analogy. As well as dealing at length with the theoretical background for analogies, practical directions are given concerning the technique. The second method described is that of using an electric potential field using either a thin flat plate or an electrolytic tank. Further analogies of the resistance and capacitance net type are also fully described.

The authors of the chapter on Photoelasticity are to be congratulated on their lucidity and simplicity in the presentation of the basic concepts, experimental technique, and interpretation of results. The nature of polarized light, the effect of wave and quarter wave plates and the stressed model are explained clearly and simply. The various types of polariscope plane, circular, reflexion, spectroscopic and doubling are shown and a simple loading rig described. The relation between isoclinics and stress trajectories and the isochromatics or fringes, the derivation of $p - q$ and, in particular, the edge stress, the use of compensators for fractional fringe orders and the determination of the fringe constants are all clearly dealt with. The method of obtaining $p + q$ by thickness variation using Schaid's inferometer and the De-Forest strain gauge extensometer are described. Various examples of the application of photoelastic technique are given such as holes and notches in flat bars, contact between rollers and gear teeth, crane hooks, etc. The method of stress freezing to obtain stress analysis in three-dimensional problems is described, together with examples of its application. The final chapter gives a description of radiography leading up to X-ray analysis.

N. E. GUNDRY

X-ray Identification and Crystal Structures of Clay Minerals. Edited by G. W. BRINDLEY. (London: The Mineralogical Society.) Pp. 345. Price 35*s*. net.

When the Clay Minerals Group of the Mineralogical Society was formed in 1947 it was felt that a new set of thoroughly reliable X-ray diffraction data for all clay minerals should be prepared, as the A.S.T.M. index and previous publications dealing with clay minerals were out of date. The book under review does far more than fill this gap. It is an authoritative review of present-day information on methods, data, and structural interpretation. Many of the results and interpretations have not been published before. There are altogether eleven contributors, including American and French mineralogists. Seven chapters of the book deal with the following broad groups of clay minerals: kaolin, montmorillonite, mica, chlorite, vermiculite, sepiolite, palygorskite-attapulgite, while three chapters review oxides and hydroxides of aluminium and iron, non-clay minerals in clays, and phase changes in kaolin upon heating. The remaining chapters are concerned with technique, interpretation, and with detailed discussions of the random characteristics of clay diagrams, due (*a*) to interleaving of layers of different thickness and (*b*) to random shifts in the superposition of the layers. In accordance with its character and emphasis on crystal structures the book gives only as much information on chemical, optical, thermal, base exchange and other properties as can be correlated with crystal structures.

It is not possible to discuss all the interesting or novel features of the book here but the following may be mentioned.

The iron and magnesium analoga of kaolin are discussed and new data for chamosite given. Mullite can usually be detected on kaolin samples after heating to 950° C, but in halloysite it is only detected above 1 100° C; correspondingly γ-alumina, which occurs as an intermediate phase and usually disappears above 1 000° C in heated kaolin, remains in halloysite up to 1 150° C. A formula is derived which relates changes in the *b*-dimension of the unit cell of montmorillonite to isomorphous replacements in the octahedral and tetrahedral layers, and in the mica chapter useful calculations are made for the intensities of the basal reflexions as functions of the octahedral replacements. In the chapter on random stratification the theory of Hendricks and Teller is extended and graphical solutions given for a number of complexes known or likely to occur in soil colloids or other sediments.

In conclusion, the reviewer feels that this is an excellent book with much matter of interest to the beginner as well as to the advanced research worker. The editor and the Clay Minerals Group are to be congratulated on their accomplishment.

G. NAGELSCHMIDT

Pottery and Ceramics. By E. ROSENTHAL, Dr. Ing., A.M.I.E.E. (Harmondsworth: Penguin Books.) Pp. 304. Price 1*s*. 6*d*. net.

The importance of ceramics to the physicist in his home is well illustrated by a sectional drawing of a house that Dr. Rosenthal has imaginatively placed at the beginning of his text—a score of important items in the construction and equipment of the house are made of ceramic material. The importance of ceramics to the physicist in his laboratory will be seen by reference to the final chapters on electrical porcelain and low-loss ceramics. Conversely, the importance of physics to the ceramic technologist will be appreciated by the many references to the physical behaviour of clay, both before and after firing, scattered throughout the book. Highly theoretical branches of ceramic technology can have only brief mention in a book of this nature. The physicist, however, will see the application of his branch of science in the sections of the book on particle size measurement, drying, temperature measurement, and the electrical properties of the special ceramic bodies used in the electrical industry. Should the physicist wish to make a quick general survey of the ceramic industry as a whole, he could not do this more expeditiously, accurately, or cheaply than by reading this Penguin.

A. T. GREEN

Notes and Comments

The Institute of Physics

At the Annual General Meeting of the Institute of Physics held on 16th May, 1951, Prof. W. E. Curtis (Professor of Physics, King's College, University of Durham, Newcastle-upon-Tyne) was re-elected President. Dr. J. Topping was elected a Vice-President and Mr. O. W. Humphreys and Dr. S. H. Piper were elected Ordinary Members of the Board. The Honorary Treasurer, Mr. E. R. Davies, and the Honorary Secretary, Dr. B. P. Dudding, were re-elected. Dr. A. B. Wood and Mr. C. G. Wynne join the Board as the new representatives of the Physical Society.

The Board's Report for 1950 which was adopted at the meeting, again records continued progress. The effect of the establishment of the Graduate grade, the raising of the minimum ages and lengthening of the periods of experience for the Fellowship and Associateship are reflected in the way the numbers in the different grades have changed compared with previous years.

The system of granting official recognition, after inspection by the Institute, to technical colleges whose standard of teaching physics is considered adequate has resulted in substantial improvements in their facilities in several cases. Thirty-three Colleges have now been recognized.

For the first time, candidates were presented for the new Higher National Certificate in Applied Physics which is issued jointly by the Ministry of Education and The Institute of Physics. The Report records the active part the Institute has taken in the many discussions on Higher Technological Education; it submitted its observations to the Minister of Education on the report issued by the National Advisory Council for Education in Industry and Commerce.

The Report states that by the end of the year the circulation of the British Journal of Applied Physics was very satisfactory and promised well for its future, it having only been established last year.

Among the professional matters which received consideration during 1950 were the conditions of service and salaries of physicists, the economic status and general conditions of teachers of physics in schools, consulting fees, insurance and radiation hazards, and careers in physics.

A conference on "Some Aspects of Fluid Flow" was held in October and the proceedings are being published in book form. The growing use of the facilities of the Institute's House in Belgrave Square, S.W.1, is shown by the almost doubling of the number of meetings held in it compared with the previous year.

The Institute's fifteen local Branches and specialist subject Groups held 112 meetings during the year, several of which took the form of 2–5 day conferences, often in collaboration with other organizations. In addition most of the Branches and Groups arranged visits to laboratories and works and some held receptions in the Institute's House.

Elections to The Institute of Physics

Fellows: J. A. Hall, M. Ishaq, P. Parker, S. Rajaraman, C. S. Rao, J. F. Stirling.

Associates: N. C. Bagchi, W. B. Chivers, S. I. Evans, J. A. Farnham, P. Garwood, T. H. L. Graff, J. C. Hawkes, E. S. Hutchinson, A. J. Kennedy, A. A. R. L. M. Morris, N. J. Rumsey, E. W. Saker, G. G. Schaeffer, J. H. Shackley, O. H. Turner, E. L. P. de Valence, R. J. Wakelin, F. J. B. Wall.

Thirty-six Graduates, 4 Subscribers and 34 Students were also elected.

Guide to research activities.—The Department of Scientific and Industrial Research has just issued a brochure entitled "Brief Guide to the Research Activities of D.S.I.R. and the Research Associations." The object of this publication is to guide enquirers to the specialized sources of information existing in the Research Associations and in the D.S.I.R. Research Stations. It is hoped that, in the future, the scope of this guide will be extended to include details of other specialized sources of scientific and technical information in the United Kingdom. A companion volume to this publication entitled *Scientific Research in British Universities* is also in preparation and will be published this year. Nearly 1 000 subjects are classified according to the U.D.C. system to which there is also an alphabetical index. Copies of the brochure are obtainable from the Technical Information Service, D.S.I.R., Charles House, 5–11 Regent Street, London, S.W.1.

Journal of Scientific Instruments

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SPECIAL ARTICLE

Plasma oscillations*

By D. GABOR, Dr. Ing., M.I.E.E., F.Inst.P., Imperial College of Science and Technology, London, S.W.7

A comprehensive review is given of the state of knowledge on plasma oscillations in the narrower sense, and of those electronic oscillations which are determined by the internal dynamics of the discharge rather than by the outer circuit. Experimental and theoretical work is reviewed and a new mathematical formulation is given for oscillations in "irrotational" streams. If the electrons and ions originate outside a magnetic field, or if they issue from cathodes where the magnetic field has no normal component, the curl of the total or Schwarzschild momentum will be zero everywhere. In this important case the self-consistent equations of any number of electron or ion streams can be completely formulated in a series of differential equations for certain vector stream-functions; one function for each stream. Comparatively few cases have been discussed so far with the proper boundary conditions. The most important targets for research are pointed out.

In their classical investigations on low pressure gas discharges (1924), Langmuir and Mott-Smith[1] divided the discharge space into "sheaths" and "plasmas." The sheaths are thin layers, surrounding the electrodes and the walls, with strong gradients of potential, while the plasmas are almost neutral electrically, as they contain positive and negative charges in nearly equal numbers. The name "plasma oscillation" was also coined by Langmuir, in collaboration with Tonks,[2] for a phenomenon which was already discovered some years earlier by Penning.[3] Since that time this useful term has acquired a meaning considerably more general than the original. In accordance with current usage it will be applied to all oscillation phenomena, regular or irregular, which are determined by the internal dynamics of the discharge phenomenon rather than by the outer circuits.

It will be all the more justifiable to extend the field of this survey because, in the author's opinion, plasma oscillations in the narrower sense do not hold out any great practical promise. Researches by many authors, extending over more than ten years, have led so far to one device only which can be considered as practical; the "plasma oscillator" described by Wehner.[4] It might be perhaps possible, by patient work, to increase the efficiency which was about 1% "under favourable conditions," in the numerous experiments of Emeléus and his collaborators.[5] But by the very fact that the frequency of the plasma oscillations is determined mainly by the internal conditions in the discharge (in the first instance by the charge density), and not by the electrode geometry, it appears extremely difficult if not impossible to keep this frequency in the narrow limits required by modern microwave techniques, and achieved, without great difficulty, by electronic devices.

Yet the mechanism by which plasma oscillations are generated is of great scientific and practical interest. On the positive practical side, this mechanism is not essentially different from the one in the electron beam amplifiers invented by Haeff[6] and independently by Pierce and Hebenstreit.[7] On the negative side, oscillations of this type are almost certainly at the root of the "excess noise" phenomena, observed in magnetrons and in other devices. On the scientific side, it is at least very likely that solar and stellar radio noise will be ultimately explained in these terms. And finally, there is increasing evidence that plasma oscillations are never entirely absent in gas discharges, and that they will give the key to the understanding of several of the paradoxical phenomena discovered by Langmuir almost thirty years ago, which have hitherto defied all attempts at explanation.

EXPERIMENTS ON PLASMA OSCILLATIONS

A striking feature of all experiments on plasma oscillations is the contrast between the extreme simplicity of the experimental arrangements, and the complication of the phenomena which they reveal. Let us consider first the very simple device, used by Langmuir and others, shown in Fig. 1. The tube, containing usually low pressure mercury, has a straight tungsten filament as cathode, with a coaxial, cylindrical anode. A small probe is arranged between the two; it is usually movable. This and the cathode, or the anode, are connected to a pair of Lecher wires, bridged by a detector. The voltages applied are usually of the order 15–100 V, the currents range from a few milliamperes to about 100 mA. Under almost all conditions oscillations in the decimetric range can be detected, though with very variable frequencies and amplitudes.

Fig. 1. Simple arrangement for detecting plasma oscillations as used by Langmuir and Tonks, Emeléus and others

Armstrong and Emeléus[5] have simplified this device even further, by leaving out the probe and connecting only the cathode or the anode with the inner conductor of a coaxial line, leaving the outer conductor floating. But the apparent simplicity of this device is wholly deceptive. Among many other things, the magnetic field

* Based on a lecture given before the Electronics Group of The Institute of Physics, 13 March, 1951.

of the heating current in the filament is very important. When Armstrong and Emeléus interrupted the heating current with a rotary switch 2 000 times/sec, so that the cooling was entirely negligible, the oscillations in the off-periods were found to be only small fractions of those in the on-periods.

Perhaps the clearest results so far obtained are those of Merrill and Webb,[8] Fig. 2. They used an indirectly heated, large diameter oxide-coated cathode, with negligible magnetic field, and limited the emerging electron beam to a rectangle of about 4×6 mm. They went to great trouble to be able to move the probe by very small steps. The whole discharge tube and the oven surrounding it was freely suspended, to avoid vibrations, and the tilt was varied by thermal expansion of a supporting wire. These precautions paid, because they discovered an important phenomenon which had escaped Tonks and Langmuir; the remarkably sharp localization of the regions of plasma oscillation and of electron scattering.

The phenomenon of "high scattering" was discovered by Langmuir[9] in a tube of the type shown in Fig. 1, but without the Lecher wires. Applying his probe method, previously worked out in collaboration with Mott-Smith,[1] Langmuir found that there were three distinct electron groups present in the discharge space between the filament and the anode. He called these "primary," "secondary" and "ultimate" electrons. The primary electrons had energies very nearly equal to the voltage applied to the anode. The conclusion was that they were accelerated in a very thin sheath surrounding the cathode, which acted like a sort of ideal grid, and contained practically the whole potential drop. The rest of the discharge space was filled with a plasma of almost uniform potential; the anode drop was negligible. The secondary electrons had energies of the order of 10 V; they were evidently produced by ionizing collisions of the primary electrons with gas molecules. The ultimate electrons were the most numerous, of the order $10^{10}/cm^3$; these formed an electron gas with a very nearly Maxwellian distribution of energies, corresponding to temperatures of 10–30 000° C or 1–3 V. These must have originated from secondary electrons which had lost almost all their energy in collisions, and were finally almost trapped by the equal and opposite positive ion charge, which allowed them to escape only at a slow rate to the anode and to the walls of the discharge tube.

Langmuir's probe methods enabled him to measure the velocity distribution of the primary electrons with very great accuracy, and he found that they had a spread round the nominal energy corresponding to the total voltage drop of a few tenths of a volt only—about what one would expect if they had suffered no collisions at all and carried their original temperature with them. This was the case near the cathode at all currents, and throughout the discharge space at small currents. But at higher currents, of the order of a score of milliamperes the striking phenomenon of "high scattering" started; the electrons acquired an energy spread of the order of several volts, and, most surprisingly, there was an appreciable fraction which had energies exceeding the total voltage drop in the tube. Langmuir believed that he had excluded the possibility of oscillations, but after a suggestion by Dittmer,[10] he and Tonks returned to the problem and were able to pick up oscillations of very high frequency, even if the anode was held at a completely steady potential, on the measuring probe itself, and on other auxiliary electrodes.

But the exact localization of the oscillations and their connexion with the phenomenon of "high scattering" remained unknown, until Merrill and Webb[8] approached it with their more refined technique. They used the same movable probe alternatingly for the analysis of electron velocities and of the oscillations. They found that, though oscillations could be picked up everywhere, they had a sharp peak in a sheath usually only a few tenths of a millimetre deep, somewhere about midway between the cathode and the anode (*B*, Fig. 2). Surprisingly, the high scattering appeared not at this point, but about ¼ mm nearer to the cathode, at *A*. They interpreted the relation between the two by assuming that for some reason which could not be explained at that time, the oscillating electric field was restricted to a narrow sheath at the position *A*. But no oscillations can be picked up at this spot, because the electron beam in this region is still completely steady, and there is no alternating component in the current which, in conjunction with the alternating field could produce an oscillatory output. Hence *velocity modulation* or "high scattering" will be measurable at *A*, but no oscillations. But a short distance beyond *A*, at *B*, *bunching* has

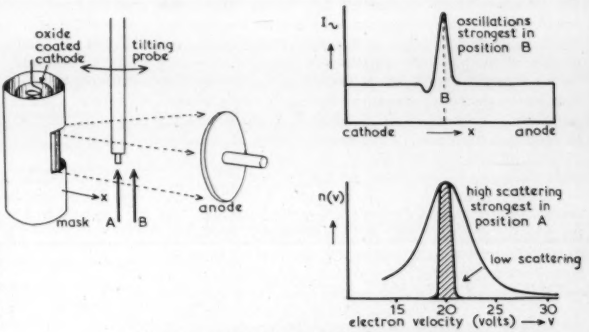

Fig. 2. Experiments of Merrill and Webb, 1939

occurred, the current possesses an alternating component and strong oscillations are recorded in the outer circuit. This explanation was novel in 1939; it appears very natural to us now, since we have become familiar with the Klystron. We can also conclude at once that the velocity modulation must have been either very deep, which is certainly not the case, or its frequency must have been ill-defined because Merrill and Webb were unable to pick up further maxima.

The narrowness of sheath *A* in which the electrical oscillations are concentrated remained unexplained for a long time. Quite recently Bohm and Gross[11] have proposed a semi-quantitative explanation, to which reference will be made later.

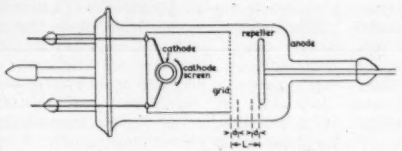

Fig. 3. Wehner's plasma oscillator, 1950

The first practical "plasma oscillator" was described by Wehner[3] and is shown in Fig. 3. It can be said that Wehner succeeded in producing the phenomena in their simplest, very nearly one-dimensional form, by using a device somewhat more complicated than those previously described. The source of the primary electron beam is here not a hot cathode, but a plasma, which is produced in a metallic box, containing a hot cathode, with a fine-mesh plane gauze at one end. The box and the grid are nearly at cathode potential. This type of beam-source was previously developed by the author[12] in the Research Laboratory of the British Thomson-Houston Co. Ltd. If the cathode is sufficiently strongly heated, large currents can be extracted from the grid without any ionization taking place inside the box, which is filled with a non-luminous plasma at a very low electron temperature. Practically the whole potential drop occurs at the grid, which becomes the source of a very uniform parallel electron beam. In Wehner's tube this is reflected to and fro several times between the grid and a "repeller," parallel to it, and also nearly at cathode potential, at a distance of 6 mm. The anode is the metallic casing of the tube. The gas pressure is about 5 μ. Oscillations of 1–4 000 Mc/s can be picked up at the repeller, their frequency can be regulated by the anode current or voltage, and it is claimed that they are very stable.

The theory of this device is comparatively simple if one accepts the result of Merrill and Webb that a bunching zone will be spontaneously formed at some distance from the cathode, i.e. in this case from the grid. In fact this zone can be *seen*, according to Wehner, as a slightly darker sheath in the luminous discharge in the space between the grid and the repeller. In some cases a second narrow dark zone appears nearer to the repeller. Wehner interprets this as a second zone of strong oscillations, which acts as the *catcher*, so that the device can be interpreted as a *Klystron*, with the difference that the oscillators (the Rhumbatrons) are supplied by the plasma itself. Probe measurements indicate that the frequency is equal to the plasma frequency calculated by Langmuir's formula (see later) from the electron density at the position of the "bunching" zone. In other cases the second dark layer moves up to the repeller and merges with the thin dark sheath surrounding this electrode. In this case Wehner interprets the device as a *reflex Klystron*.

Apart from the experiments described here a very large amount of material has been collected especially by Emeléus and his collaborators Humphreys, Neill and Armstrong,[5] but it will be sufficient to keep these few data in mind in order to understand the essentials of the theory.

A quite different line of experiments, of great practical and theoretical importance, has been started by Haeff,[6] and by Pierce and Hebenstreit.[7] These "multiple beam" or "slipping stream" tubes may be regarded as sufficiently well known to make a description unnecessary. A third important line of experiment relates to the noise in the positive column of discharge tubes, with or without a magnetic field.[13]

GENERAL CLASSIFICATION OF THEORIES

The complication and variety of the phenomena described and referred to suggests clearly enough that a theory which could deal adequately with all their features would be nothing less than a general theory of electronic and gas discharges, excluding only perhaps the more trivial theory of low frequency phenomena, in which the outer circuit plays an essential part.

This is evidently a too ambitious programme, and the best that can be done at present is to approach it from several sides, starting with simplified models, in which one or the other complicated feature of the general problem has been deliberately suppressed. There are two such main approaches, both of historical and practical importance: (1) from the side of the theory of *regular* electron streams, without random motion; (2) from the side of the theory of the plasma in *complete thermal disorder*.

The first approach is the more highly developed. It can be said that it allows at least an almost complete mathematical formulation, and that only purely mathematical difficulties prevent us from accounting, on the basis of known equations, for at least a very great part of the phenomena observed in electron tubes in which no gas is present. But it would be quite wrong to assume that we can account for everything, and that in order to complete the theory it will be sufficient to superimpose in the end the random motion of the electrons, as a small disturbance. A clear warning is given by the case of the magnetron. Starting with the assumption of a regular electron stream, with zero initial velocity emitted by the

cathode, it can be "proved," as Brillouin[14] has done that the stationary solution must be a "single stream solution," and this must be unavoidably the "Hull–Brillouin solution," in which the electron cloud rotates around the cathode almost like a solid body, and has almost uniform density distribution. This is not essentially changed if afterwards account is taken of the initial velocity spread as a small disturbance. But if, as the author has done,[15] the opposite assumption of an "infinite stream" solution is made, taking the initial velocity distribution into account from the start, one obtains a quite different law of density, and a strong slip of different electron layers on one another. Experiment (Marton and Reverdin)[16] certainly decides against the Hull–Brillouin solution; it shows that there must be essentially what amounts to a double stream, but it does not favour the "complete disorder" theory either. It appears to be nearest to an intermediate solution, also found by the author,[15] under the assumption that the axial disorder is suppressed.

The second approach, from the side of the plasma in thermal equilibrium, is very important and indispensable for the understanding of the gas discharge phenomena which we have described, but it is evident that in its pure form it cannot take us far enough. The plasmas occuring in these and other discharge devices are evidently far from equilibrium; they are traversed by strong electron streams, and these are generated in sheaths, at the boundaries; else one could only obtain the very weak thermal radiation from these devices, and the effects are, in fact, in most cases many orders of magnitude stronger. The best which the theory of the homogeneous or almost homogeneous near-equilibrium plasma can give us is the *laws of propagation* of plasma waves, and it is in this direction that the greatest progress has been made, especially by the work of Denisse,[17] and by Smerd and Westfold.[18] The weakest point is still the behaviour of plasmas near the boundaries, i.e. the theory of sheaths and plasmas, and the generation of oscillations. In this respect the greatest advance has been made by Bohm and Gross.[11]

Before undertaking the formidable mathematical work required, it is evidently very desirable to know that we are building on a safe foundation. But here an important doubt must first be cleared up. *All* theories so far proposed consider the electrons in what is known in physics as "Hartree's self-consistent field approximation"; that is to say only *one* electron is considered as a particle, the effect of the others on it is replaced by their smoothed, averaged space charge and current distribution. In other words, the N-electron problem is replaced by a one-electron problem. This can hardly be avoided, because the N-electron problem is even a little more refractory than the notorious N-body problem of celestial mechanics. But how far is this admissible, i.e. how far are we entitled to neglect the *interaction of individual charged particles*? This is a problem of formidable mathematical difficulties, and though most theoretical authors (Thomas,[19] Landau,[20] Chapman and Cowling[21]) agree well between themselves, there was some reason to disbelieve their results. Langmuir[9] has shown that the Maxwell-distribution of electron velocities re-establishes itself in gas discharges at distances from a source of disturbance, such as an electrode, or the wall, which are sometimes at least 1 000 times smaller than the mean free paths calculated from the theoretical formulae. Though this dilemma has been known for the last twenty-five years, no experiments have been undertaken to clear it up until quite recently. It appears now, from experiments of Mr. E. A. Ash which are still in progress, that the interaction is at least not very much stronger than the theoretical value, and almost certainly not strong enough to explain by itself the "Langmuir paradox." This indicates that the Maxwell-distribution in very strongly diluted gases and at relatively low electron concentrations, below about $10^{12}/\text{cm}^3$, is probably only apparent; it may well be an average over turbulent oscillations in *macroscopic, not molecular disorder*. As the phenomenon of electron temperatures forms the very basis of our present understanding of low pressure discharges, this is certainly a strong encouragement to proceed with the theory of plasma oscillations on the present theoretical basis.

OSCILLATIONS IN REGULAR ELECTRON STREAMS

We call electron streams regular which are emitted, or could be emitted by any number of cathodes with zero initial velocity. This is a more stringent condition than might be realized at first sight. The reason is Lagrange's circulation theorem, the strongest simplifying device in the dynamics of electron streams. In order to formulate this, we must first introduce the concept of the "total momentum" of an electron, due to Larmor and to Schwarzschild. These authors have found that the general methods of Lagrangian and Hamiltonian dynamics can be applied to the motion of electrons in electromagnetic fields only if the mechanical momentum mv is replaced by

$$\boldsymbol{p} = m\boldsymbol{v} + e\boldsymbol{A} \tag{1}$$

where $\boldsymbol{A}$ is the vector potential, from which the magnetic induction $\boldsymbol{B}$ is derived by $\boldsymbol{B} = \text{curl } \boldsymbol{A}$. Let us now define the circulation C as the line integral of $\boldsymbol{p}$ over a closed circuit

$$C = \oint \boldsymbol{p} \,.\, d\boldsymbol{s} = m\oint \boldsymbol{v} \,.\, d\boldsymbol{s} + e\oint \boldsymbol{A} \,.\, d\boldsymbol{s} = m\oint \boldsymbol{v} \,.\, d\boldsymbol{s} + e\Phi \tag{2}$$

The last expression is obtained by applying Stokes' theorem to the second integral. Φ is the magnetic flux through the circuit. The circulation thus defined is m times the circulation as defined in hydrodynamics, plus e times the magnetic flux threaded by it. Lagrange's theorem states that if we draw such a circuit through the positions of electrons at any instant, and follow them in their flight, *the circulation remains invariant.*

It is this theorem which enables us to formulate the dynamics of regular electron streams with comparative ease, because we can follow the electrons from the instant when they started at the cathode with zero velocity. It has been shown (Cherry,[22] Gabor[23]) that if there exists no magnetic field at right angles to the cathode, the circulation will be zero everywhere, and this leads to

$$\operatorname{curl} \boldsymbol{p} = 0 \tag{3}$$

for such beams, which are called "*irrotational*." This does not mean that the electrons have no rotation, i.e. vortex motion, but their vorticity will be such as to compensate exactly the vortex of $\boldsymbol{A}$, which is the magnetic induction $\boldsymbol{B}$. The case of electrons starting in a normal magnetic field is somewhat more complicated, and will be dealt with later.

The basis of electron dynamics is Lorentz's law

$$d(m\boldsymbol{v})/dt = e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{4}$$

Using equation (1), the equation of electromagnetic theory $\boldsymbol{E} = -\operatorname{grad}\phi - \dot{\boldsymbol{A}}$, and the vector identity

$$\frac{d\boldsymbol{v}}{dt} = \frac{\partial \boldsymbol{v}}{\partial t} + \operatorname{grad}(\tfrac{1}{2}v^2) - (\boldsymbol{v} \times \operatorname{curl} \boldsymbol{v})$$

Equation (4) can be written in the form

$$\frac{\partial \boldsymbol{p}}{\partial t} + \operatorname{grad}(\tfrac{1}{2}mv^2 + e\phi) = \boldsymbol{v} \times \operatorname{curl} \boldsymbol{p} \tag{5}$$

For irrotational beams the right-hand side is zero. The result can be further simplified if we solve equation (3) by putting

$$\boldsymbol{p} = \operatorname{grad} W \tag{6}$$

where the function $W(x, y, z, t)$ is called the "action function." Substituting this into equation (5) we obtain at once

$$\frac{\partial W}{\partial t} + e\phi + \operatorname{grad} \tfrac{1}{2}mv^2 = \text{const.} \tag{7}$$

If, using equation (1) and (6), we express the velocity $\boldsymbol{v}$ in terms of W and $\boldsymbol{A}$ as

$$\boldsymbol{v} = (\operatorname{grad} W - e\boldsymbol{A})/m \tag{8}$$

equation (7) becomes a partial differential equation for the action function W alone, which is known as the Hamilton–Jacobi equation. This is not a very convenient equation analytically, being of the first order but of the second degree, but in principle it enables us to calculate all electron trajectories if the fields are prescribed, i.e. if the potentials ϕ and $\boldsymbol{A}$ are known.

If, however, as is the case in oscillations of the type in which we are interested, the field is not prescribed but itself determined by the electron motion, it is better to proceed a little differently and to write equation (7) in the equivalent form

$$\left(\frac{m}{e}\right)_i \left(\frac{\partial \boldsymbol{v}_i}{\partial t} + \operatorname{grad} \tfrac{1}{2}v_i^2\right) = \boldsymbol{E} \tag{9}$$

Here we have introduced a suffix i on the left-hand side, providing for the general case in which there are several streams of ions and electrons sharing the same space, which may have different ratios m/e, or at least different velocities $\boldsymbol{v}_i$ at the same point, (velocity groups). The electric field intensity $\boldsymbol{E}$, however, is common to all of them.

We now combine this with Maxwell's equations for the electromagnetic potentials, in the form

$$\left.\begin{aligned} \Box \boldsymbol{A} \equiv \Delta \boldsymbol{A} - \frac{1}{c^2}\ddot{\boldsymbol{A}} &= -\mu_0 \boldsymbol{i} \\ \Box \phi \equiv \Delta\phi - \frac{1}{c^2}\ddot{\phi} &= -\rho/\epsilon_0 \end{aligned}\right\} \tag{10}$$

ϵ_0 and μ_0 are the permittivity and the permeability of empty space, connected by the relation $\epsilon_0\mu_0 c^2 = 1$. $\boldsymbol{i}$ and ρ are the current density and the charge density, produced by all the partial streams. Assuming that no ionization or recombination takes place we have for every one of these streams a conservation equation

$$\operatorname{div} \boldsymbol{i}_i + \dot{\rho}_i = 0 \tag{11}$$

For the resulting current and density this is true even if there exists ionization or recombination. From equation (10) we obtain

$$\Box \boldsymbol{E} \equiv \left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} = \frac{1}{\epsilon_0}\left(\operatorname{grad} \rho + \frac{1}{c^2}\frac{\partial \boldsymbol{i}}{\partial t}\right) \tag{12}$$

We now solve the continuity equations (11) by introducing for every kind of ion, or for every velocity group i a *vector stream function* $\boldsymbol{S}_i$, by putting

$$\boldsymbol{i}_i = \epsilon_0 \frac{\partial \boldsymbol{S}_i}{\partial t} \quad \rho_i = -\epsilon_0 \operatorname{div} \boldsymbol{S}_i \tag{13}$$

If the gaussian system is used ϵ_0 must be replaced by $1/4\pi$ in the last two equations.

By means of this we can express the velocity of the ith group in the form

$$\boldsymbol{v}_i = -\dot{\boldsymbol{S}}_i/\operatorname{div} \boldsymbol{S}_i \tag{14}$$

We now apply the dalembertian operator $\Box$ to both sides of equation (9), and using equations (12) and (14) obtain

$$\left(\frac{m}{e}\right)_i \left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left\{\operatorname{grad} \tfrac{1}{2}\left(\frac{\dot{\boldsymbol{S}}_i}{\operatorname{div} \boldsymbol{S}_i}\right)^2 - \frac{\partial}{\partial t}\left(\frac{\dot{\boldsymbol{S}}_i}{\operatorname{div} \boldsymbol{S}_i}\right)\right\} = -\left(\operatorname{grad}\operatorname{div} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Sigma \boldsymbol{S}_i \tag{15}$$

There are as many equations as there are ion or velocity groups, one for every stream function $\boldsymbol{S}_i$. They can be called the *self-consistent dynamical equations of irrotational electron streams*.

It can be said with some confidence, that these rather complicated equations contain implicitly the greater part of the phenomena which have been observed or can be observed in vacuum devices, subject only to the condition of irrotationality (i.e. no magnetic field normal to

the cathodes), and with the reservations which have been made regarding the importance of random motion. They are non-linear, and it can be expected that they will account also for the phenomena of amplitude limitation and intermodulation, which have not yet been theoretically investigated. But their complicated form makes a general discussion difficult. It will be sufficient for the present if we consider instead the much simpler case of *small disturbances* superimposed on a steady stream pattern, in which the ith partial stream has a velocity V_i and a density D_i. Let s_i be the stream function of the small disturbance. Equation (15) now becomes

$$\left(\frac{m}{e}\right)_i\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left\{\text{grad}\left[\frac{V_i}{D_i}(\dot{s}_i + V_i \,\text{div}\, s_i)\right] + \frac{\partial}{\partial t}\left[\frac{1}{D_i}(\dot{s}_i + V_i \,\text{div}\, s_i)\right]\right\} = -\epsilon_0\left[\text{grad div} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\Sigma s_i \quad (16)$$

The interesting fact may be noted that once the steady stream is prescribed, the differential equation of the small disturbances is *ipso facto* given, without any reference to the electric or magnetic fields which have produced the basic stream pattern. But it must be emphasized that this is true for irrotational streams only.

As equation (16) is a linear equation, the complete solution can be obtained by superposing simple particular solutions of the form

$$s(x, y, z, t) = s_0(x, y, z)e^{j\omega t} \quad (17)$$

i.e. harmonic standing waves. Consider first the extremely simple, though somewhat unreal, case of a uniform electron cloud at rest. This is possible only if the charges are neutralized by ions, but for simplicity we can assume the ions as so heavy that they are not affected by the oscillating fields. We have now a medium uniform in space, and the analysis can be conveniently carried out in terms of particular solutions of the form

$$s(x, y, z, t) = s_0 e^{j(\boldsymbol{k}\,.\,\boldsymbol{r} - \omega t)} \quad (18)$$

that is to say, in terms of *progressive waves*. $\boldsymbol{r}$ is the radius vector (x, y, z), $\boldsymbol{k}$ is the wave vector, which points in the direction of propagation, normal to the phase-fronts. Its scalar value is

$$k = 2\pi/\lambda$$

and is called the propagation constant.

Many important properties of the medium can be derived if one knows the relation between the vector $\boldsymbol{k}$ and the frequency ω, which is called the *dispersion equation*. One obtains from this, first of all the *phase velocity*

$$V_p = \omega/k \quad (19)$$

which is normal to the wavefronts, in the direction of k, and the *group velocity*

$$V_g = \mathrm{d}\omega/\mathrm{d}k \quad (20)$$

In the case of a uniform electron cloud at rest, with neutralized charge, $V = 0$, $D = en = constant$ where n is the number of electrons per unit volume; substituting the wave given by equation (18) in equation (16) the dispersion equation is found to be:

$$(c^2k^2 - \omega^2)\omega^2 s_0 = \omega_p^2[c^2\boldsymbol{k}(\boldsymbol{k}\,.\,s_0) - \omega^2 s_0] \quad (21)$$

Here ω_p is a constant frequency, given by

$$\omega_p^2 = \frac{e^2 n}{m\epsilon_0} = \frac{4\pi e^2 n}{m} \quad (22)$$

The first expression is valid in any rationalized system, such as M.K.S. or Lorentz–Heaviside in which the dielectric constant of the vacuum is ϵ_0, the second, more familiar formula is valid in the Gaussian system, used by almost all authors on this subject. $f_p = \omega_p/2\pi$ is called, after Langmuir, the "*plasma frequency*"; $f_p = 9\,000\sqrt{n}$ c/s for electrons if n is the concentration per cm^3. But it may be noted that this frequency was known long before, by Rayleigh and many other authors, as the "*critical frequency*," and it still goes by this name in propagation theory.

It can be seen from equation (21) that the relation between k and ω is different according to the angle between the wave vector $\boldsymbol{k}$ and the stream vector $\boldsymbol{s}$, which is parallel to the direction in which the electrons oscillate. Two special cases are of particular interest:—

Transversal waves, $\boldsymbol{k}\,.\,s_0 = 0$. In this case the following dispersion law is obtained:

$$\omega^2 = \omega_p^2 + c^2k^2 \quad (23)$$

This appears in a more familiar form if one defines a refractive index μ in the usual way, as the quotient of c and the phase velocity $V_p = \omega/k$. This is given by

$$\mu^2 = 1 - \omega_p^2/\omega^2 \quad (24)$$

Darwin,[24] who has devoted particularly searching investigations to this law, calls this Sellmeyer's formula. The meaning of ω_p as a "critical frequency" is evident; frequencies smaller than this are not propagated in the quiescent plasma. It may be noted that a uniform translation would not affect this formula; the propagation law for transversal waves is influenced only by random motion or by a magnetic field.

Longitudinal waves, $\boldsymbol{k} \parallel s_0$ In this case we obtain simply

$$\omega^2 = \omega_p^2 \quad (25)$$

i.e. the only possible frequency of longitudinal waves is the plasma frequency itself, with any value of k, i.e. with any wavelength. The group velocity is zero. This means that the energy of plasma waves is attached to the electron cloud, it is not propagated if the cloud is at rest, and if there is a drift the oscillations are carried along with the electron stream. This is as it must be, because the quiescent or uniformly translated electron stream evidently possesses no source of energy to cover losses.

Things become very different if one considers at least two partial electron streams in the same space, with

velocities V_1, V_2 and densities $D_1 = en_1$, $D_2 = en_2$. Considering now, for simplicity, all velocities V_1, V_2, s_1, s_2 as parallel to one direction, equation (16) immediately gives two equations for the two partial streams

$$\left.\begin{aligned}(\omega + V_1k)^2 s_1 &= \omega_1^2(s_1 + s_2)\\ (\omega + V_2k)^2 s_2 &= \omega_2^2(s_1 + s_2)\end{aligned}\right\} \tag{26}$$

where ω_1 and ω_2 are the two "partial" plasma frequencies

$$\omega_1^2 = 4\pi e D_1/m \qquad \omega_2^2 = 4\pi e D_2/m$$

The two homogeneous linear equations (26) are compatible only if their determinant vanishes, and this gives

$$\frac{\omega_1^2}{(\omega + V_1k)^2} + \frac{\omega_2^2}{(\omega + V_2k)^2} = 1 \tag{27}$$

This is the dispersion law for two parallel, interpenetrating, uniform electron streams, first given by Haeff,[6] and by Pierce.[7] It can be readily generalized for any number of parallel beams.

Haeff investigated this equation, and found that in certain, rather wide limits it possesses complex roots for the propagation constant k, corresponding to real values of ω; moreover, some of these roots stood for solutions which increased exponentially in the drift direction. He took this for an explanation of the experimentally-observed amplification of small signals, and in fact the agreement was remarkably good. But, as Twiss has pointed out recently,[25] this oversimplified explanation is open to serious objections. By the same argument one could show that there is a complex frequency corresponding to real k-s, that is to say solutions periodic in space, within quite wide limits of the wavelength, should be unstable and run away exponentially in time; an evident fallacy in view of the well-known stability of slip-stream tubes. The root of the error is, of course, that the dispersion relation of equation (27) does not in itself indicate what solutions are physically possible, as it contains no reference to the boundary conditions.

Comparatively little work has been done as yet on the small-amplitude theory of slipping streams, taking the all-important boundary conditions into account. Papers by Pierce,[26] MacFarlane and Hay,[27] and by Bunemann[28] may be mentioned as important exceptions, but much remains to be done. It is evident that without rather special boundary conditions, especially at the start and at the termination of the beams, no electromagnetic energy could ever be extracted from these longitudinal oscillations. They have the important general characteristic that they produce no magnetic field, hence no Poynting vector and no electromagnetic output. There is merely a cyclic conversion of kinetic and electrical energy taking place in them, which, under homogeneous conditions remains an internal affair of the beam or beams. But by special terminations it is possible to extract electromagnetic energy from the fluctuating space charge, in just the same way as in velocity modulation tubes.

The weaknesses of the present imperfect theories become particularly obvious when it is attempted to apply them to the problems of solar or stellar radio noise (for example, Haeff[6a]). In the solar or stellar atmospheres one can hardly postulate energy-extracting circuits similar to those used in microwave tubes; it must be explained how the longitudinal waves change their character in non-uniform regions so that an electromagnetic flow arises, at least part of which will ultimately escape into empty space. No homogeneous theory can do this, nor any theory which deals with longitudinal waves only. Bohm and Gross[11] have sketched out some possible mechanisms, but we do not possess as yet a single three-dimensional model of a disturbance in a stellar atmosphere which satisfies reasonable boundary conditions, and which could be shown to radiate.

It appears that in stellar atmospheres magnetic fields play at least an important, possibly an indispensable, part in the generation of oscillations. It is therefore of importance to recall once more that equations (15) and (16) are based on the assumption of irrotational streams, which is equivalent to the assumption that they have issued from a cathode where the magnetic field had no normal component. This condition is almost always fulfilled in electronic devices, also in travelling wave and slipping-stream tubes, as experience has shown that a magnetic field at the cathode gives rise to strong excess noise. But in the case of cosmic radio sources vortices of p cannot be excluded. Hence it is of importance to extend the theory of electron streams to the general case. This will be done elsewhere. For the present it will be sufficient to note that the dispersion equation for plasmas without thermal motion in the presence of a magnetic field has been treated by Appleton[29] in his magneto-ionic theory, also by Bailey and Martyn,[30] and by Huxley and Ratcliffe.[31] In a magnetic field H a new characteristic frequency appears in the dispersion equation, the "cyclotron frequency"

$$\omega_c = \frac{eH}{mc} \tag{28}$$

Another important difference is that now it is no longer possible to separate the oscillations into purely transversal and purely longitudinal waves; they are coupled by the magnetic field, and an electromagnetic output (a Poynting vector) can appear even under uniform conditions, as shown by Bailey.[32] A little more will be said of his theories in the next section.

OSCILLATIONS IN PLASMAS WITH RANDOM MOTION

In the previous section it was assumed that the velocity vector could assume only a limited number of directions and values at any point of space. If we drop this assumption, it appears natural to describe the state of the plasma, as in the kinetic theory of gases, by a number of

distribution functions, one for every kind of ions, so that

$$f_i(x, y, z, v_x, v_y, v_z)dxdydzdv_xdv_ydv_z$$

is the number of ions of the kind i in the space element $dx\,..$ and in the velocity range $dv_x\,..$ But it must be remembered that this is a *statistical* description, and if the oscillations have very high frequencies and/or very short wavelengths, we may not find in a small range of position and velocity, during a short time interval, a number of ions sufficiently large for a statistical description. But in fact the theory has not yet been extended to the point when this assumption could lead to noticeable errors.

Assume now only one kind of mobile charges, electrons, the ions will be considered stationary as before. The conservation equation for the electrons can now be written in the form

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\,.\,\mathrm{grad}_r f + \frac{e}{m}(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\,.\,\mathrm{grad}_v f = \left(\frac{\partial f}{\partial t}\right)_{coll.} \quad (29)$$

This is Boltzmann's *transfer equation*. grad_r stands for the gradient of f in configuration space, grad_v in velocity space. The equation is exact if one accepts the statistical description, because all possible errors appear in the "collision term" at the right. But the concept of collision, a clear and useful notion in the kinetic theory of gases, becomes a very doubtful one if extended to a gas composed of ions and electrons, which act on one another with Coulomb forces, falling off with the inverse square of distance only. The "collision cross-section" becomes logarithmically infinite if the dimensions of the electron gas go to infinity. The situation is not quite so bad in plasmas, where the ions screen off the field of the remoter electrons, but considerable difficulties persist. The Boltzmann transfer equation with collision term corresponding to Coulomb forces has been treated by Landau[19] and by Cahn,[33] who has confirmed the result previously mentioned that the interaction, as expressed by the collision term accepted by most theoreticians, is not sufficient to account for the rapid establishment of Maxwell distribution in most cases (Langmuir's paradox).

While the correct form of the collision term in equation (29) is still rather doubtful if the equation is to hold for relaxation processes, the situation is easier in the field of plasma oscillations, as there are some good reasons to assume that, for the high-frequency electron oscillations at least, the electron-electron and electron-ion collisions can be neglected. This was suggested in 1938 by A. Vlasov,[34] who first established the dispersion equation in plasmas with random motion on the basis of equation (29). But instead the simpler explanations of Bohm and Gross[11] will be followed here, all the more as they have revealed an important physical feature of this phenomenon which was overlooked by Vlasov.

Bohm and Gross limit themselves from the start to longitudinal oscillations. Let us consider such a wave, k, ω in a co-ordinate system running with the wave, with the phase velocity $V_p = \omega/k$, which is assumed large compared with the thermal velocities of the electrons. The field in the system running with the wave can be described by a static potential ϕ. Imagine this, as shown in Fig. 4, as a profile, with the electrons whose velocity in the laboratory system is v, running past it with a mean relative velocity

$$u_0 = v - V_p$$

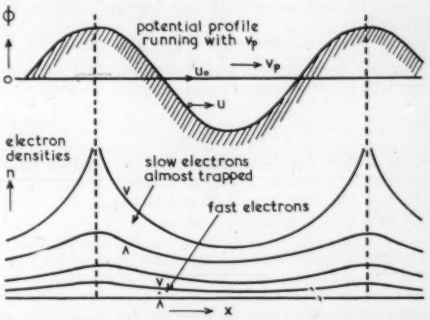

Fig. 4. Deriving the dispersion relation after Bohm and Gross (1949)

Only those will be able to run freely over the potential crest whose kinetic energy

$$\tfrac{1}{2}mu^2 = \tfrac{1}{2}mu_0^2 - e\phi \quad (30)$$

is still positive at the crest. The others will be *trapped*, and forced to run with the wave, while performing oscillations in the wave trough. It is one of the chief merits of Bohm and Gross to have pointed out the importance of the trapping process.

Considering now only long waves, with very large phase velocity, the trapped electrons can be neglected. One can now easily work out the contribution which each velocity group, $f(v)dv$, makes to the space charge. As shown in the lower part of Fig. 4, electrons will be slowed down near the crest; their contribution to the space charge is inversely proportional to their relative velocity u, which follows from the last equation. The total density is

$$n_- = n_0 \int_{-\infty}^{\infty} \frac{f(v)\mathrm{d}v}{(1 - 2e\phi/mu^2)^{\frac{1}{2}}} \quad (31)$$

Substituting here u from the last two equations, taking into account the ion density, equal to n_0, and applying Poisson's equation. The result is

$$4\pi e n_0 \left\{ 1 - \int_{-\infty}^{\infty} \frac{f(v)(v - V_p)\mathrm{d}v}{[(v - V_p)^2 - 2e\phi/m]^{\frac{1}{2}}} \right\} = -\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} \quad (32)$$

and if this is to be a sinusoidal wave, the right-hand side must be $k^2\phi$. Assuming the amplitude to be so small

that the slowing down of electrons at the crest is negligible compared with their large relative energy, of the order $\frac{1}{2}mV_p^2$, Bohm and Gross obtain the dispersion relation in the form of an integral equation

$$k^2 = \omega_p^2 \int_{-\infty}^{+\infty} \frac{f(v)\mathrm{d}v}{(v - V_p)^2} \tag{33}$$

which becomes a relation between ω and k if we replace V_p by ω/k. The same equation has been obtained also by Vlasov, for the case of small k-s. Equation (33) can be approximately solved under the assumption that the mean kinetic energy $\frac{1}{2}m\overline{v^2}$ is the same, $\frac{3}{2}kT$, as if there were no plasma-waves present. One then obtains the dispersion relation in the explicit form

$$\omega^2 = \omega_p^2 + \overline{v^2}k^2 = \omega_p^2 + \frac{3kT}{m}k^2 \tag{34}$$

This is the dispersion law of Bohm and Gross, and also of Vlasov for long waves. A similar result, but without the factor 3, had been already previously obtained by J. J. Thomson,[35] who started from Maxwell's macroscopic transfer equation. It will be better to distinguish the macroscopic transfer equation, which forms also the starting point of Bailey's work as a *transport* equation. The method of the transport equation is a somewhat coarse approximation to the transfer method; as Gross has shown recently[37] it can give errors more serious than wrong values for a constant. But it has the advantage, as demonstrated by Bailey's investigations,[32] that at least approximate results can be obtained in cases when the Boltzmann equation would lead to hopeless complications.

The dispersion relation (34) has the same form as equation (23) for transversal waves, with the difference that it is now the mean square velocity $\overline{v^2}$ of the electron gas which takes the place of c^2. It confirms the previous result that if there is no thermal motion there is only one frequency, ω_p, and the group velocity is *nil*. But if random motion exists, equation (34) gives the simple relation

$$V_p V_g = \overline{v^2} \tag{35}$$

By its derivation equation (34) is valid only for waves with large phase velocity and small k. Vlasov, who left trapped electrons out of account obtained a dispersion curve, as shown in Fig. 5, which agrees with the Bohm and Gross law only near the plasma frequency. Its meaning near the origin is not clear. This was pointed out also by Landau,[36] who drew attention to the fact that as the transfer equation is 6-dimensional, a definite dispersion law will be obtained only under certain restrictive conditions. Bohm and Gross have made it clear that this restriction is just equivalent to neglecting the trapped electrons.

What happens if they are not neglected, i.e. if it is assumed that there are velocity groups which run approximately with the wave? The answer is that in this case *just anything* can be obtained. This is not a fault of the theory, but an essential feature of the potentially rich field of plasma waves and oscillations. Almost any oscillation pattern can be obtained if we launch the wave, together with appropriate electron

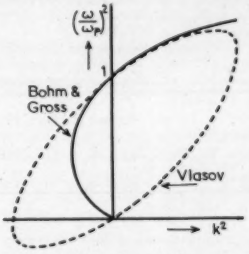

Fig. 5. The dispersion relation according to Bohm and Gross, and to Vlasov

groups, under the right initial conditions. Examples of this are given by Bohm and Gross in their first two papers. A third, more recent, paper[11] gives also at least a semi-quantitative explanation of the phenomena discovered by Merrill and Webb,[8] and this, though not completely satisfactory, can be considered as the utmost limit to which the theory of plasma oscillations has been able to make liaison with the experimental results.

CONCLUSION

The theory of irrotational electron streams without random motion rests on a sufficiently safe basis to account for most phenomena observed in discharge tubes without gas filling. Exceptions are probably magnetrons, in which the electrons spend sufficiently long transit times for at least partial disorder to be established. A satisfactory theory of vortex motion is also outstanding. But apart from these exceptions it can be said that this field of physics is now largely reduced to problems of mathematics.

The theory of plasmas with random motion rests on a less safe basis, because of the doubts which still surround the collision term. But even those branches of the theory which are more safely established have not yet been sufficiently developed mathematically. Perhaps the most urgent problem is that of a satisfactory model for radiating stellar atmospheres.

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ORIGINAL CONTRIBUTIONS

A non-destructive X-ray method for the determination of the thickness of surface layers

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A method is described for the non-destructive determination of the thickness of surface layers. It has been applied to the examination of layers produced by the abrasion of single crystals and of metallic films deposited on polycrystalline bases.

1. INTRODUCTION

Most existing techniques for the determination of the thickness of surface layers require two measurements, either before and after deposition *or* before and after removal of the layer (those measurements carried out without the layer are "blank experiments"). Methods free from these restrictions are generally limited to magnetic materials.[1] The X-ray technique described below can, in many cases, be applied without a blank experiment to measurements of layers on single crystals or polycrystalline bases. Results obtained in a study of abraded surface layers will be discussed elsewhere: this paper gives an account of the experimental technique.

2. THEORY OF METHOD

The intensity of reflexion of a parallel, monochromatic X-ray beam is measured as a function of the angle (ϕ) between the trace of the surface of the specimen and the trace of the reflecting planes in the plane of incidence. The principle is illustrated in Fig. 1 for a single crystal with a surface layer. Both the incident beam and the beam reflected by the matrix crystal are absorbed by the

material in the layer. The amount of absorption depends on the path length in the layer, i.e. on the angle ϕ. When ϕ equals the Bragg angle (θ) for the reflecting planes, the reflected beam is completely absorbed and the intensity falls to zero. It will be shown elsewhere that for a crystal with an absorbing layer

$$I/I_0 = \frac{K}{\mu(1+R)} \exp\{-\mu' t[\operatorname{cosec}(\theta+\phi) + \operatorname{cosec}(\theta-\phi)]\} \quad (1)$$

where K = volume reflexion coefficient of the matrix,
μ = linear absorption coefficient of the matrix,
μ' = linear absorption coefficient of the layer,
t = thickness of the layer,
R = width of reflected beam (W)/width of incident beam (W_0) = $\sin(\theta-\phi)/\sin(\theta+\phi)$,
I = intensity (energy per unit area) of the reflected beam,
I_0 = intensity of the incident beam.

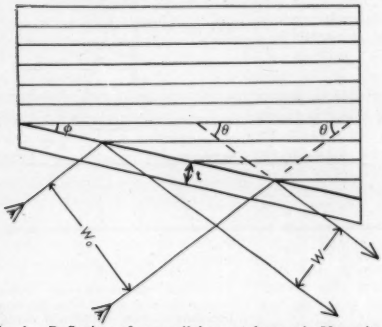

Fig. 1. Reflexion of a parallel monochromatic X-ray beam from a set of reflecting planes inclined to the surface of a crystal with a surface layer

This relation can be put in the form

$$\log_e[(1+R)I/I_0] = -\mu' t[\operatorname{cosec}(\theta+\phi) + \operatorname{cosec}(\theta-\phi)] + \log_e K/\mu \quad (1a)$$

The value of $\mu' t$ can be obtained by plotting the experimentally determined values of $\log_e[(1+R)I/I_0]$ against $[\operatorname{cosec}(\theta+\phi) + \operatorname{cosec}(\theta-\phi)]$; and, if μ' is known, the thickness of the layer can be determined. The same relation applies to surface layers on polycrystalline bases.

It appears from the experiments, however [see Section 7(*b*)], that a relation of type (1) applies not only to reflexions absorbed by material different from that of the matrix, but also to reflexions absorbed by any projections on the surface of the matrix. If the deposited layer is sufficiently thick, this latter effect will be negligible. For greatest accuracy, however, the effective value of $\mu' t$ for the matrix can be determined before deposition of the layer, or the effect of the projections may be made negligible by choice of a radiation which is only relatively slightly absorbed by the matrix. It may also be possible in certain cases to measure independently the effective value of $\mu' t$ due to projections by using two radiations, one with wavelength between the absorption edges of the substances of the matrix and layer, and the other with wavelength on the same side of both absorption edges.

3. THE SPECTROMETER

The spectrometer (Fig. 2) allows rotation of the specimen about horizontal and vertical axes.

A collimated beam of X-rays, *A*, falls on a deconcentrating monochromator, *B*,[2] adjusted by translation and rotation by means of screw *C* into the best reflecting position. The most uniform part of this beam (usually about 1 mm in width) is selected by slit *D*, and strikes the specimen *E*. *E* is mounted on the horizontal axis of the dial *F*; *F* is fixed to a screw motion slide *G*, which enables the surface of the specimen to be brought to the axis of the horizontal circle, *H*. The specimen holder fits into the slide, *I*, so that the specimen can be removed and replaced in the same position. *E*, *F*, *G* form an arm *J* of the circle, *H*, so that the specimen can be rotated about the vertical axis of *H*. Another arm, *K*, carries a Geiger counter, *L*, and quenching unit, *M*, a slit system, *N*, and a holder, *O*, for films or absorbing foils. The movements of the arms *K*, *J*, and of the dial *F* are vernier controlled, and can be read to 1 min of arc.

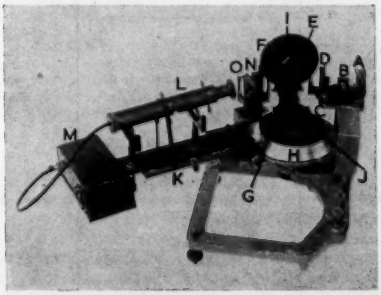

Fig. 2. The surface layer Geiger counter spectrometer

4. VARIATION OF ϕ

For a single crystal base, a variation of ϕ between limits $\pm\omega$ can be achieved by rotating the crystal (by means of dial *F*) about the normal to the set of reflecting planes inclined at an angle ω ($\sim\theta$) to the surface of the crystal.[2] For a polycrystalline specimen, ϕ may be varied for a given reflexion by rotating the specimen about the vertical axis (circle *H*), or about the horizontal axis (dial *F*), provided that this is inclined at an angle $\sim\theta$ to the normal to the surface of the specimen.*

* The latter method is preferable as it eliminates any possible variation of *K* due to preferred orientation [see footnote, Section 5(*b*).

5. CALIBRATION OF DIAL

The dial F is calibrated in terms of ϕ, and the intensity of reflexion is then measured for a number of values of ϕ.

(*a*) *Single crystal base.* For a given setting of the dial F, the corresponding value of ϕ can be obtained by measuring the widths of the incident and reflected beams and using the relation

$$R = W/W_0 = \sin(\theta - \phi)/\sin(\theta + \phi) \qquad (2)$$

The incident and reflected beams are photographed and the widths can be measured on a travelling microscope. For small values of R, however, the widths cannot be measured accurately, and an extrapolation is used. If the dial reading measured from the position of unit concentration ($R = 1$), is called ψ, it can be shown[2] that

$$\tan\phi = \sin\psi \tan\omega \qquad (3)$$

Combining (2) and (3),

$$\sin\psi = \frac{1 - R}{1 + R} \cdot \frac{\tan\theta}{\tan\omega} \qquad (4)$$

The straight line of $\sin\psi$ against $(1 - R)/(1 + R)$ is convenient for extrapolation. The position of unit concentration, which determines the zero value of ψ, is found from a graph of the dial angle against $(1 - R)/(1 + R)$, at the point where $R = 1$. An example of the two curves used in a calibration for a ground crystal of fluorite is shown in Fig. 3.

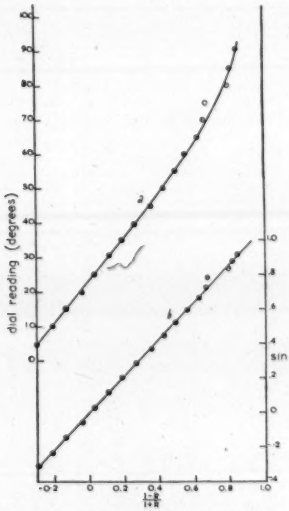

Fig. 3. Graphs used for calibration of dial readings in terms of R for a crystal of fluorite

(*a*) Dial readings against $(1 - R)/(1 + R)$ to find $\psi = 0$.
(*b*) $\sin\psi$ against $(1 - R)/(1 + R)$, for extrapolation.

(*b*) *Polycrystalline base.* For polycrystalline specimens, the widths of the reflected beams are more difficult to measure, because of long exposure times, the "umbrella" effect, and the possible large grain size of the specimen. It is more convenient to mount the specimen with axis of rotation at a known angle to the surface. In this way the angle ω is known and equation (3) can be applied directly, provided the zero position of ψ for unit concentration is known.

Parallel-sided specimens may be mounted on a wedge of known angle, for which the dial is calibrated. Other specimens may be mounted on wedges of unknown angle; the dial is calibrated for the resultant wedge by rotating the dial F and determining visually the minimum of the apparent surface area of the specimen viewed through the slits in front of the counter. This position of the dial corresponds to maximum concentration. At 90° from this position the angle ψ is zero, provided the axis of the dial F is placed so that it bisects the angle between the direction of the incident and reflected beams.* The angle ω is obtained by rotating the arm K carrying the slits about the vertical axis, until the apparent area is zero (Position I, Fig. 4).

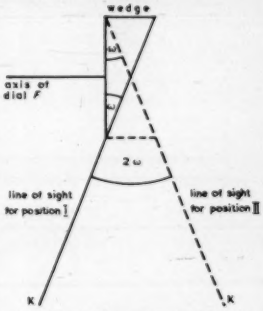

Fig. 4. Determination of ω

Horizontal section through wedge. Continuous and dotted lines represent positions I and II of the wedge respectively.

When the dial is rotated through 180°, a second similar position may be found (Position II, Fig. 4) for the arm K. The angle between the two positions of the counter arm K is equal to 2ω. In this simple manner, ω and the zero setting for ψ can be determined to within ~ 30 min of arc. This is sufficiently accurate for most purposes.

6. MEASUREMENT OF INTENSITY

(*a*) *Geiger counter method.* The total energy per unit height of beam (E) is measured for the reflected beams for various settings of the dial (i.e. of ϕ) and for the incident beam (E_0) after the removal of the specimen. I/I_0 is obtained from the relation $E/E_0 = I/I_0 \cdot R$. For these measurements, the slits controlling the height of the beam entering the counter are adjusted to give

* It should be noted that the axis of rotation must always be perpendicular to the reflecting planes if equation (4) is to be applied. For a polycrystalline specimen the reflecting planes of the particular grains contributing to the reflexion all have the same orientation in the specimen; hence the reflexion coefficient K is constant for all values of ϕ even though the specimen may have a preferred orientation.

a reasonable counting rate even for the weakest total intensities. The adjustment of these slits remains unchanged throughout the experiments. Several measurements are taken at each setting of ψ. A single crystal specimen is adjusted, for each reading, into the best reflecting position by means of the spectrometer arm, *J*.

For CuKα radiation, counters (15 cm long) filled with argon at atmospheric pressure and a few millimetres of alcohol were found to be very stable and efficient. The intensities are recorded on a counting rate meter similar to that described by Kip and others.[3] This instrument is calibrated by means of standard foils. During the intensity measurements the X-ray output is stabilized to better than 1% over short periods of time by manual variac control of the input voltage and tube current. Counting rates of about 2 500 counts/min were found convenient. The larger intensities were reduced by the insertion of standard foils. The error of each individual reading is about $\pm$ 1%.

(*b*) *Photographic method.* Alternatively, the reflected and incident beams can be recorded on films placed in the holder (*O*); these films are subsequently photometered.[2,4] The error of individual readings is about $\pm$ 2%.

7. APPLICATIONS

(*a*) *Abraded surface layer on single crystal.* A typical set of experimental results is shown in Fig. 5 for a crystal of lithium fluoride ground with fine carborundum at 22° 39′ to the (200) planes. The value of $\mu' t$ is found to be $0{\cdot}031 \pm 0{\cdot}001$. The interpretation of this and similar results will be fully discussed elsewhere.

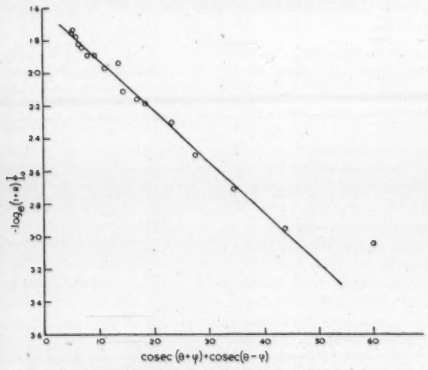

Fig. 5. Experimental points obtained from a ground crystal of lithium fluoride (CuKα radiation; $\theta = 22° 39'$). The slope of the straight line fitted to the points gives $\mu' t = \cdot 031 \pm \cdot 001$

(*b*) *Layer of aluminium deposited on polycrystalline copper slab.* The experimental points obtained for a specimen of copper covered with a layer of aluminium are shown in Fig. 6. The value of $\mu' t$ is equal to $0{\cdot}30 \pm 0{\cdot}01$. The value of $\mu' t$ due to the presence of projections was found to be $0{\cdot}047 \pm 0{\cdot}005$ (from a similar curve before deposition of aluminium, Fig 6), which is small compared with the value for the layer. Assuming the absorption coefficient to be equal to that of the fully dense metal, the thickness of the layer of aluminium is $19{\cdot}3 \pm 1{\cdot}2 \times 10^{-4}$ cm which is in agreement, within the experimental accuracy, with the value ($17{\cdot}5 \pm 1 \times 10^{-4}$ cm) determined by weighing beforehand.

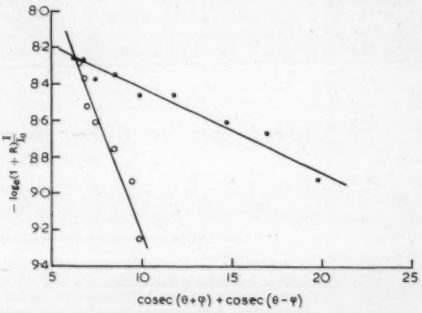

Fig. 6. Experimental points obtained from a specimen of copper with (○) and without (●) a layer of aluminium deposited on its surface (CuKα radiation; $\theta = 21° 45'$). The slopes of the straight lines fitted to the points give values of $\mu' t = 0{\cdot}30 \pm 0{\cdot}01$ and $\mu' t = 0{\cdot}047 \pm 0{\cdot}005$ respectively

8. DISCUSSION

This method is applicable to all layers with reflexion coefficients different from that of the matrix. The advantages of this method over most other X-ray methods are that it is non-destructive, can be performed without a blank experiment,[5,6] and is applicable to single crystal bases. The disadvantage of the method is the effect of the projections, which makes the determination of t from the experimental value of $\mu' t$ uncertain. In many cases, however, t may be large compared with the projections (see example (*b*) above); and if necessary, the value of $\mu' t$ due to the projections alone might be determined by measurements at two suitable wavelengths. Values of $\mu' t$ down to 0·004 have been measured accurately ($\pm$ 2%) and it is likely that this limit can be reduced to 0·002 or 0·001. For layers with high absorption coefficients (e.g. $\mu' \sim 200$), thicknesses down to 1 000 A might be determined in favourable cases. By suitable choice of radiation layers up to $\frac{1}{10}$ mm thick can be measured. Provided that the effect of the projections can be determined, the accuracy of the method compares well with that of other non-destructive techniques.[1]

Another non-destructive X-ray method, in which the

intensity of reflexion from a polycrystalline matrix is compared with that from the layer, has been described by Eisenstein.[7] This method suffers from the same disadvantage as that described above, and the lower limit of thickness which can be measured is unlikely to be less than 1 000 A.

9. ACKNOWLEDGMENTS

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A vacuum pre-treatment apparatus for filtering, dehydrating and degassing oil

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An apparatus for the vacuum pre-treatment of oils has been designed and constructed; the method of using it is described. An assessment has been made of the efficiency of the apparatus, which is capable of removing up to 1% added water in one or two treatments depending on the oil viscosity, with a consequent improvement in the electrical properties (d.c. resistivity and power factor) of the oils. Loss of volatile oil components has been shown to be very small, and oxidation of oil during treatment appears to be unlikely. The apparatus has been used to study the rates at which degasified oils of different types re-dissolve air. Under static conditions at 20° C, absorption in the initial stages is very rapid, becoming more gradual later. The rates of re-solution of air between one and six hours are in the inverse order to the viscosities of the four oils examined.

There are numerous applications where lubricating and insulating oil must be essentially free from moisture, insoluble solid materials and sometimes from dissolved or dispersed gases; this applies particularly to certain oils used to impregnate electrical insulating materials and to oils for refrigerators. Oil is always liable to become slightly contaminated during transportation and storage, although this may be of no practical significance for many applications. When a particularly high standard of purity is required, however, it is usual for the manufacturer of the equipment with which the oil is used to purify the oil specially for his purpose immediately before use. In the majority of cases, this pre-treatment consists of filtration and spraying into a vacuum chamber whilst the oil is hot. When oil is pre-treated in this way before use, it is often necessary to ascertain from tests on samples what the properties of the oil will be after treatment. Measurements of the properties of the untreated oil in such cases are of little practical value. It is therefore desirable to apply a standardized pre-treatment to laboratory samples before carrying out tests, the results of which would be affected by slight contamination of the sample. The laboratory treatment should simulate, as far as possible, the full-scale process employed. The principal effect upon the oil of such treatments[1] is to remove volatile contaminants such as moisture, organic solvents, etc., and insoluble solid materials. The dielectric loss is therefore lowered, the electric strength increased,* and, according to Berberich,[3] the curvature of the graph of the logarithm of the power factor against temperature is reduced; the curve tends to be more nearly a straight line. The presence of moisture gives rise to curves concave to the power factor axis.

An apparatus suitable for carrying out such pre-treatments has been designed and constructed after consultations with the research staff of Messrs. British Insulated Callender's Cables Ltd., Wood Lane, London. The apparatus described below is a modified version of that employed by this Company.

* The Electrical Research Association has shown[2] that the presence of cotton fibres in a dry oil decreases its electric strength by up to 30%, the reduction being about 20% in the case of press-board fibres. When fibres are added to oil containing moisture, the reduction may be as much as 90%. The effect therefore appears to be most serious when fibres and moisture are present together in an oil.

EXPERIMENTAL AND RESULTS

The principles involved in pre-treatment. It has been stated by Clark[4] that it is generally recognized that degasification demands careful technique if all the dissolved air is to be removed. The application of a vacuum on a volume of still oil de-aerates the oil only after a long time. A review of the various methods for measuring the solubility of gases in liquids, and of the factors influencing solubility is given by Markham and Kobe.[5] The available data[4] indicate that the solubility of air in mineral insulating oil changes with pressure in exact accordance with Henry's Law (i.e. the solubility is directly proportional at a given temperature to the partial pressure of the gas), at least at pressures up to one atmosphere. Other investigators[6,7] have also found that the relationship still applies, although more approximately, over a wider range of pressures. In order that the processes of degasification and dehydration shall be facilitated, it is necessary to expose the maximum possible surface area of hot oil to vacuum. In the present apparatus, these processes are combined with the filtration step which may be needed to remove insoluble contaminants from the oil. The pre-heated oil foams through a sintered glass filter, and the foam descends slowly through a heated spiral, being exposed to vacuum for the whole of this time.

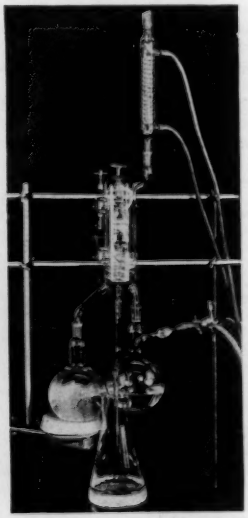

Fig. 1. An apparatus for the filtration and vacuum treatment of insulating oil samples. Complete assembly

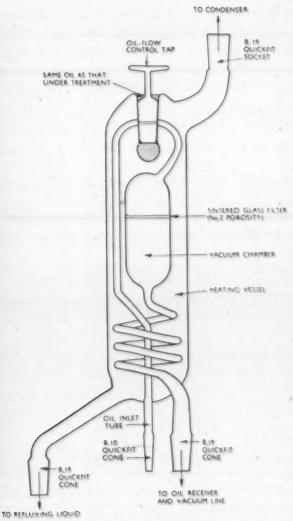

Fig. 2. An apparatus for the vacuum pre-treatment of oils to remove gases, moisture and insoluble contaminants

Description of the apparatus and method. The complete apparatus is shown in operation in Fig. 1, and a diagram of the treatment vessel is given in Fig. 2. The oil sample is drawn directly from the container up the oil inlet tube, and pre-heated prior to filtration during its passage through the heating vessel, which is merely a tube (of diameter about 3 in) through which the vapour of boiling carbon tetrachloride is passing. The temperature within this vessel therefore remains constant at about 77° C. The rate of flow of the oil to the sintered-glass filter is regulated by means of the control tap, which is lubricated and sealed by a small volume of the same oil as that under treatment (see Fig. 1). The sintered-glass filter is manufactured by Messrs. James A. Jobling and Co. Ltd. (Pyrex brand) and is of their No. 2 porosity, average pore diameter 40–50 μ. It serves not only to remove insoluble contaminants which may be present but also to break up the oil into foam. This foam gradually breaks down as it falls through the heated spiral. The rate of flow of oil is adjusted so that all (or very nearly all) the bubbles of air have collapsed by the time the oil reaches the receiving flask, which is of 1 000 ml capacity. By experience, it has been found that the most suitable rate of flow is about 250 to 1 000 ml of oil/hour, depending on the oil being processed. The more viscous oils require the slower rates of flow for complete degasification and dehydration. (In this connexion, it may be noted that Gemant[8] has found that the solubility of air in mineral oils decreases with increasing viscosity of the oil.) The apparatus has been found to be suitable for the treatment of oils within the viscosity range 45–900 sec (Redwood I/140° F). A vacuum pump, capable of reducing the pressure to 0·1 to 1·0 mm of mercury, is connected via a three-way tap, to a nozzle on the receiving flask. The three-way tap is necessary for releasing the vacuum. If this tap is opened whilst there is a layer of oil above the filter, the pressure above the filter is restored to atmospheric. If and when it becomes necessary to pre-treat a further quantity of the same oil, a reduction in the pressure in the vacuum chamber causes the oil above the filter to be forced through with unusual speed. In these circumstances the degasification and dehydration of the oil may not be complete, and to avoid this difficulty it is recommended that care should be taken to release the vacuum only when there is no oil on the filter. This can be arranged by closing the flow-regulating tap a short time before the vacuum is released.

A flask, heated by a hot-plate, and containing boiling carbon tetrachloride, is connected to the lower end of the

heating vessel, whilst at the upper end (top right in Fig. 1) is connected a reflux condenser. The heat supplied by the hot-plate should be so regulated that the reflux zone is in the lower portion of this condenser. Other solvents have been used for pre-treatments at other temperatures, but in general it is believed that carbon tetrachloride is satisfactory.

The apparatus can normally be cleaned satisfactorily with a warm solvent such as trichlorethylene, which should also be forced under gentle pressure in a reverse direction through the sintered filter in the event of this becoming clogged. If more drastic treatment is required, the apparatus may be cleaned with chromic acid.

The apparatus in use, and inspected at the Research Laboratories of British Insulated Callender's Cables Ltd., involved the use of a large, thermostatically-controlled oven in place of the small vapour jacket described above. It was understood that they were constructing a new apparatus, in which the oven would be replaced by a glass heating vessel, lagged externally, and surrounded by resistance wires for supplying the necessary heat. It was considered that the arrangement described above would be less unwieldy, and more reliable in use. Furthermore, the number of ground-glass joints is reduced to a minimum. Also there is no possibility of "hot spots" or localized regions of high temperature, as there might be in the case of direct electrical heating.

Pre-treatment of oils. Three oils, *A*, *B* and *C* (see Table), were intentionally contaminated with distilled water, which was dispersed throughout the samples by mechanical shaking for a period of one hour. The oils as sampled, and after contamination, were then subjected to the pre-treatment described above. In one case it was found necessary, after electrical measurements had been made, to repeat the process. The d.c. resistivity at 500 V (measurement made after the application of direct voltage for 1 min) and power factor at 1 000 V 50 c/s of each oil were measured in a cell the electrode spacing of which was 1·5 mm, and crackle tests (British Standard Specification No. 148, appendix L) were carried out, before and after the addition of water, and after pre-treatment.

The results are summarized in Table 1, and indicate that the two oils *A* and *B* contained some insoluble and/or volatile contaminants which were removed by a single treatment. After intentional contamination with water, the electrical properties were found to be extremely poor. One vacuum treatment was, however, sufficient to cause a substantial improvement in the electrical properties. The lower viscosity oil *A* was found to be slightly better in d.c. resistivity and power factor than it was before contamination, although it appeared to break down on the application of 1 000 V (50 c/s) to the test cell. The higher viscosity oil *B*, after vacuum treatment, was still inferior to the same oil before contamination. This may be explained, however, by the fact that suspended droplets of water were deliberately left in the oil as drawn through the apparatus, to assess the efficiency of dehydration. It is possible that if the contaminated oil had been allowed to stand until the globules of water settled out, and had then been subjected to treatment, the process would have eliminated all the dissolved and dispersed water. A second vacuum treatment was, in fact, necessary to raise the quality of the oil to above that of the uncontaminated oil.

Oil *C*, of viscosity 98 sec Redwood I/140° F was a highly refined naphthenic product and had very good electrical properties. It was contaminated as above with 1% water and subjected to vacuum pre-treatment in

Effect of vacuum pre-treatment on the electrical properties of various oils

Oil	*Condition of oil*	*Power factor (tan δ) at 60° C and 50 c/s*	*D.c. resistivity (ρ) at 60° C (10^{12} Ωcm)*	*Crackle test*
A. Refined naphthenic oil. Viscosity, Redwood I/140° F 389 sec	1. As sampled	0·0024	6	—
	2. After vacuum pre-treatment	0·0016*	10	—
	3. As sampled, but contaminated with 0·4% water	Too high to measure	ca 0·0001 (at 6 V)	—
	4. As (3) above, but after vacuum pre-treatment	ca 0·002†	6·7	—
B. Refined naphthenic oil. Viscosity, Redwood I/140° F 907 sec	1. As sampled	>0·02††	ca 2††	Positive
	2. After vacuum pre-treatment	0·0011**	16	Negative
	3. As sampled, but contaminated with 1% water	—	—	—
	4. As (3), but after one vacuum pre-treatment	Too high to measure	ca 0·1	Positive
	5. As (3), but after two vacuum pre-treatments	0·00095	18	Negative
C. Highly refined naphthenic oil. Viscosity, Redwood I/140° F 98 sec	1. As sampled	<0·00005	>50 000	—
	2. After addition of 1% water, and vacuum pre-treatment	<0·00005	ca 50 000	—

* The reading slowly increased from 0·0015 to 0·0017.
† The oil broke down when tested at 1 000 volts.
†† The readings were very unsteady.
** The reading slowly increased to this value.

order to ascertain whether the high electrical quality of the oil itself could be fully restored, and also to investigate the severity of the treatment with regard to the oxidation stability of the product. The results given in Table 1 indicate that the water was effectively removed, and that the conditions of the treatment were not such as to cause a deterioration in electrical properties due to oxidation.

The possibility that the more volatile oil components might be lost during vacuum pre-treatment was investigated by subjecting a transformer oil, type B30 class (*D*, viscosity 45 sec Redwood I/140° F), to the treatment. A trap immersed in solid carbon dioxide/acetone was inserted in the vacuum line, and after one cycle of the oil, this trap was found to contain only a very small amount (less than 0·05% based on the amount of oil undergoing treatment) of a cloudy liquid, which undoubtedly contained water as well as oil condensate. It may be safely assumed that more viscous oils would show even smaller losses due to volatility.

Rates of solution of air in degasified oils. Since, in the above experiments, the study of the effects of water-elimination was the main object of the investigation, no precautions were taken to prevent air from re-dissolving in the oils prior to the electrical measurements. The effects of dissolved air upon the power factor and resistivity of the oils were assumed to be of little importance, since the temperature and time factors were not considered to be such as to permit noticeable oxidation of the oils. In this connexion, Whitehead[2] has stated that it is difficult to say what direct effect dissolved gases have upon electrical breakdown.

For some purposes, it may be necessary to investigate the properties of a dehydrated oil in a completely degasified condition, and it is therefore important to know to what extent, if any, contact with the atmosphere is permissible during the transfer of the oil from the receiver vessel to another apparatus, or during measurements. Depending on the rates of re-solution of air by the oil, and on the effect of small quantities of dissolved air upon the oil properties, it may be necessary to conduct such transfer operations, etc., *in vacuo*.

There is little published evidence concerning the rates at which degasified oils of different types re-dissolve air. It is, however, likely that the composition of the air dissolved by oil differs from that of the atmosphere, since the solubility of oxygen in oil is greater than that of nitrogen at normal temperatures. In the present context, therefore, the word "air" refers to the combined volumes of oxygen and nitrogen, not necessarily in the proportions in which they are found in the atmosphere. The apparatus described above was therefore used in a brief study of this problem. The four oils *A*–*D* referred to above were subjected to treatment in the apparatus, the receiver vessel of which was modified in such a way that it could be removed from the apparatus whilst the degasified oil sample was held under vacuum. A two-way tap was also incorporated, which permitted venting to the atmosphere, and subsequent connexion to a combined gas burette-manometer system containing dibutyl phthalate. After removal from the treatment apparatus, the oil receiver and gas-measuring system were immersed in a thermostat in which the temperature was controlled to 20° ± 0·1° C. A similar assembly, containing the same volume (100 m*l*, of surface area approximately 70 cm²) of oil sample, under air at atmosphere pressure, was placed in the same thermostat to act as a control for correcting the volume changes due to minor variations in atmospheric pressure and temperature during the course of the measurements.

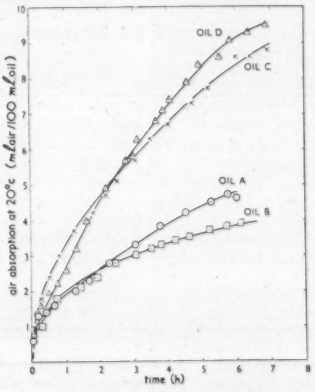

Fig. 3. Absorption of air by degasified oils

It was found that the initial rates of re-solution of air in the static, degasified oil samples were very rapid, and were followed by slower and more measurable rates. It was impossible with the present apparatus to carry out measurements of rates of air absorption during the 10–15 sec following venting to the atmosphere, as the actual absorption of air by the samples was confused by (1) volume fluctuations due to the pressure equilibration on venting to the atmosphere, and (2) the change in temperature, and hence volume, of the ambient air (at 19–22° C) as it entered the thermostatted parts of the apparatus. "Zero-time" was therefore taken at about 15 sec after venting, when the first accurate volume measurements were made. The results obtained for the four oils are plotted in Fig. 3; it should be noted that, since measurements during the initial stages of the experiments were not possible, it is probable that the curves plotted in Fig. 3 are in error with respect to their displacement from the time axis. Nevertheless, it is clear that the four oils differ appreciably in the rates at which they re-dissolve air during the periods of measurement. It is noteworthy that oil *D* (the least viscous of the four oils) is approaching saturation only after a period of 7 h. The air solubility in this case is, therefore, of the order

of 10% vol./vol., in good agreement with literature values[8] varying from 7·3 to 10·6% for a series of transformer and other oils. The actual rates of absorption, which are free from the displacement error referred to above, are plotted as a function of time in Fig. 4. The rapidity of the initial absorption, even during the first hour of the experiment, is clearly demonstrated. The rates of absorption change more or less abruptly

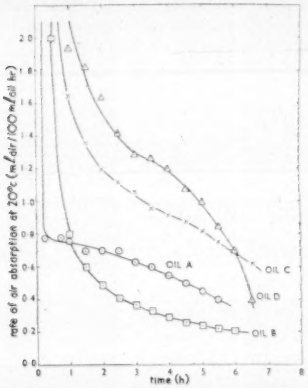

Fig. 4. Rates of air absorption as a function of time

after this initial period, and a gradual decrease becomes apparent in each case. The rate curve for oil *A* crosses that for oil *B* at about 1 h, and the curve for oil *C* crosses that for oil *D* at about 6 h. In the intermediate period where the rates were most accurately measurable, the rates are in the order of increasing magnitude *B*, *A*, *C* and *D*. Reference to the properties of the oils, as given in Table 1, etc., shows that the viscosities (Redwood I/140° F) are in the reverse order of magnitude.

These results may be interpreted qualitatively in the following way. The initial rapid uptake of air is regarded as the saturation of the surface layers of oil under static conditions in the receiver vessel. This saturation takes place at widely different rates, as can be seen by the striking variation of the rates of re-solution during the first two hours (Fig. 4). The subsequent slower absorption rate which was measured over a period of several hours corresponds to the slow diffusion of air downwards from the saturated zone below what may be termed the "diffusion threshold." This diffusion process would be expected to be governed largely by the viscosity of the oil.

DISCUSSION

It seems evident from the investigation of the rates of re-solution of air that the measurements of the electrical properties of the four oils were carried out when the oils contained appreciable amounts of re-dissolved air. This was of little consequence in the present work where another more important factor, viz. the effect of water, was under investigation. However, if the properties of an oil are likely to be affected by the presence of small amounts of dissolved oxygen and nitrogen, it may be important to take rigorous precautions to prevent any contact with the atmosphere after the oil has been degasified.

Depending on the importance of small amounts of dissolved gases, some contact may, of course, be permissible. It can be seen from Fig. 3 that the volume of air re-dissolved after 1 h is about 20–30% of that re-dissolved after 7 h under static conditions, although the possibility of a "displacement error" in these figures should be borne in mind. From the results in Fig. 4 it is clear that the rates of re-solution are highest immediately after the degasified oils are exposed to the atmosphere, and that under static conditions the least viscous oil requires up to ca 7 h before it reaches the state where it is saturated with air.

Besides the main uses outlined above, it is clear that the vacuum pre-treatment apparatus has other possible applications. Interest has been increasing recently amongst a number of transformer manufacturers in connection with the use of degasified transformer oil in high voltage transformers, and it is considered that it may be possible to use the present apparatus to investigate the effects of degasification on the properties of transformer oils.

ACKNOWLEDGMENTS

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Differential methods of leak detection

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An experimental investigation of the differential method of detecting leaks in high vacuum systems has shown that it is possible to attain a sensitivity 20 times greater than that of the corresponding single gauge method. The differential pressure change may be produced either by differential condensation of gases such as butane or by chemical absorption of carbon dioxide. With the latter method, using extremely simple equipment, a sensitivity of 10^{-3} $l\mu$/sec may be obtained. With differential condensation and ionization gauges the sensitivity is only a factor 10 less than that of the mass spectrometer leak detector.

1. Introduction

The great increase in the number, complexity, and size of high vacuum plant in the past decade has led to the expenditure of considerable effort in the development of rapid and systematic methods for detecting leaks. Methods most firmly established at the present time can be divided into two groups: those in which capital cost is of minor importance compared with simplicity and certainty of operation, and those in which capital cost is the vital consideration. Mass spectrometer methods[1] are undoubtedly the most satisfactory where their employment can be justified economically. In this paper, however, it is proposed to consider the methods in the second class, and to show how an appreciation of the fundamental problems involved can lead to the development of simple and sensitive apparatus which is satisfactory for the majority of problems.

The methods to be considered are the so-called "dynamic methods" in which a gas or vapour is sprayed over the suspected part, a leak then being indicated by the presence of this "probe" material inside the vacuum. For such methods the essential requirements are first to ensure that every part of the suspected system is covered by the probe gas for an adequate time and, second, to be able to detect a small partial pressure of this gas in the presence of a large partial pressure of air or other residual gases or vapours. The sensitivity of all dynamic methods of leak detection is limited by real fluctuations of pressure inside the vacuum system and by inherent instabilities in the leak detector instrument. The chief advantage of the mass spectrometer is that its inherent instability is so small that a partial pressure change of 10^{-8} μ can be measured. Furthermore, since the instrument can be made unresponsive to the pressure (and therefore to the pressure fluctuations) of all other gases, this sensitivity can be obtained even when the total pressure is 10^5 times higher than that of the probe gas. It follows that any alternative method which has a limited response to system fluctuations whilst at the same time having a reasonable sensitivity to the probe gas will tend to have the same advantages as the mass spectrometer. Such a method is the so-called differential Pirani gauge which was first described by Jacobs and Zuhr.[2]

2. Principle of the differential method

For this method two identical gauges were attached to the same point in the vacuum system, one of the gauges having a liquid air trap. The probe gas was condensible, so that whenever a leak was covered, the trapped gauge experienced a diminution of pressure, the pressure in the compensating gauge remaining sensibly constant. There was thus a differential change in the indications of the two gauges which could be estimated by standard bridge methods. Fluctuations of the permanent gas background pressure, on the other hand, affected both gauges equally and therefore left the bridge balance unchanged. This system is represented schematically in Fig. 1, and the symbols used are defined in the appendix. S_a

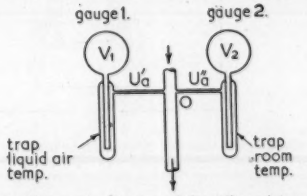

Fig. 1. Schematic diagram of the differential method

is the system pumping speed (for air) at the gauge and U'_a and U''_a the conductances of the gauge tubulations. K_a and K_p are the gauge calibration constants for air and probe gas respectively. If a leak occurs, the amount of air Q_a passing through in 1 sec is given by the Knudsen equation:[3]

$$Q_a = (A_a + B_a P'_a)P'_a \qquad (1)$$

where P'_a is the pressure of air in the vicinity of the leak (normally equal to atmospheric pressure) and A_a and B_a the appropriate constants for this small hole with an air through-put.

When the leak is covered with the probe gas the amount of probe gas passing through in 1 sec is given by:—

$$Q_p = (A_p + B_x P'_x)P'_p \qquad (2)$$

where A_p now refers to the probe gas, and $P'_p = xP'_a$ is

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the partial pressure of the probe gas above the leak (normally this will be equal to either atmospheric pressure or the vapour pressure of the probe gas, whichever is the smaller). The subscript $_x$ refers to the air-probe gas mixture. In the diffusion pump the corresponding flow equations are:—

$$Q_a = S_a P_a \tag{3}$$

$$Q_p = S_p P_p \tag{4}$$

Eliminating Q_a and Q_p from equations (1) to (4) we have

$$\frac{S_a P_a}{S_p P_p} = \frac{A_a + B_a P_a'}{A_p + B_x P_x'} \cdot \frac{P_a'}{P_p'} \tag{5}$$

This is a general equation from which can be computed the partial pressures P_a and P_p of the air and probe gas in the low pressure parts of the system.

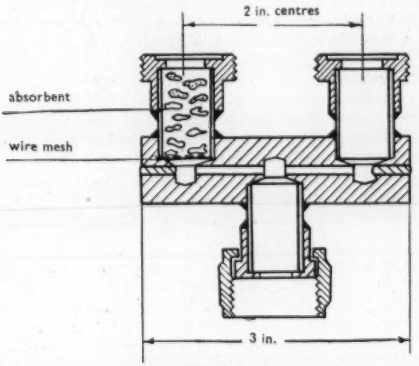

Fig. 2. Gauge mounting for differential absorption method of leak detection

Referring to Fig. 1, the pumping speed at the differential gauge will be S_a for air and, because of the introduction of the liquid air trap, $S_p \doteqdot \sqrt{\frac{M_a}{M_p}}(S_a + U_a')$ for the probe gas. If it is assumed that the flow through the leak is purely viscous then A_a and A_p may be neglected and therefore from equation (5)

$$\frac{P_a}{P_p} \doteqdot \frac{S_p}{S_a} \cdot \frac{B_a}{B_x} \cdot \frac{P_a'}{P_p'}$$

$$\doteqdot \frac{S_a + U_a'}{S_a} \cdot \frac{\eta_x}{\eta_a} \sqrt{\frac{M_a}{M_p}} \cdot \frac{P_a'}{P_p'} \tag{6}$$

If the ratio ϕ, the substitution sensitivity factor, is defined as:

$$\phi = \frac{\text{Apparent pressure change on substitution}}{\text{Air pressure before substitution}}$$

then, on the assumption that the partial pressure of the probe gas is zero in the trapped gauge we may write

$$\phi = \frac{K_p P_p}{K_a P_a} \tag{7}$$

From (6) and (7).

$$\phi = \frac{K_p}{K_a} \cdot \frac{\eta_a}{\eta_x} \cdot \left(\frac{M_p}{M_a}\right)^{\frac{1}{2}} \cdot \frac{P_p'}{P_a'} \cdot \frac{S_a}{S_a + U_a'} \tag{8}$$

The conditions for maximum response are clearly shown in equation 8. The vapour pressure of the probe should be greater than atmospheric (then $P_p' = P_a' =$ atmospheric pressure), this gas should have a high molecular weight and a low viscosity.

To achieve effective condensation in gauge I without also appreciably lowering the pressure in gauge II, U_a' must be not greater than S_a. (U_a' cannot be reduced indefinitely as the gauge time constant V_1/U_a' must be small.) The partial pressure of air must be the same in both gauges for both static and varying values of pressure at the point o, and therefore the gauge gas flow time constants, V_1/U_a' and V_2/U_a'' must be equal (see definitions in appendix).

Experiments in this laboratory have shown that condensation is by no means the only way in which a differential response to a probe gas can be effected. For example, the reaction between carbon dioxide (used as a probe) and calcium hydroxide (used instead of a cold trap) has several practical advantages.* Furthermore differential ionization gauges can sometimes be used instead of differential Pirani gauges. In this paper a description is given of the differential method, and the performance attainable by differential condensation or differential chemical absorption is compared with that of the corresponding single gauge method. Results for the mass spectrometer leak detector are also given.

3. EXPERIMENTAL PROCEDURE

(*a*) *Apparatus used.*—Ordinary commercial gauges were used throughout the work (metal-cased Pirani type by Metropolitan-Vickers Ltd., and type 29D2 ionization gauges, by The Edison Swan Electric Co. Ltd., although the precise form of the gauge is not important). Fig. 2 shows a gauge-head for the differential absorption method. This unit consists of a brass block bored out to hold the absorbent and to carry two gauges. With this arrangement it is easy to make the symmetrical gas flow conditions which satisfy the requirement $V_2/U_a'' = V_1/U_a'$. The bridge circuit used with the differential Pirani gauge is shown in Fig. 3. The supply voltage was adjusted so that the operating temperature of the gauges was approximately 100° C, a value which gave adequate sensitivity combined with simplicity of power supply. Preliminary work on the differential ionization gauge was carried out with commercial gauge control units with a common d.c. amplifier, in the

* British Patent No. 645526.

manner shown in Fig. 4. A control unit developed especially for this work, in which common power supplies and common reference potentials were used for the gauges, gave a somewhat better performance. A photoelectric amplifier with a high speed pen recorder was used to measure the amplitude of fluctuations.[4] A typical record is shown in Fig. 7. Laboratory experiments were carried out either on the fine side of a type O3 pump or in the backing space between the O3 and O2. ("Metrovac" 3 in and 2 in oil diffusion pumps.)

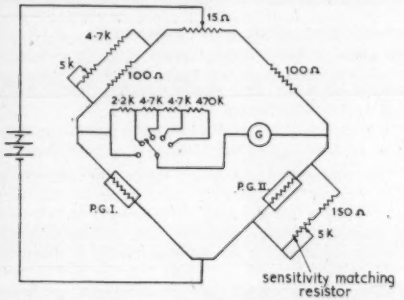

Fig. 3. Bridge circuit for the differential Pirani method

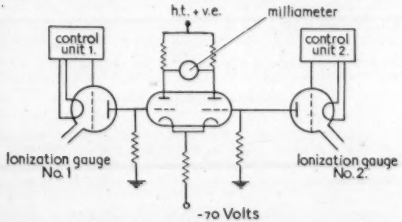

Fig. 4. Differential ionization gauge control: schematic diagram

The apparatus is shown in Fig. 5. A variable constriction was placed between the two pumps so that the pressure in the backing space could be varied.

(*b*) *Experimental results.* (1) *Residual fluctuations.*— By using an ionization gauge and operating the automatic recorder with a suppressed zero, it was shown that for 2 in, 3 in, and 8 in diffusion pumps the amplitude of the gas pressure fluctuations was of the order 2% of the mean gas pressure and that, so long as the pumps were operated in their normal range of through-put, the percentage fluctuations remained constant. From the evidence available it appears that pressure fluctuations are caused by changes in pumping speed and that this is a general characteristic of the oil diffusion pump. The effect of these fluctuations may be reduced by constricting the gauge tubulation, by using gauges which have an inherent time constant and by using the gauges connected differentially. The amount of reduction which can be made in a practical case is shown in Fig. 6,

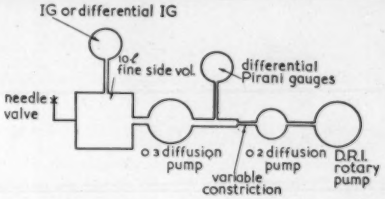

Fig. 5. Pumping plant used in leak detection experiments

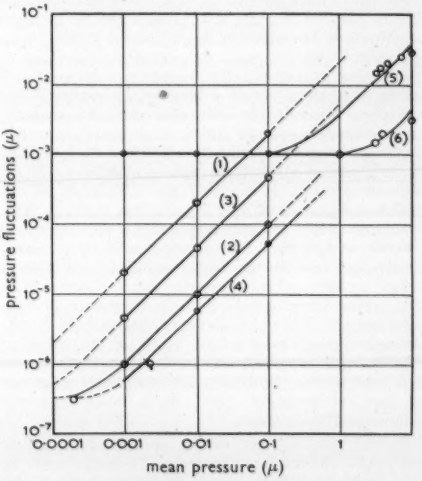

Fig. 6. Fluctuations of single and differential gauges as a function of mean pressure

(1) Single ionization gauge
(2) Differential ionization gauge } short time constant.
(3) Single ionization gauge
(4) Differential ionization gauge } 5 second time constants.
(5) Single Pirani gauge.
(6) Differential Pirani gauge.

where the mean apparent fluctuations of air pressure/min are plotted against the mean pressure. The measurements were made on the plant shown in Fig. 5. The figures for the ionization gauge were obtained on the fine side of the O3 pump, and those for the Pirani gauge on the pipe connecting the O2 and O3. Curves (1) and (2) refer to gauges with a time constant of the order of

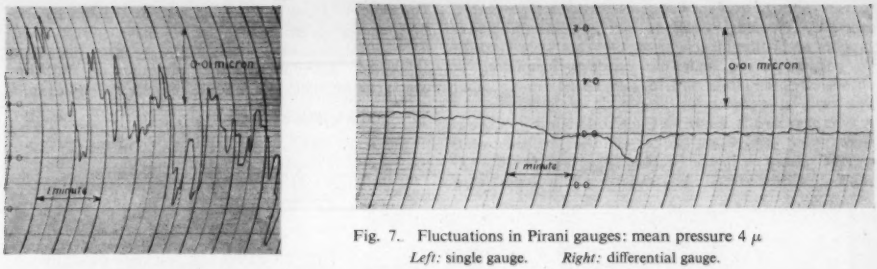

Fig. 7. Fluctuations in Pirani gauges: mean pressure 4 μ
Left: single gauge. *Right:* differential gauge.

0·1 sec. To obtain curves (3) and (4) the gauge tubulations were constricted so as to increase the time-constants to 5 sec. The curves (5) and (6) are for the single and differential Pirani gauges with time constants of approximately 2 sec. Curves (1) and (3) show that the apparent amplitude of the pressure fluctuations is reduced by a factor of 4 when the gauge time-constant is increased to 5 sec. Curve (2) shows that the reduction is by a factor of 20 when short period differential ionization gauges are used. By using the differential method in conjunction with gauges having 5 sec time-constants the apparent pressure fluctuation is reduced to 2·5% of the real value, i.e. to 1 part in 2 000 of the mean gas pressure. As the ultimate pressure is reached, the advantages of the differential method become less and less marked. This is because at very low pressures a limit is imposed by the inherent instabilities of the gauges and their control circuits, whereas at the higher pressures the limit is caused by real changes in pressure coupled with the fact that perfect gauge matching was not achieved in these experiments. Since the inherent fluctuations in the ionization gauge were found to be of the order of 2×10^{-7} μ as compared with 10^{-3} μ in the Pirani gauge, it is evident that the differential ionizations gauge can be used at pressures 5 000 times lower than the differential Pirani system.

(2) *Reaction between carbon dioxide and calcium hydroxide.*—When the carbon dioxide passes over the leak there is a rapid change of bridge balance, the initial condition being quickly restored when the carbon dioxide is removed. (The rate of change appears to be determined entirely by the time constants of the gauges and flow system.) If, however, the carbon dioxide is held over the leak for a prolonged period, a saturation effect takes place such that after about 20 min the initial deflexion is reduced to zero. However, so long as the carbon dioxide is not kept over the leak for more than 2 or 3 min (as it never should be in practice) no saturation is observed. Calcium hydroxide does not appear to deteriorate when stored in direct contact with the atmosphere, but the absorbing power drops by 50% if it is kept under vacuum for about 6 weeks. This is probably due to the change from hydroxide to oxide by removal of water, and means that for routine work pellets should be changed once a month.

3. SUBSTITUTION SENSITIVITY FACTOR

The substitution sensitivity factor (as defined in section 2) has been measured for the different detectors by opening a needle valve to produce a definite air pressure at the gauge, then covering the leak with the probe gas and noting the apparent change in pressure. Confirmatory measurements have been made with real leaks in welded joints. (For the differential methods, the air pressure before substitution must be determined with one gauge only.) The values for ϕ are given in the table. Since the gauge is always placed where the flow past it is diffusive, the gas flow contribution to ϕ is the same for all designs of pumping plant, and the values given in this table are applicable quite generally.

Substitution sensitivity factors

	ϕ			
	Pirani gauge		*Ionization gauge*	
probe gas	*single*	*differential*	*single*	*differential*
butane	1·0	2·0	10·0	11
ether meth.	0·7	1·7	5·0	6
carbon dioxide	0·3	1·3	1·0	2
hydrogen	0·4	0	−0·4	0

Butane (C_4H_{10}), which is available commercially as "Calor gas," has been found to be the most effective of the condensible vapours. It does, however, produce explosive mixtures in the atmosphere unless adequate ventilation is provided. The surprisingly high substitution sensitivity factor for the ionization gauge with butane can be explained in terms of its low viscosity, high molecular weight, and large ionizing cross-section (see Section 2).

4. PRACTICAL DESIGN OF LEAK DETECTION PLANT

As has already been indicated the requirements for high sensitivity in any leak-detection plant are (1) that the ratio of mean gas pressure to apparent pressure fluctuations at the detector should be high, (2) that the overall time constant of the system should be not greater

than 5 sec, and (3) that the substitution sensitivity factor should be high. These conditions are readily satisfied by the mass spectrometer leak detector but less easily by the differential methods. For small leaks, a fine side method using differential ionization gauges and a diffusion pump with a butterfly valve capable of reducing the pump speed until $v/s = 5$ sec is relatively straightforward. This method is preferable in cases where the majority of leaks are small and operators capable of using ionization gauges are available. For the more common case, where the size of leak is not previously known and the operator is relatively unskilled, Pirani gauges are to be preferred. As Figs. 6 and 8 show, it is impossible to attain a high sensitivity [$L/\Delta L = P_a/\Delta P_s$ (appendix)] with this gauge unless the pressure can be increased to about 1 μ. To attain this pressure with small leaks, the speed of the pump must be reduced considerably and the testing of vessels of moderate size can then be accomplished best by using the backing space technique. For this technique, which is shown schematically in Fig. 5, the vessel under test is evacuated by a fast diffusion pump, and the time constant for transference of leakage gas to the backing space can therefore be negligibly small. By keeping the volume of the backing space and gauges to about 0·4 *l* it becomes possible to limit the conductance of the backing line to 0·1 *l*/sec without making the time constant greater than 5 sec. Thus, for the plant shown, a compression ratio of 250 has been achieved and a total leak rate of $0{\cdot}1 \times 1$, i.e. 0·1 $l\mu$sec, will produce the desired 1 μ pressure at the gauge. Since the differential Pirani gauge has residual fluctuations of 10^{-3} μ, a probe gas giving unity substitution sensitivity factor would ensure the detection of leaks as small as 10^{-4} $l\mu$sec, i.e. one-thousandth of the total. The same gauge, used on the fine side, to test a 100 *l* vessel would not give this performance unless the total leak were 25$l\mu$ and the minimum detectable value would then be 250×10^{-4} $l\mu$sec. Based on these considerations small plants consisting of a type O3B and an O2 diffusion pump in cascade with a D.R.1 type rotary pump have been built for routine testing purposes. Differential absorption with Pirani gauges is used, the gauge being placed between the two diffusion pumps. A variable constriction allows the pumping speed at the gauges to be set at either 0·1 or 0·5 *l*/sec. A flap valve on the fine side of the O3B and an auxiliary pumping line permit the test pieces to be evacuated without the necessity of cooling the diffusion pumps, thus speeding up the cycle of operation. The hood technique is used for proof testing (i.e. the test vessel is covered with a hood which can be completely filled with carbon dioxide). A fine jet of carbon dioxide is used for the actual location of leaks. With two of these plants one process worker, in a normal 44 h week, can proof test and locate all leaks greater than 10^{-3} $l\mu$sec in about 100 component parts of vacuum systems.

To illustrate the utility of the differential technique in a case where the normal laboratory technician's method (benzene rag and Geissler tube) had resulted in two weeks of wasted effort the following notes are given: "A 200 *l* welded steel tank with many side flanged joints was evacuated by a type O3B diffusion pump and a D.R. 1 type rotary pump. There was no provision for leak detection and a differential Pirani gauge with a liquid air trap was simply mounted on the phosphorous pentoxide trap of the rotary pump. Using Calor gas as the probe 5 leaks were found in less than 2 h after setting up the equipment. The smallest leak (0·01 $l\mu$sec) was found before the largest (10 $l\mu$sec)."

5. COMPARISON OF DIFFERENT METHODS

Using the definition of minimum detectable leak given in the appendix, the graphs of Fig. 8 have been calculated from sensitivity tests carried out in this laboratory. In each test the detecting element was attached at the most advantageous position, the gas flow time constants and pressures were adjusted for maximum sensitivity, and the leak was completely covered with the probe gas. The calculated values of minimum detectable leak are for vessels not greater than 100 *l* volume evacuated by types O3B and O2 pumps in series. For less ideal

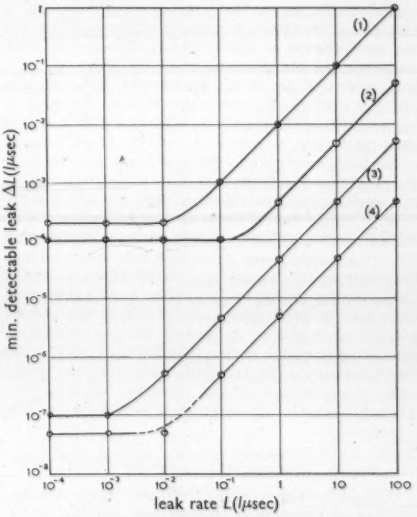

Fig. 8. Comparative sensitivities of different methods of leak detection under optimum gas flow conditions

(1) Single Pirani gauge: hydrogen probe.
(2) Differential Pirani gauge: CO_2 absorption.
(3) Differential ionization gauge: condensation of butane.
(4) Hot cathode mass spectrometer: helium probe.

conditions such as would be encountered in ordinary practice, it is desirable to apply a factor of safety of ten to each curve in Fig. 8. With volumes greater than 100 *l* larger pumps would be required so that the results for the smaller plant may not apply. For example, the increased backing space volumes and the thermal decomposition of the oil will not normally allow the conductance at the detector to be reduced to 0·1 *l*/sec.

6. CONCLUSIONS

The advantages of the differential method from the point of view of capital cost, simplicity of manufacture, operation and maintenance hardly need emphasis. For small total leak rates the performance to be obtained, using differential ionization gauges and condensation of butane, is not very much inferior to that obtainable with mass spectrometer leak detectors of the type described by Nier.[5] The principal conclusion drawn from this work, however, is that for general laboratory technique and routine vacuum testing the differential Pirani method with CO_2 absorption is a most practical and certain method. It has adequate sensitivity for almost all dynamic vacuum systems, and has none of the objections inherent in the use of liquid and gaseous organic probe gases, nor does it require the use of liquid air. However, it must be emphasized that, for the measurement of small pressure changes, the basic sensitivity of the differential Pirani gauge is small compared with the mass spectrometer leak detector, and that it relies on pressure amplification for much of its sensitivity. Consequently, unless the flow system is designed so that it is possible to build up the pressure at the gauge to something between 1 and 10 μ, whilst at the same time keeping the gas flow time constant down to something less than 10 sec, a good deal of the sensitivity of the differential Pirani method will be lost. With the differential ionization gauge pressure amplification is not so necessary, and a fine side technique is usually satisfactory. In vacuum test plants it is a relatively simple matter to use by-pass valves and variable constrictions: in field work, however, it is not usually possible to choose or to control the flow conditions because no provision for so doing has been made in the original design. The standardization of plant design with the requirements of leak detection borne in mind thus remains one of the outstanding problems in vacuum engineering.

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APPENDIX

Definitions used in specifying sensitivity of leak detection methods

1. *Units* pressures in microns.
   leak rates in $l\mu$sec. (Litre-micron/second).
   volumes in litres.
   speeds and conductances in litres/second.
2. P_a Mean air pressure at the point being considered.
3. ΔP_a Fluctuations of mean air pressure at the same point.
4. ΔP_{app} Apparent fluctuations in gauge resulting from real fluctuations ΔP_a at gauge inlet point. ΔP_{app} is equal to the *mean* rate of drift of the pressure indicator/min, or the maximum drift in any period of 10 sec, whichever is the greater. The values are expressed as apparent *air* pressure changes for single gauges, or as differential *air* pressure changes for differential gauges.
5. P_s Apparent change of pressure when a leak producing air pressure P_a at the gauge is covered with probe gas.
6. ϕ *Substitution sensitivity factor.* As discussed in Section 2 this is the ratio of apparent pressure change upon substitution/air pressure before substitution.

   i.e. $\phi = P_s/P_a$
7. ΔP_s *Minimum detectable substitution change.* This is the minimum detectable value of P_s and is given by

   $$\Delta P_s = \Delta P_{app}/\phi$$
8. ΔL *Minimum detectable leak.* This is simply the product of the net speed of the constriction on the high pressure side of the gauge and the minimum detectable substitution change, i.e.

   $$\Delta L = S_a \Delta P_s = \frac{S_a \Delta P_{app}}{\phi}$$

   S_a = conductance (for air) at the detector.
9. L = Actual leak rate (real and apparent).
10. $L/\Delta L$ = Ratio total leak to minimum detectable leak

    $$= \frac{P_a}{\Delta P_{app}\phi} = \frac{P_a}{\Delta P_s}$$
11. *Overall time constant.* The time required for the indicator to reach $(1 - 1/e)$ of its final reading after applying probe gas to the leak.

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A problem in radial heat flow

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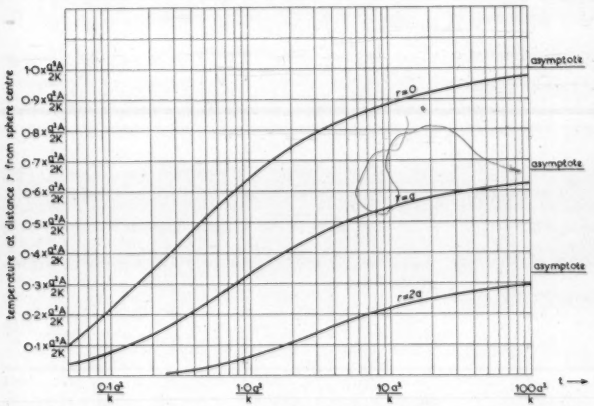
The heat equation is solved for the case of constant heat/unit volume/unit time, generated in a spherical region of a homogeneous infinite medium. Curves are drawn showing the transient temperature: (1) at the centre of the spherical region; (2) at the surface of the spherical region; and (3) at a distance from the centre of twice the radius. The transient temperature at any point of the medium may readily be calculated from the expressions derived, as, apart from material constants, these expressions involve only tabulated functions.

The application of h.f. fields to complicated substances gives rise to differential heating since the components usually possess different electrical and thermal characteristics. This effect may be used for the disinfestation and sterilization of biological materials, but the question has arisen as to whether this phenomenon always explains the magnitude of the observed effects. Indeed, often it seems unlikely on physical grounds. A mathematical approach does not appear to have been attempted hitherto apart from a discussion by Wenk,[1] who reasons that the theory of point heating as suggested by Haase and Schliephake,[2] and others, to explain certain biological effects is untenable because of the very large ratio of surface to volume of the organisms involved. An estimate of the limits imposed by purely physical considerations may be obtained, however, by the solution of the appropriate heat equations, with certain reasonable assumptions, and a knowledge of the relevant material constants. Such information as is available[3] indicates that living organisms, cells, bacteria, etc., are enclosed in a very thin skin which has a relatively low electrical conductivity but a high specific capacitance; thus surface heating seems unlikely to be important.

The present paper derives the solution of the heat equation when constant heat/unit volume/unit time is generated in a spherical region of a homogeneous infinite medium. As the resultant temperature differences are small, the effects of radiation and convection are negligible, and therefore have not been taken into account. In the figure curves showing the resultant transient temperature (1) at the centre of the spherical region; (2) at the surface of the spherical region; (3) at a distance from the centre of twice the radius—are given. The transient temperature at any point of the medium may readily be calculated from expressions derived here, as, apart from the material constants, these expressions involve only tabulated functions. The more general problem of different constant rates of heating generated in the spherical region and in the remainder of a homogeneous infinite medium may then be obtained by the superposition of the effect of a constant rate of heating throughout the medium upon the results obtained in the present paper. A particular case of the more general problem gives a first approximation to the practical case of a bacterium suspended in a liquid. A point to be noted is that although mathematically there is no restriction to the size of the spherical region, the application of the results is not justified in practice unless the size of the spherical region is much greater than that of the particle size of the surrounding medium. The relationships between the field strength, frequency, electrical properties and heat production for a range of conditions have been studied by Thomas.[3] His paper provides the



Differential heating of sphere in infinite medium. Curves giving transient temperatures at distance r from sphere centre when $r = 0$, a and $2a$ (a = radius of sphere)

K = thermal conductivity } (sphere and medium possess equal
k = thermal diffusivity } K and k).
A = excess rate of heat production in sphere over that in medium (constant).
t = time from start of heating.

data required to enable the general solution given below to be applied to practical cases.

It will be observed that the method of solution adopted in the present paper is that of the Laplace transformation. This is considered by the author to be preferable to extending the solution of the corresponding problem for an instantaneous spherical heat source, as given by Taylor-Jones, Morgan and Wheeler.[4] The method of treatment here given illustrates the ease and generality with which the theory of Laplace transformation may be used in the solution of partial differential equations subject to boundary conditions. It is true that Carslaw and Jaeger[5] have obtained a partial solution of the problem under consideration, but they have not carried their work as far as that of the present paper.

NOTATION

a radius of spherical region within which heat is produced at a constant rate.

r distance of any point from the centre of the spherical region above.

A constant rate of heating/unit time/unit volume.

K thermal conductivity.

k diffusivity.

u $\dfrac{1}{\sqrt{k}}$

γ $\dfrac{a^2}{k}(= u^2a^2)$.

ρ density.

c specific heat.

Suffixes 1 and 2: suffix 1 denotes point within region $0 \leqslant r < a$; suffix 2 denotes point within region $r > a$.

t time, measured from the moment of application of constant rate of heating.

$V_1(t)$ temperature at a point r within the region $0 \leqslant r < a$, as a function of t and r.

$V_2(t)$ temperature at a point r within the region $r > a$, as a function of t and r.

$\mathfrak{L}\{\ \}$, $\mathfrak{L}^{-1}\{\ \}$ Laplace transform notation.

$$\mathfrak{L}\{V(t)\} = \bar{V}(p) = \int_0^\infty e^{-pt}V(t)dt$$

$$\mathfrak{L}^{-1}\{\bar{V}(p)\} = V(t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{pt}\bar{V}(p)dp$$

$\bar{V}(p)$ is the Laplace transform of $V(t)$ with respect to t.

$V(t)$ is the inverse Laplace transform of $\bar{V}(p)$.

$\bar{V}_1(p)$ $\mathfrak{L}\{V_1(t)\}$

$\bar{V}_2(p)$ $\mathfrak{L}\{V_2(t)\}$

α_1 defined by $\sqrt{\alpha_1} = 1 - \dfrac{r}{a}$

α_2 defined by $\sqrt{\alpha_2} = 1 + \dfrac{r}{a}$

β_1 defined by $\sqrt{\beta_1} = \dfrac{r}{a} + 1$

β_2 defined by $\sqrt{\beta_2} = \dfrac{r}{a} - 1$

$\operatorname{erf} x$ $\dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^x e^{-u^2}du$

$\operatorname{erfc} x$ $\dfrac{2}{\sqrt{\pi}}\displaystyle\int_x^\infty e^{-u^2}du = 1 - \operatorname{erf} x$

$O(1/x)$ order of $1/x$ for large values of x.

$r \to +a$ r tends to a from values of r greater than a.

$r \to -a$ r tends to a from values of r less than a.

i $\sqrt{-1}$

$h_1, h_2, \phi, \eta_1, \eta_2$ functions of time defined in the text.

MATHEMATICAL SOLUTION

Mathematically the problem may be specified as follows:—

Heat is produced for time $t > 0$ at the constant rate A/unit time/unit volume in the spherical region $0 \leqslant r < a$ of an infinite medium whose initial temperature is zero. It is required to find the temperature at all points of the medium for time $t > 0$.

Let V, ρ, c, K and k denote the temperature, density, specific heat, thermal conductivity and diffusivity respectively, and let $u = \sqrt{(\rho c/K)} = 1/\sqrt{k}$. Further let the suffix 1 refer to the region $0 \leqslant r < a$ of the infinite medium and the suffix 2 to the region $r > a$ of the infinite medium.

We need the solution of the following boundary value problem:—

$$\rho_1 c_1 \frac{\partial V_1}{\partial t} = K\left[\frac{\partial^2 V_1}{\partial r^2} + \frac{2}{r}\frac{\partial V_1}{\partial r}\right] + A \qquad \left.\begin{matrix} 0 \leqslant r < a \\ t > 0 \end{matrix}\right\} \quad (1)$$

$$\rho_2 c_2 \frac{\partial V_2}{\partial t} = K\left[\frac{\partial^2 V_2}{\partial r^2} + \frac{2}{r}\frac{\partial V_2}{\partial r}\right] \qquad \left.\begin{matrix} r > a \\ t > 0 \end{matrix}\right\} \quad (2)$$

$$V_1 = 0,\ V_2 = 0 \text{ when } t = 0 \quad (3)$$

$$\left.\begin{matrix} V_1 = V_2 \\ \dfrac{\partial V_1}{\partial r} = \dfrac{\partial V_2}{\partial r} \end{matrix}\right\} \text{ when } r = a \quad (4)$$

$$V_1 \text{ finite as } r \to 0 \text{ and } V_2 \text{ finite as } r \to \infty \quad (5)$$

Equations (1) and (2) may more conveniently be written in the form—

$$\frac{\partial^2 V_1}{\partial r^2} + \frac{2}{r}\frac{\partial V_1}{\partial r} - u^2\frac{\partial V_1}{\partial t} = -\frac{A}{K} \qquad \left.\begin{matrix} 0 \leqslant r < a \\ t > 0 \end{matrix}\right\} \quad (1a)$$

$$\frac{\partial^2 V_2}{\partial r^2} + \frac{2}{r}\frac{\partial V_2}{\partial r} - u^2\frac{\partial V_2}{\partial t} = 0 \qquad \left.\begin{matrix} r > a \\ t > 0 \end{matrix}\right\} \quad (2a)$$

Denote $$\bar{V}(p) = \int_0^\infty e^{-pt}V(t)dt = \mathfrak{L}\{V(t)\}$$

$$V(t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{pt}\bar{V}(p)dp = \mathfrak{L}^{-1}\{\bar{V}(p)\}$$

The subsidiary equations are—

$$\frac{\partial^2 \bar{V}_1}{\partial r^2} + \frac{2}{r}\frac{\partial \bar{V}_1}{\partial r} - u^2 p \bar{V}_1 = -\frac{A}{Kp} \qquad 0 < r < a \quad (6)$$

$$\frac{\partial^2 \bar{V}_2}{\partial r^2} + \frac{2}{r}\frac{\partial \bar{V}_2}{\partial r} - u^2 p \bar{V}_2 = 0 \qquad r > a \quad (7)$$

$$\left.\begin{aligned} \bar{V}_1 &= \bar{V}_2 \\ \frac{\partial \bar{V}_1}{\partial r} &= \frac{\partial \bar{V}_2}{\partial r} \end{aligned}\right\} \text{ when } r = a \quad (8)$$

$\bar{V}_1$ finite as $r \to 0$ and $\bar{V}_2$ finite as $r \to \infty$ (9)

Equations (6) and (7) may be written in the form—

$$\frac{\partial^2}{\partial r^2}(r\bar{V}_1) - u^2 p (r\bar{V}_1) = \frac{-rA}{Kp} \qquad 0 < r < a \quad (6a)$$

$$\frac{\partial^2}{\partial r^2}(r\bar{V}_2) - u^2 p (r\bar{V}_2) = 0 \qquad r > a \quad (7a)$$

The solution of (6*a*) with $\bar{V}_1$ finite as $r \to 0$ is—

$$r\bar{V}_1 = C_1 \sinh u\sqrt{(p)}r + (rA/Ku^2p^2) \quad (10)$$

The solution of (7*a*) with $\bar{V}_2$ finite as $r \to \infty$ is—

$$r\bar{V}_2 = C_2 e^{-u\sqrt{(p)}r} \quad (11)$$

C_1 and C_2 are found from the application of the boundary conditions of equation (8) to equations (10) and (11). They are—

$$C_1 = \frac{-aA}{Ku^2}\frac{u^4a^4}{(ua\sqrt{p})^5}(1 + ua\sqrt{p})e^{-ua\sqrt{p}}$$

$$C_2 = \frac{-aA}{Ku^2}u^4a^4\frac{[\sinh ua\sqrt{(p)} - ua\sqrt{p}\cosh ua\sqrt{p}]}{(ua\sqrt{p})^5}$$

We therefore have—

$$\bar{V}_1 = \frac{A}{Ku^2p^2} - \frac{aAu^4a^4}{rKu^2}\frac{(1 + ua\sqrt{p})}{(ua\sqrt{p})^5}e^{-ua\sqrt{p}}\sinh\left(\frac{r}{a}ua\sqrt{p}\right) \quad (10a)$$

$$\bar{V}_2 = \frac{-a}{r}\frac{A}{Ku^2}u^4a^4\frac{[\sinh ua\sqrt{(p)} - ua\sqrt{p}\cosh ua\sqrt{p}]}{(ua\sqrt{p})^5}e^{-\frac{r}{a}ua\sqrt{p}} \quad (11a)$$

As $\mathfrak{L}^{-1}\left\{\frac{1}{p^2}\right\} = t$, it follows that—

$$V_1(t) = \frac{At}{Ku^2} - \frac{\gamma^2}{2}\frac{a}{r}\frac{A}{Ku^2}h_1(t) \quad (10b)$$

$$V_2(t) = \frac{-\gamma^2}{2}\frac{a}{r}\frac{A}{Ku^2}h_2(t) \quad (11b)$$

where

$$h_1(t) = \mathfrak{L}^{-1}\left\{\frac{2[1 + \sqrt{(\gamma p)}]}{(\gamma p)^{5/2}}e^{-\sqrt{(\gamma p)}}\sinh\frac{r}{a}\sqrt{(\gamma p)}\right\} \quad (10c)$$

$$h_2(t) = \mathfrak{L}^{-1}\left\{\frac{2[\sinh\sqrt{(\gamma p)} - \sqrt{(\gamma p)}\cosh\sqrt{(\gamma p)}]}{(\gamma p)^{5/2}}e^{-\frac{r}{a}\sqrt{(\gamma p)}}\right\} \quad (11c)$$

$\gamma = u^2a^2$

It is well known[6] that if

$$\mathfrak{L}\{\phi(t)\} = \Phi(p) \text{ then } \mathfrak{L}\{\gamma\phi(\gamma t)\} = \Phi(p/\gamma) \quad (12)$$

where γ is any real positive number. The application of this theorem to equations (10*c*) and (11*c*) gives—

$$\gamma h_1(\gamma t) = \mathfrak{L}^{-1}\left\{\frac{2(1 + \sqrt{p})}{p^{5/2}}e^{-\sqrt{p}}\sinh\frac{r}{a}\sqrt{p}\right\} \quad (13)$$

$$\gamma h_2(\gamma t) = \mathfrak{L}^{-1}\left\{2(\sinh\sqrt{p} - \sqrt{p}\cosh\sqrt{p})\frac{e^{-\frac{r}{a}\sqrt{p}}}{p^{5/2}}\right\} \quad (14)$$

Now $\mathfrak{L}^{-1}\left\{\frac{2(1 + \sqrt{p})}{p^{5/2}}e^{-\sqrt{p}}\sinh\frac{r}{a}\sqrt{p}\right\}$

$$= \mathfrak{L}^{-1}\left\{\left(\frac{1}{p^{5/2}} + \frac{1}{p^2}\right)(e^{-\sqrt{(\alpha_1 p)}} - e^{-\sqrt{(\alpha_2 p)}})\right\}$$

$$= \phi(\alpha_1, t) - \phi(\alpha_2, t) \quad (15)$$

where

$$\sqrt{(\alpha_1)} = 1 - r/a$$

$$\sqrt{(\alpha_2)} = 1 + r/a$$

$$\phi(\alpha, t) = \mathfrak{L}^{-1}\left\{\left(\frac{1}{p^{5/2}} + \frac{1}{p^2}\right)e^{-\sqrt{(\alpha p)}}\right\}$$

It follows from equations (10*b*), (13) and (15) that—

$$V_1(t) = \frac{At}{Ku^2} - \frac{1}{2}\frac{a}{r}\frac{a^2A}{K}[\phi(\alpha_1, t/\gamma) - \phi(\alpha_2, t/\gamma)] \quad (16)$$

It is known[7] that—

$$\mathfrak{L}^{-1}\left\{\frac{e^{-\sqrt{(\alpha p)}}}{p^2}\right\} = \left(t + \frac{\alpha}{2}\right)\operatorname{erfc}\sqrt{\left(\frac{\alpha}{4t}\right)} - \sqrt{\left(\frac{\alpha t}{\pi}\right)}e^{-\alpha/4t} \quad (17a)$$

$$\mathfrak{L}^{-1}\left\{\frac{e^{-\sqrt{(\alpha p)}}}{p}\right\} = \operatorname{erfc}\sqrt{\left(\frac{\alpha}{4t}\right)} \quad (17b)$$

$$\mathfrak{L}^{-1}\left\{\frac{e^{-\sqrt{(\alpha p)}}}{p^{3/2}}\right\} = \sqrt{\left(\frac{4t}{\pi}\right)}e^{-\alpha/4t} - \sqrt{\alpha}\operatorname{erfc}\sqrt{\left(\frac{\alpha}{4t}\right)} \quad (17c)$$

where α is real and $\geqslant 0$ and $\operatorname{erfc} x = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$

Further[8] if—

$$\mathfrak{L}^{-1}\{\Phi(p)\} = \phi(t)$$

then $$\mathfrak{L}^{-1}\left\{\frac{\Phi(p)}{p}\right\} = \int_0^t \phi(\tau)d\tau \quad (18)$$

The direct application of this theorem to (17*c*) gives—

$$\mathfrak{L}^{-1}\left\{\frac{e^{-\sqrt{(\alpha p)}}}{p^{5/2}}\right\} = \int_0^t\left[\sqrt{\left(\frac{4\tau}{\pi}\right)}e^{-\alpha/4\tau} - \sqrt{\alpha}\operatorname{erfc}\sqrt{\left(\frac{\alpha}{4\tau}\right)}\right]d\tau \quad (19)$$

The application of the theorem of equation (18) to equations (17*b*) and (17*a*) gives immediately—

$$\int_0^t \operatorname{erfc}\sqrt{(\alpha/4\tau)}d\tau = (t + \alpha/2)\operatorname{erfc}\sqrt{(\alpha/4t)} - \sqrt{(\alpha t/\pi)}e^{-\alpha/4t}$$

Also

$$\frac{2}{\sqrt{\pi}}\int_0^t \sqrt{(\tau)}e^{-\alpha/4\tau}d\tau = \frac{\alpha^{3/2}}{2\sqrt{\pi}}\int_{\sqrt{(\alpha/4t)}}^{\infty}\frac{e^{-x^2}}{x^4}dx$$

$$= \sqrt{\alpha}\left[\frac{\alpha}{3}\operatorname{erfc}\sqrt{(\alpha/4t)} + \frac{4t}{3\sqrt{\pi}}\sqrt{(t/\alpha)}e^{-\alpha/4t} - \frac{2\alpha}{3\sqrt{\pi}}\sqrt{(t/\alpha)}e^{-\alpha/4t}\right]$$

after two integrations by parts.

The substitution of the above expressions for the integrals on the right-hand side of equation (19) gives

$$\mathfrak{L}^{-1}\left\{\frac{e^{-\sqrt{(\alpha p)}}}{p^{5/2}}\right\} = \sqrt{\alpha}\left[\frac{4t}{3\sqrt{\pi}}\sqrt{(t/\alpha)}e^{-\alpha/4t} + \frac{1}{3}\sqrt{\left(\frac{\alpha t}{\pi}\right)}e^{-\alpha/4t} - \left(t+\frac{\alpha}{6}\right)\operatorname{erfc}\sqrt{\left(\frac{\alpha}{4t}\right)}\right] \quad (19a)$$

The addition of equations (17a) and (19a) gives $\phi(\alpha, t)$, defined by equation (15). This is—

$$\phi(\alpha, t) = \frac{4t^{3/2}}{3\sqrt{\pi}}e^{-\alpha/4t} + (\alpha/3 - \sqrt{\alpha})\sqrt{(t/\pi)}e^{-\alpha/4t} + [t + \alpha/2 - \sqrt{\alpha}(t+\alpha/6)]\operatorname{erfc}\sqrt{(\alpha/4t)} \quad (20)$$

The temperature at any point within the region $0 \leqslant r < a$ for time $t > 0$ is then given by equation (16) with $\phi(\alpha, t)$ defined as above.

In order to find the temperature at $r = 0$, it is more convenient to allow $r \to 0$ in equation (10a) rather than in the general expression derived above for the temperature within the region $0 \leqslant r < a$. This gives—

$$(V_1)_{r=0} = \frac{A}{Ku^2p^2} - \frac{A}{Ku^2}\left(\frac{1}{p^2} + \frac{ua}{p^{3/2}}\right)e^{-ua\sqrt{p}}$$

As $\mathfrak{L}^{-1}\left\{\frac{1}{p^2}\right\} = t$, and $\gamma = u^2a^2$, it follows from equations (17a) and (17c) that—

$$(V_1)_{r=0} = \frac{a^2A}{2K} \times \left[1 + (2t/\gamma - 1)\operatorname{erf}\sqrt{(\gamma/4t)} - \frac{1}{\sqrt{\pi}}\sqrt{(4t/\gamma)}e^{-\gamma/4t}\right] \quad (21)$$

where $\operatorname{erf} x = 1 - \operatorname{erfc} x = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

The steady state temperature at $r = 0$ may be found by allowing $t \to \infty$ in the above expression for $(V_1)_{r=0}$. For large values of t—

$$\frac{2t}{\gamma}\operatorname{erf}\sqrt{(\gamma/4t)} - \frac{1}{\sqrt{\pi}}\sqrt{(4t/\gamma)}e^{-\gamma/4t} \text{ is } O\left(\frac{1}{\sqrt{t}}\right)$$

and therefore tends to zero as $t \to \infty$. The steady state temperature at $r = 0$ is therefore given by—

$$\underset{t\to\infty}{Lt}\,(V_1)_{r=0} = a^2A/2K \quad (22)$$

The graph of $(V_1)_{r=0}$ against t is given in the figure. An expression for $(V_1)_{r=0}$ has previously been given by Carslaw and Jaeger.[5] (An error exists in the final results as printed.)

The temperature at $r = a$ for time $t > 0$ may be obtained from equation (16) by letting $r \to a$. This gives $\alpha_1 = 0$ and $\alpha_2 = 4$, and equations (16) and (20) then become—

$$(V_1)_{r=a} = (V_2)_{r=a} = \frac{a^2A}{2K}\left[t/\gamma - \frac{4}{3\sqrt{\pi}}(t/\gamma)^{3/2} + \frac{2}{3\sqrt{\pi}}e^{-\gamma/t} \times \left[2(t/\gamma)^{3/2} - \sqrt{(t/\gamma)}\right] + (\tfrac{2}{3} - t/\gamma)\operatorname{erfc}\sqrt{(\gamma/t)}\right] \quad (23)$$

The expansion of $(V_1)_{r=a}$ in descending powers of t gives—

$$(V_1)_{r=a} = \frac{a^2A}{3K} + O\left(\frac{1}{\sqrt{t}}\right)$$

and the steady state temperature at $r = a$ is therefore—

$$\underset{t\to\infty}{Lt}\,(V_1)_{r=a} = \frac{a^2A}{3K} \quad (24)$$

The graph of $(V_1)_{r=a}$ against t is given in the figure.

The steady state temperature at any point within the region $0 \leqslant r < a$ is obtained by first expanding $\phi(\alpha, t)$ in descending powers of t giving—

$$\phi(\alpha, t) = \frac{4t^{3/2}}{3\sqrt{\pi}} + t(1 - \sqrt{\alpha}) - [2\sqrt{(\alpha)} - \alpha]\sqrt{(t/\pi)} + \left(\alpha/2 - \frac{\alpha\sqrt{\alpha}}{6}\right) + O\left(\frac{1}{\sqrt{t}}\right) \quad (25)$$

then substituting for $\phi(\alpha, t/\gamma)$ in equation (16) giving

$$V_1(t) = \frac{a}{r}\frac{a^2A}{2K}\left[\left(\frac{\alpha_2}{2} - \frac{\alpha_2\sqrt{(\alpha_2)}}{6}\right) - \left(\frac{\alpha_1}{2} - \frac{\alpha_1\sqrt{(\alpha_1)}}{6}\right)\right] + O\left(\frac{1}{\sqrt{t}}\right) = \frac{a^2A}{2K}\left(1 - \frac{1}{3}\frac{r^2}{a^2}\right) + O\left(\frac{1}{\sqrt{t}}\right)$$

and finally taking the limiting value of $V_1(t)$ as $t \to \infty$. The resultant expression for the steady state temperature at any point within the region $0 \leqslant r < a$ is therefore—

$$\underset{t\to\infty}{Lt}\,V_1(t) = \frac{a^2A}{2K}\left(1 - \frac{1}{3}\frac{r^2}{a^2}\right) \quad (26)$$

This reduces to equations (22) and (24) as $r \to 0$ and $r \to a$ respectively.

We now proceed with the general solution for the region $r > a$, which is given by equations (11b) and (14).

$$\gamma h_2(\gamma t) = \mathfrak{L}^{-1}\left\{\frac{2(\sinh\sqrt{p} - \sqrt{p}\cosh\sqrt{p})e^{-\frac{r}{a}\sqrt{p}}}{p^{5/2}}\right\} = \mathfrak{L}^{-1}\left\{\left(\frac{1}{p^{5/2}} - \frac{1}{p^2}\right)e^{-\sqrt{(\beta_1 p)}} - \left(\frac{1}{p^{5/2}} + \frac{1}{p^2}\right)e^{-\sqrt{(\beta_2 p)}}\right\}$$

where $\sqrt{(\beta_1)} = \frac{r}{a} - 1$ and $\sqrt{(\beta_2)} = \frac{r}{a} + 1$

Denote

$$\eta_1(\beta, t) = \mathfrak{L}^{-1}\left\{\frac{e^{-\sqrt{(\beta p)}}}{p^{5/2}}\right\}; \quad \eta_2(\beta, t) = \mathfrak{L}^{-1}\left\{\frac{e^{-\sqrt{(\beta p)}}}{p^2}\right\}$$

Then $\eta_1(\beta, t)$ and $\eta_2(\beta, t)$ are given directly by equations (19*a*) and (17*a*), and we have—

$$V_2(t) = -\frac{\gamma^2}{2}\frac{a}{r}\frac{A}{Ku^2}h_2(t)$$

$$= -\frac{a}{2r}\frac{a^2A}{K}[\eta_1(\beta_1, t/\gamma) - \eta_2(\beta_1, t/\gamma) - \phi(\beta_2, t/\gamma)] \tag{27}$$

with $\phi(\alpha, t)$ defined as in equation (15).

Equation (27) thus gives the temperature for all time $t > 0$ for all points in the region $r > a$.

For large values of time t—

$$\eta_1(\beta, t) = \frac{4t^{3/2}}{3\sqrt{\pi}} - \sqrt{(\beta)}t + \beta\sqrt{t}/\sqrt{\pi} - \frac{\beta\sqrt{\beta}}{6} + O\left(\frac{1}{\sqrt{t}}\right)$$

$$\eta_2(\beta, t) = t - 2\sqrt{(\beta/\pi)}\sqrt{t} + \beta/2 + O\left(\frac{1}{\sqrt{t}}\right)$$

while $\phi(\beta, t)$ is given by equation (25). The substitution of these expressions in equation (27) gives—

$$V_2(t) = \frac{a}{3r}\frac{a^2A}{K} + O\left(\frac{1}{\sqrt{t}}\right)$$

The steady state temperature for the region $r > a$ is therefore given by—

$$\underset{t\to\infty}{Lt}\ V_2(t) = \frac{a^3A}{3rK} \tag{28}$$

This reduces to equation (24) as $r \to a$ and also tends to zero as $r \to \infty$. The graph of $(V_2)_{r=2a}$ against t is given in the figure.

ACKNOWLEDGMENTS

The author wishes to express his thanks to Mr. A. Morris Thomas, of the Electrical Research Association, for suggesting the present problem and for useful discussion with regard to it, and to the Director of the Electrical Research Association for permission to publish this paper.

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Correspondence

A method of studying the diffusion of waxes through rubber

The diffusion of wax through vulcanized natural rubber has been used by technologists for some considerable time to give protection against attack by normal atmospheric agencies, e.g. ozone. The wax is generally mixed into the rubber before vulcanization in proportions exceeding the solubility at room temperature. The surplus wax slowly diffuses to the surface of the rubber and produces a wax film which probably acts as a physical barrier to the oxidizing agents. The process of the diffusion of the wax has not previously been studied and the selection of waxes for protective purposes has been mainly by empirical methods. The work with the instrument to be described was initiated with the intention of studying the mechanism and laws governing this diffusion process.

Constructional details. The instrument was designed principally for use with a vernier microscope but could be adapted for use with an ordinary microscope if necessary.

The heating chamber is constructed of three rectangular plates of sheet brass $\frac{3}{16}$ in thick shown as *a* in Fig. 1, these being of slightly larger size than the ordinary microscope slide. Each plate has a $\frac{1}{2}$ in diameter hole in the centre. The middle section is recessed to take a microscope slide and cover slip *b*, between which the specimen is placed. The upper plate is hinged at one end *c* and fastened by a screw *d* at the other, thus providing access to the chamber. The heating chamber is sealed by glass windows *e* in the top and bottom sections;

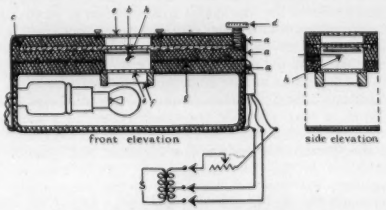

Fig. 1. Schematic diagram of apparatus

below the bottom window is an annular ring of a heat insulating material which holds the polaroid *f*, serving as polarizer. The lamp housing is open at the sides to enable better dissipation of the heat generated by the 5 W lamp.

The electrical energy is supplied by a low voltage transformer, so reducing the insulation difficulties which might be involved with higher voltages. The heater *g*, which is wound with nickel chrome tape, is clamped between the bottom and middle sections and is insulated from them by mica sheets. A rheostat controls the current from a 12 V supply. The stage is clamped to the base of the travelling microscope, the eyepiece of which is fitted with a disk of polaroid acting as the analyser. A thermocouple *h*, insulated from the metal stage by glass beads, is located in the chamber of the heating stage.

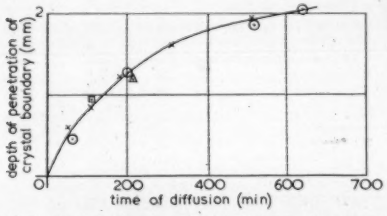

Fig. 2. Rate of advance of wax crystal boundary at 50° C

Experimental. In the case of the diffusion experiments the translucent rubber mix is vulcanized between the slide and cover slip in an air oven. One of the edges of the rubber is confined by a piece of aluminium with a straight edge; after vulcanization the aluminium is removed and molten wax introduced alongside the straight edge produced in the rubber. The wax is cooled rapidly to limit its diffusion into the rubber during this process. The system is then placed in the stage heater which is at a temperature somewhere below the wax melting point. It is kept at this temperature for some specified time and then allowed to cool to room temperature. The wax, which has diffused into the rubber, crystallizes as the temperature falls and its solubility in the rubber decreases. The crystals farthest from the interfacial boundary form a plane front through the section of the rubber parallel to the interfacial boundary. The crystals are in all the cases examined optically anisotropic and can be viewed by polarized light. A photomicrograph of the specimen after the diffusion has taken place is shown in Fig. 3. The measurement of the position of the wax crystal boundary in the rubber for different periods of diffusion enables the isothermal rate of penetration of the wax at that concentration to be studied. The concentration of the wax required to produce crystals at room temperature under similar conditions can be found. This provides all the necessary data for the estimation of the diffusion coefficient at this temperature and concentration.

Another method of determination of the diffusion coefficient is to heat a similar two-phase system in a thermostatically controlled water bath for a certain period and then find the relation between wax concentration and distance of penetration in the following manner. In a preliminary experiment the wax solubility-temperature relationship is determined by finding the temperature at which the wax crystals just disappear and reappear for known concentrations

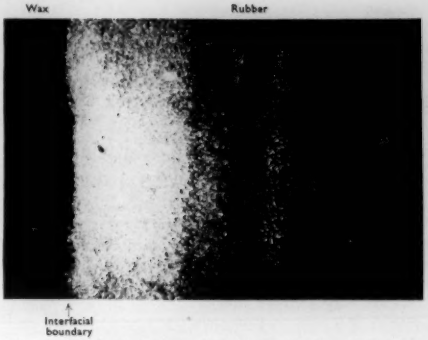

Fig. 3. Photomicrograph of wax crystals in rubber after diffusion process (magnification × 20)

of wax in rubber. The two-phase system can then be examined in the microscope stage and the positions measured at which crystals are just visible at various temperatures. These results, used in conjunction with those of the preliminary experiment, yield a relation between concentration and distance of penetration from which values of the diffusion coefficient can be deduced. This method has the advantage of allowing the variation of diffusion coefficient with concentration to be studied.

Using a magnification of about 60, individual wax crystals can be examined, so that the determination of melting points can be made and the actual process of formation and development of crystals can be observed when the temperature falls slowly. In practice the procedure in the determination of melting points is to allow temperature to fall by small intervals, maintaining each level steady for a short period to enable the sample to reach a uniform temperature throughout.

Reproducibility of measurements on diffusion systems. Some experimental results are shown in Fig. 2, which records the advance with time of the wax in the rubber. Experiments were carried out on specimens made at different times and tested on different occasions by the same operator. The results obtained with the various specimens are indicated on the graph.

Several inexperienced operators have measured the crystal penetrations in a number of diffusion systems. Generally for the low penetrations there was remarkable agreement between operators but as the diffusion progresses the crystal boundary becomes somewhat more difficult to locate precisely. Nevertheless, over a series of measurements of different penetrations, a statistical analysis of the results shows that there is no significant difference between results by the various operators. The standard deviation of the results of the measurements by these operators at the lowest penetration is 0·002 cm, whereas at the highest penetration the value is 0·007 cm.

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Research Association of British Rubber Manufacturers, and
Physics Dept., Northern Polytechnic, London.
L. L. BEST

New Books

Inventories of Apparatus and Materials for Teaching Science Vol. II. (Paris: UNESCO) (London: H.M. Stationery Office.) Pp. 145. Price 12*s.* 3*d.* net.

To help the teaching of science in schools and colleges in the war-damaged and under-developed parts of the world Unesco has prepared a series of inventories of apparatus and materials for teaching science. They are being published in three volumes, the second of which is devoted to the courses in Universities. The information concerning chemistry and physics relates to undergraduate courses provided by British Universities, whereas for botany, geology and zoology the corresponding courses in the colleges of the United States have served as models.

The physics syllabuses are given in detail, along with lists of laboratory experiments. It would seem that they are based on the degree courses at the University of Manchester. There follows a very complete list of apparatus and supplies for demonstration and laboratory experiments, as well as for the workshop.

The information should be invaluable to those planning or building new laboratories. But the claim made in the Preface that "By diffusing information of this comparative kind on the training of students, the inventories may well contribute also to bringing about equivalence of degrees and diplomas in science and engineering, irrespective of the universities and institutions where they are awarded" is surely too facile.

J. TOPPING

Glass. Edited by J. HOME DICKSON, M.Sc. (London: Hutchinson's Scientific and Technical Publications.) Pp. 300. Price 25*s.* net.

Criticism of this book on grounds of inadequacy is disarmed somewhat by a statement in the Preface that the treatment does not claim to be exhaustive. Nevertheless, one cannot but feel that in some of the chapters restrictions of space have forced the authors to compress so much information into so few words that parts have been reduced almost to catalogue form. This might have been avoided to some extent by omitting the discussion of the laws of refraction and of the aberrations of lenses in Chapter V, thereby allowing more space for other chapters, particularly Nos. I to IV, which might have been expanded with advantage. A small amount of additional space might have been made available, also, by avoiding duplication of treatment of certain details by the different authors. Little criticism is offered of the material presented, though the treatment of "decolourizing" (pp. 42, 43 and 99) is out of date and perpetuates two erroneous and wholly inadequate ideas on the nature of decolourizing. A useful glossary of technical terms, combined with the Index, is a commendable feature though, like the Oxford Dictionary, it does not define "illiants" (p. 42). This may be one of the comparatively few and mainly unimportant misprints, one of which, however, needs specific mention, namely, $(1 - r)p$, instead of $(1 - r)^p$ on p. 91, line 4.

The book is welcome. It provides much valuable information in a small compass, and is well illustrated. The authors and the publishers are to be congratulated on an attractive production which will be interesting to the layman and should be of considerable value to the student of glass and glass technology.

H. MOORE

The Elements of Thermodynamics. By CHARLES FABRY. Translated by G. F. HERRENDEN HARKER. (London: Frederick Muller Ltd.) Pp. vii + 223. Price 15*s.* net.

This is the first English edition of a well-known French work first published in 1928 and now in its sixth French edition, from which the present volume has been translated. The conceptual aspects of physics are probably most marked in the thermodynamics branch and the exposition of the subject both suffers and benefits more than most other branches from the varying powers of different teachers. Once the student has gone beyond a certain stage nothing is more helpful in clarifying his ideas than the unfamiliar presentation of a subject. For this reason an English edition of the late Professor Fabry's *Elements of Thermodynamics* should be welcomed. His approach to many of the basic ideas is rather different to the general run of text books in use in Britain and in most respects this new approach is very illuminating. The work is hardly suitable as a text-book in itself, but as ancillary reading it will be valuable particularly as the student will have to exercise his knowledge of the subject in order to satisfy himself that the methods used by Professor Fabry are as valid as those with which he has been accustomed.

The book is really an essay in thermodynamics and as such there are one or two points to which the present reviewer would take exception, not on matters of fact but of emphasis and clarity of exposition. But the attraction of an essay is precisely that what one reader dislikes may be very much enjoyed by another. After reading this book one feels that similar essays in the exposition of other branches of physics would also be worthwhile. Indeed, this kind of thing is so exceptional that in conclusion it must be pointed out that in describing it as an essay the reviewer does not imply that it is popularizing non-mathematical work. On the contrary, the mathematical treatment is fully developed. What the author has done is to show that it is possible to write in the language of scientific analysis as interesting and discursive an essay as could be written in usual literary language.

R. S. SILVER

Fundamentals of Acoustics. By L. E. KINSLER and A. R. FREY. (London: Chapman and Hall, Ltd.) Pp. vii + 516. Price 48*s.* net.

The authors specify that in this book the aim is "to present, in as clear and concise a form as possible, the fundamental principles underlying the generation, transmission and reception of acoustic waves." They stress also the importance of bearing in mind the close association between acoustics and communications engineering, since the design and analysis of many acoustical devices are facilitated by converting mechanical or acoustical quantities into equivalent electrical analogues. They have partly succeeded in their aim. Much of the treatment is good, but there are many fragments which will cause confusion to the student. In particular, the treatment of electrical analogues is scattered in different parts of the book; for example, motional impedance is introduced immediately after the discussion of simple harmonic motion and the definition of acoustic impedance appears in the midst of the treatment of Helmholtz resonators. It would be far better if these conceptions were introduced in a clear manner early in the treatise. As if aware that there will be criticism of one of the shortcomings

of the book, the authors state that no attempt has been made to include a complete bibliography. This is very regrettable in such a costly book where the student should be given guidance for further study and some indication at times of the origin of some of the theoretical and practical work. No reference of any kind appears in the first 242 pages, and in all only about 18 references are given. An excellent feature of the book which should be invaluable to student and teacher is a selection of examples at the end of each chapter, many of them dealing with modern developments in acoustics. On the whole one feels the treatment could have been re-arranged so as to present a more coherent argument. After a chapter devoted to the discussion of simple harmonic motions one meets in the second chapter the remark "an important type of periodic motion that frequently appears in nature is simple harmonic motion." There are excellent chapters on loudspeakers and microphones, but one would have liked more detailed treatment of reciprocity methods of calibration of microphones. There is a general discussion on psycho-acoustics; it is unfortunate that, in this book as in many other books on acoustics, the faculty of binaural location is attributed to appreciation of intensity difference and phase difference rather than intensity, quality, and time differences. The relatively easy location of complex sounds as compared with the difficulty in locating pure tones can be explained much more effectively in terms of the latter hypothesis.

During the last few years a number of books on acoustical theory and practice have appeared from the publishers. With the reservations stated this is a useful and stimulating addition to the series. T. S. LITTLER

The Microphysical World. By WILLIAM WILSON, Ph.D., D.Sc., F.R.S. (London: Methuen and Co. Ltd.) Pp. vii + 216. Price 5*s*. net.

This excellent and reasonably priced book is one of the series of Home Study Books published by Methuen and Co. Ltd., under the general editorship of Dr. B. Ifor Evans. It is "addressed to the intelligent layman" and sketches the development of atomic theory. The inclusion of so much material in such a small volume has been made possible by the author's concise style. With the occasional mathematical expression of special interest incorporated in footnotes, the book is quite readable and it is evident to the reader that he is being interested by an acknowledged expert. This maintenance of interest should certainly enable the author to succeed in his object of whetting the appetites of his readers for the more serious study of associated subjects. The inclusion of the chapter number on each page would aid the use of cross-references. R. HARESNAPE

Fundamental Mechanisms of Photographic Sensitivity, being the Proceedings of a Symposium held at the University of Bristol in March, 1950. Edited by J. W. MITCHELL. (London: Butterworth's Scientific Publications Ltd.) Pp. viii + 347. Price 63*s*. net.

The developments in the theory of photographic sensitivity which have taken place in recent years was the main theme of the conference held at the University of Bristol in 1950 under the chairmanship of Professor N. F. Mott. Forty-eight papers were presented and these are now published in this volume in the following five groups: physical properties of silver halides; production and properties of silver halide grains in photographic emulsions; photographic sensitivity; latent image formation; nuclear track emulsions.

The volume does not include summaries of the discussion on the various papers but the Editor has attempted to review the position as it appeared to him at the conclusion of the conference in a paper entitled "Lattice defects and latent image formation in silver halides."

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The dielectric behaviour of some types of human tissues at microwave frequencies. By H. F. Cook.

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Procedure in industrial physics and its implications for education*

By R. S. SILVER, D.Sc., F.Inst.P.

An attempt is made to analyse and illustrate the general method of procedure in industrial research. Attention is mainly directed to the methods of thought which are operative and which are considered to govern decisions on experimental work and on its application in design. It is suggested that the fact that theoretical concepts in industrial research need not be regarded as valid in an absolute sense outside the context of the particular problems, is of considerable importance. It arises because of the pressure of time and the need for short-cut methods and quick action. It therefore allows a freedom of concept, which carries with it the demand and responsibility to conceptualize as clearly and as widely as possible. The way in which existing training in rigid mathematical physics assists or hinders this process of thought is discussed. It is concluded that an advanced theoretical training is of considerable value, although it is suggested that a supplementary advanced education in non-technical subjects is also desirable.

The aim of this article is to express a strong warning against the tendency to specialize too greatly on technical problems in the teaching of physics. The present industrial and economic position of the country has introduced an almost febrile atmosphere, in which demands are made on all sides for a much increased and a much more rapid production of scientists and technicians. The emphasis behind these demands is no doubt justified, but there seems a real risk that the methods of intensive training advocated by some might defeat their own ends. This danger has been noticed by many and there have been frequent expressions of opinion that too early specialization must be avoided, and that a good education in the humanities as well as sciences is also desirable. It is to be feared, however, that such statements are regarded by others as naïve expressions of academic piety, innocent of the realities of modern life. So far as I can judge the state of the present conflict, by trends which come to my notice, the ultra-practical, emphatically technical side is at present in the ascendant. In particular it has been very disturbing to find many university people, sincerely anxious to meet the wishes of industry, suggesting in all helpful seriousness that they should drop such and such a subject from the student's curriculum.

I consider therefore that it will be useful to present the case for a generalized education, not merely as an expression of something that is desirable from the humanist or citizenship point of view, but as something which directly affects the competence to achieve success in industrial research. The substance of this case is that, when the actual nature of the activities of an industrial research worker is closely examined, it becomes clear that to accomplish these activities successfully, he requires more than mere technical training.

The theme of the foregoing sentence is well known as a generally held opinion, and again it is treated too often as a pious platitude. It seems most desirable that some attempt should be made to show, by a specific analysis of industrial research procedure, that it is fundamentally true and that it can be ignored or dismissed only at our peril. In this context I am not concerned with procedure in the sense of administrative arrangements, which may vary from firm to firm and from industry to industry, nor with the various interesting discussions which have appeared from time to time as to the responsibility which it may be desirable for scientific staff to exert in the affairs of the firm. I am concerned with what an industrial research worker is doing when he is doing industrial research!

To conclude this introduction, it is desirable to emphasize that it is with the research worker that I am concerned. I am unable to say precisely how far the views expressed here would be applicable to operative- and process-control scientists or engineers. It may be suggested, however, that wherever the nature of their work involves—as it almost always does—investigative effort, the present ideas would be applicable, to some extent at least.

A layman, if he were asked what he thought scientists did, either in industry or in the academic world, would probably reply along the lines that they tried to find out how things really worked, and to make them work better, or to make new things. Certainly the way in which a non-scientific employer will normally express his wishes to a scientific employee is equivalent to saying—"we want you to find out exactly what is happening here so that we can design to meet it." It is clear that the notion of *truth* is involved, which has presumably been carried over to the layman from the days when science was regarded as a part of philosophy and was thought to be concerned with establishing the truth about the nature of the universe. It would be a fascinating aside to discuss at this point whether the methods of science are at all competent for such a task. Far too many people have assumed glibly that science is competent to do so, and have adjusted their scheme of moral and religious values, their whole ideas of man's significance and of his existence, to suit what they have thought of as truths established by science. To discuss this attractive aside would be out of place here. It is valuable to mention it, however, because it might

* Based on a paper read at a Conference of the Education Group of the Institute of Physics on 5 April, 1951.

reasonably be urged that even if it were admitted that science may not be competent to determine truth in that wide and universal sense, it is nevertheless fully appropriate to the establishment of truth for the material world and particularly for the world of industry.

Descending from these heights of speculation, I ask directly what is the meaning of "truth" in the industrial research context? This can only be answered by considering the functions of industrial research, which again require a statement of the functions of industry itself. Put very briefly, it may be said that the functions of industry are to provide such goods and services as mankind in its wisdom or unwisdom desires and to provide them with the least possible expenditure of human effort and material resources.

In the field to which I am limiting myself, therefore, the question regarding truth comes to mean what things are true for me when I want to meet specific desires with minimum expenditure. An obvious answer would be that I must know the way things work, i.e. I must know how materials behave so that I can put them together in a way to meet the desires with which I or my employer are concerned. Suppose now that some statement is made that such and such a thing or part behaves in such and such a way—as a general statement. If now by process of reasoning I arrive at a conclusion that when this thing, element, or part, possessing these stated properties, is used in my assembly, the desired result will be achieved, and on the basis of that I go ahead and construct the assembly and then find that the desired result is not in fact achieved, what is the significance? There are, I suggest, three possibilities: (i) that my reasoning is fallacious; (ii) that the general statement regarding the behaviour of the particular part was untrue; or (iii) that the assembly modified the behaviour of that particular part. Of course the departure of the result from that desired might be due to all of these three causes each occurring to some extent.

I have gone through the foregoing rather elaborate analysis of a procedure to show up something which we do not often have sufficient humility, or awareness of what we are doing, to admit. That is the insecurity of what happens when the converse occurs. By the converse I mean when the assembly does give the desired result. Too often it is assumed that therefore, (i) the reasoning is correct; (ii) that the general statement regarding a particular part is true; and (iii) that its truth is not modified by the rest of the assembly. It is important to realize that there is no real basis for these beliefs. The desired result might be obtained when all of these things were wrong because an error in reasoning might compensate for a faulty general statement, or a faulty general statement might be modified by the assembly so as to be correct within the assembly. To put it more generally, the errors in all three might so compensate each other as to bring about the expected and desired result.

Now in practice what I have discussed in terms of one part within an assembly holds for every part, so that really there is an assembly of N parts about each of which certain behaviour properties are assumed and the assembly is so organized—designed is the word normally used—on the basis of a chain of reasoning, so as to intend certain desired results. If the desired result is obtained there is a tendency to assume that all the assumptions were correct and that the chain of reasoning was correct. It is not normally admitted that there may be a process of mutual compensation which enables the desired results to be obtained out of incorrect assumptions and incorrect reasoning. This preference would presumably be defended on probability grounds. It would be argued that in any practical case the number of assumptions implicit and explicit is so large that the chance of mutual compensation of erroneous assumptions to bring about a desired result is negligible. One is confident in having rules of reasoning which are universally accepted, and it is therefore only the correctness or incorrectness of *assumptions* which are in question. One tends to say, therefore, that the assumptions are true if the behaviour of a constructed assembly when predicted on the basis of the assumptions agrees with experimental observations.

It will be obvious that this is a very limited kind of truth, although none the less important for its limitations. Indeed it may be all the more effective *because* of its limitations, because what is really being said is that this is true within the ambit of our interests. Whether or not it is true in any more absolute sense is beside the point. Armed with this limited truth one can construct things appropriate to one's needs, which themselves will have been defined in limited terms.

It may be useful to illustrate what has been said by a practical example. I was concerned for some time with marine engineering, and one problem which came my way was the design of drainage systems for Admiralty feed-water heaters.[1, 2] Now the desired object in a feed-water system is to condense all the steam. The drainage must not allow any of the steam to pass away uncondensed, otherwise much useful heat is lost, the latent heat being a very large proportion. On the other hand it must only just condense, i.e. one must avoid a pool of liquid water inside the vessel of any appreciable depth which would blanket off some of the tube surface. The problem is, therefore, to design a drainage orifice such that it will pass the required amount of liquid water with only a very small head above the orifice. Now for cold water such a thing is a matter of simple hydraulics. In the feed-water heater case, however, the water is at its saturation temperature and as soon as it flows into the orifice, owing to the Bernoulli fall in pressure some of it again flashes into steam. The specific volume of the steam is so much greater in relation to that of the liquid that the flow may be much less than occurs with cold water. Attempts at drainage systems give very erratic results and design was usually by trial and error. An elementary adiabatic treatment of the formation of

vapour to be expected from reduction in pressure gives results which are ridiculously far away from what occurs in practice. Thus, for example, it was found that while a short orifice passed much less saturated water than it would cold water, it still passed about five times the amount of saturated water which was predicted by the simple adiabatic theory. In the papers referred to there is given a theory which satisfactorily accounted for the observed results and which enabled design to be done much more rapidly and effectively. Now this theory had the following characteristics. It was assumed that there was in the discharge nozzle a central core of liquid with a vapour envelope between the surface of the core and the wall of the nozzle. I regarded the formation of liquid as a rate process governed by the possible heat transfer rate from the centre of the core of liquid to its surface. The precise mathematical problem was hopelessly complicated. A number of subsidiary simplifying assumptions were made to permit an analytic treatment. The predicted results were in very good agreement with practice.

Now one predicted conclusion of the theory was that the nozzle could show a critical pressure analogous to that which is obtained in the flow of gas through a nozzle, this critical pressure occurring when the velocity of discharge reaches the velocity of sound in the medium. Among the simplifications was the assumption that the core and the envelope could be treated as moving with uniform velocity, which is obviously rather dubious. The theory predicted a critical pressure and a critical velocity—which, however, was extraordinarily low, namely of the order of only 20 m/sec. This is equivalent to saying that the velocity of sound or of pressure propagated in such a system is predicted as being only of the order of 20 m/sec. When one considers that in the liquid phase alone one would expect a velocity of sound of 1 500 m/sec and in the envelope alone a velocity of 400 m/sec, the result seems astonishing. Yet we did further experiments in which we proved that in fact a critical pressure did exist at velocities of about 20 m/sec. The actual value depended on the length of the nozzle—which was also predicted by theory.

I have chosen this example because it has, from a comparison of predictions with experimental results, every appearance of correctness, and yet if I were asked if I really believed that in practice there is such a core and such an envelope as distinct phases I would be very chary of saying "Yes." It shows up quite clearly that the criterion of truth for industrial design purposes is not whether you think the postulated picture is true in an absolute sense but whether, within the ambit of one's interests and when handled by one's particular methods of reasoning, it works.

The points which the example illustrates is that in industrial research one is concerned with assumptions not singly, but in a mutual matrix, mutually conditioning, and with the exercise of reason within and about that matrix. One is not concerned with the absolute validity of any single assumption outside the matrix. One has to develop a type of theoretical procedure—both in logic and in mathematics—which is appropriately flexible for these purposes. Does the usual education in physics and mathematics, with its emphasis on rigidity, help in that process?—or does it hinder?

Before attempting to give you my answers to these questions I must go on to emphasize the need for theoretical work in industry. There is an idea current with many people that theoretical training is superfluous for industry. We have all heard statements like an ounce of practice being worth a pound of theory, and will have listened to the practical man being boosted. I should be the last to deny his value. But in actual industrial practice the so-called practical man spends half his time being theoretical! Confront any group of twenty practical engineers with a fractured driving shaft or a damaged ship's screw. Several different theories of how the damage occurred will rapidly be enunciated. From a population of twenty engineers there will probably be about seven different theories put forward—for not more than three people are likely to hold the same view. The function of the real theoretician in industry is to reduce such chaos and to reduce the time which would be necessary to prove or disprove these various suggested theories by experiments or practical trials. He would proceed in the foregoing case to develop mathematically the consequences of each of the suggested theories and he would show that they could be divided into three groups: (i) those where the orders of magnitude of effects predicted were much too low to cause the effects observed; (ii) those where the orders of magnitude of effects predicted were much too large—so that the particular failure under discussion should have been accompanied by hundreds of others; (iii) those where the orders of magnitude of effects predicted are correct.

If he is still left with several competing theories in class (iii) his next step is to examine the trends or variations predicted by the theories. That is, he takes some variable x and by mathematical treatment finds that some of the theories predict that the effects are an increasing function of x while others give decreasing functions of x. And he will have chosen x because he knows from other experience or experiments whether effects do increase or decrease with x. Hence he gets a further elimination of possibilities. It is up to his theoretical ingenuity, if there still remain competing theories, to deduce conditions where they give different predictions and then test them against experience, or, if none is available, to conduct a discriminatory experiment. It is also up to his ingenuity and experience to decide when—in the context of his interests—it is not worth while attempting to discriminate further. This is an important point to which I shall return in a moment.

I have illustrated this procedure for convenience in relation to the problem of explaining damage to a

working part, but the same sort of thing applies throughout—in every design problem, in every attempt by a firm to construct something different from what it has done in the past—and therefore throughout the whole of industry. And in present conditions the short cuts, the time saving, the avoidance of trial-and-error methods, the concentration on well-defined lines of attack, which theoretical study *alone* provides, is absolutely vital for industry. No amount of experimental work, however elaborate, however careful and precise, can be of any use unless this kind of theoretical work is available to guide it, to interpret it, and to apply it. Again, contrary to what seems to be thought by many, the small firm has just as much need of high-level theoretical work as the huge concern or Government laboratory—perhaps more, because it cannot afford elaborate and widespread experiment.

Now I want to return to the point that the theoretical worker, after eliminating impossible theories, had to use his judgment and experience to decide when—in the context of his interests—it was not worth while attempting to discriminate between two alternatives. In other words, he has to be able to show, even approximately, that all relevant effects, both in magnitude and direction, will have only second-order differences in his context. He will then obviously choose the one which is simpler to work with—whatever he may think of its *absolute* validity as compared with the other, outside his context. Now this point is important because in fact, in a particular way, it *always* occurs. Even when you have only apparently one theory, there is always in fact a ghostly competitor present. This is one with the same physical basis of assumptions but with a different mathematical treatment, either more rigid or less rigid. One must be able to decide which is appropriate to one's needs—does one need this rigidity, or would it be absurd, having regard either to the hypothesis made or to the accuracy with which one can check it?

There is one final point on this question of theoretical work and mathematics. It should be obvious, but, unfortunately, it does not appear to be. The point is that the theory is never in the mathematics. Some people confuse mathematics with theory. They imagine that, if they cover sheets of paper with successions of mathematical symbols, they are being theoretical, and, unfortunately, also other people may imagine that people who write pages of mathematics are being theoretical. In fact, I have seen pages and pages of published papers where there was a welter of mathematical symbols and not a bit of theoretical work present. Everything was empirical and the pages of mathematical manipulation were only to change the empiricism from one form to another. I want to emphasize that a theory properly so called must be something which you can put into words. It is, in fact, a statement of assumptions as to what occurs. The mathematical work which follows is not in any sense a development of the theory, i.e. it does not add anything to the theory, it only works out the quantitative implication of the theory. If it appears as if the mathematical treatment has added anything, then it must be found that some other assumptions have been introduced unnoticed, i.e. some other bit of physical thought has been added to the theory without noticing it. It is most important for industrial purposes to stress this aspect of the limitation and of the value of mathematics. One does not get out any more than one puts in—but one does get out the meaning and quantitative significance of what one puts in.

When this is clearly realized and appreciated in relation to my previous remarks, it will be seen that in stressing the need for theoretical work I am stressing the need for a type of mind which can think about the processes involved and make physical assumptions—inspired guesses—as a result of cumulative experience of handling physical research problems. In addition the man must then have command of mathematics to work out the implications of his assumptions, with reasonable quantitative accuracy.

Finally one word about the question raised earlier regarding education and training. Does the usual education in physics and mathematics help in developing the appropriate type of theoretical procedure both in logic and in mathematics which will be appropriately flexible for industrial research? I should answer unhesitatingly that it does. The emphasis on rigidity in the teaching of advanced physics and mathematics is, in my opinion, absolutely essential. Although it may sound like a paradox it is absolutely true that, unless one has been adequately trained in the rigid procedure, one cannot know when it is permissible to take liberties nor can one know how to take liberties without damaging the whole structure. It should be quite obvious from what I have said that the inferior type of training which I am afraid sometimes occurs under the heading of "applied physics" in the training of technicians—that type of training in which the student is given what are supposed to be short-cut formulae which he must either learn by rote or know suitable reference books to look up in order to get them—is quite unsuitable for tackling the kind of problem I have sketched. Accordingly I would say that for industrial research a lower standard of training in physics and mathematics than was customary for a good honours degree cannot be tolerated. Some more notice might with benefit be taken of branches of physics which have tended to go a bit out of fashion, but this can only be worth while if the theoretical level to which these subjects are taught is at least as advanced as that to which nuclear and atomic physics is now taught. (Incidentally I am sometimes perturbed by the thought that the latter may be diminishing in view of the tremendous elaboration of experimental work which now occurs in these fields, but my fears may be quite unfounded.)

One thing is certain. The physicist's honours degree training in physics and mathematics should be supplemented with some work in other subjects which would

more accustom his mind to thinking out new assumptions and encouraging what might be called elasticity of imagination. To serve part-time apprenticeships in works as our engineer friends do may be of some use in giving a background and a reassurance of confidence when one first enters industry, but it is not in itself an assurance of developing the type of mind which is needed for research work as distinct from process-control work. It seems to me that there is only one way of assisting to develop these qualities I have sketched, and that is by ensuring that in addition to studying physics and mathematics the student has the widest possible non-technical education. He must be able to query, to see different ways of looking at things, to challenge conventional explanations and conventional attitudes. Since the whole weight of scientific training is towards the establishment of a body of law and therefore of convention regarding the behaviour of materials, it tends to stultify the very faculties which are most needed for its own successful prosecution. No major scientific advance has been made by a man following the already known ideas or conventions in science. In the industrial field I have shown that one is not so much concerned with the major conventions which may be generally true, but is very much concerned with studying mutual sets of logically consistent assumptions which will meet one's case. One has, therefore, to be all the more alive to new ways of looking at things, because there is more freedom to look at things in different ways when one is not called upon to justify them outside a limited range of interest. Unless scientists have the energy and flexibility appropriate to these degrees of freedom they cannot fulfil their function.

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Impressions of a discussion on technical universities

By N. CLARKE, B.Sc., F.Inst.P.

The Education Group of the Institute of Physics recently held a discussion on the possible establishment in Great Britain of a Technical University. The opening address was by Lord Eustace Percy; his remarks and those of the other contributors to the discussion are summarized and commented upon by the Deputy Secretary of the Institute of Physics in the light of the Institute's published views on the development of higher technological education.

The formation of the Education Group of the Institute of Physics, in 1949, owed much to the close interest of a number of Members of the Institute in many of the problems raised by proposals to develop the facilities for higher technological education in this country. The need for such development is widely accepted and there are few discussions on the subject in which some reference will not be found to one of the famous foreign technological institutions such as the Massachusetts Institute of Technology and the Eidgenössische Technische Hochschule, Zurich. The establishment of one or more technical universities in this country, on the model of those abroad, is advocated in some quarters as the most satisfactory solution of the British problem and the Education Group discussion, on 25th October, 1950, was devoted to this proposed policy. The Group was particularly fortunate in having for its opening speaker Lord Eustace Percy, Vice-Chancellor of the University of Durham and Chairman of the Committee whose report,[1] which commonly bears his name, first outlined a practical policy of expansion of the country's facilities for training highly qualified technologists for industry.

THE CASE FOR THE TECHNICAL UNIVERSITY

Lord Eustace Percy made his own position quite clear at the commencement of his remarks. He was opposed to the establishment of an institution of university status primarily devoted to technology and he began by examining the case for such an institution and the reasons why it has commanded distinguished support. Apart from the argument derived from mere impulse to imitate the famous institutions of America, Germany or Switzerland, there were, in his opinion, two arguments mainly used to justify the claim for a new university.

The first of these arguments, although in Lord Eustace Percy's view it was based upon little more than a vague impression, was that an attempt to increase the number of graduates in technology by developing the technology departments of existing universities sufficiently to meet the needs of industry would throw out of balance the numerical relationship of university faculties. This view, he maintained, was simply not true and he confessed that the statement in the Percy Committee Report that the universities could satisfy less than half the demand for professional engineers was wrong, being based only upon a very rough survey carried out in 1944. At that time no one had foreseen the extent of the post-war growth of the universities. In the event, the expansion of the universities to provide double the output of scientists had been achieved comparatively easily without disturbing appreciably the pre-war balance of faculties. Although the number of technological degrees conferred by British universities in 1949 was more than double the

number awarded in 1938, the technological students in 1949 still comprised only 13% of the total number of students in the universities. Arts students were no less than 44% of the total student body. Even in some universities where the percentages were substantially different from those averages and where the technological students represented a bigger proportion of the students (as, for example in the Newcastle Division of the University of Durham, where technologists were 27% and arts students 25% of the total), Lord Eustace did not think that the institutions had in any way suffered thereby.

In his view, the serious argument for a new technological university was that which had been advanced by some eminent scientists—that the bias of scientific studies in the universities was towards pure research, a bias which could not be expected to develop the practical outlook and the special qualities of mind which a technologist should have. Lord Eustace pointed out that, strangely enough, this argument was usually accompanied by a demand that training in technology should be based, at the undergraduate stage, on a wider and deeper study of the fundamental sciences than was at present given. This would mean that if technologists were to be removed from existing universities, professors of pure science would have to accompany them. Further, the practice at the Massachusetts Institute of Technology would suggest that little change of teaching methods would be made in such a new technological institution. It seemed to him, therefore, that this argument for an entirely new type of university was based upon a desire to create nothing more tangible than a different atmosphere.

THE TRADITION OF UNIVERSITIES

Lord Eustace interpolated some interesting and provocative comments upon the "atmosphere" of universities. Their tradition had been to prepare students for executive responsibility. It was a tradition which had produced men who became leaders of commerce, the Civil Service and the Colonial Service and which was a logical development of ideas dominant in higher education in this country since at least the sixteenth century. It was in strong contrast to the tradition of the universities of the Continent. It might well be, however, that this tradition was less clearly followed to-day and, perhaps, among the causes of change might be the scientist's ideal of the pursuit of knowledge for its own sake (which Lord Eustace described as subtly pernicious) and the scientist's emphasis on individual and original research. The willingness of graduates of an earlier generation to be cogs in a great administrative machine stood in contrast to the comparative reluctance of a physics or chemistry graduate of to-day to join a service, such as the Scientific Civil Service, where it was necessary to rely upon team work, by which all the executive labour of the world was necessarily done. This distaste for team drudgery was a characteristic of a large proportion of the present-day university graduates in all faculties.

Yet, said Lord Eustace, if that attitude did represent a real danger of decay in our university tradition, the tradition must be repaired and restored within the universities themselves. It was a mistake to believe that industry chiefly required good engineers or good metallurgists; its demand was probably mainly for good physicists, and industry would not wish its pure scientists to be trained in a highly academic atmosphere and its applied scientists in the atmosphere of a technological institution. Such a division, however, was in any case quite impracticable. As long as it remained the ambition of a professor of one of the pure sciences to build up a school of post-graduate research in his subject, it would be impossible to attract first-class teachers of those sciences to a new institution where they would be compelled to resign that ambition.

EDUCATIONAL EXPERIMENT

He was not denying the need for new experiments in technological education but he believed that experiments would be carried out best and quickest in an existing institution, provided that the institution was free and willing to undertake them. Lord Eustace felt that a disquieting feature of much current discussion on this subject was the tendency to consider policy without any sufficient knowledge of the existing educational resources; new ventures should not be recommended without an accurate survey of the present assets. The assets which this country had in the field of higher technological education were two—technical colleges and universities. The Report of the Percy Committee had recommended the experiment of selecting a few of the technical colleges for development into institutions of higher technology and Lord Eustace had suggested, as a personal note to the Report, that those colleges should be given the title of Royal Colleges of Technology. He regretted that he had not then had the courage to say what he now knew to be true—that no development of this kind could be hoped for as long as the technical colleges were owned and administered by local education authorities. A Royal College of Technology could not be administered by a municipal committee and the recommendations of his Committee had fallen flat owing to the obstinate reluctance of the Ministry of Education and the local authorities to select one institution from among others for special development. As long as this continued, it was not too much to say that half our assets for experiment and development in higher technological education were immobilized in a blocked account. Nevertheless, to write off those assets would be disastrous. There were examples, such as the Royal College of Arts, to show the enlivening effect of granting self-government to a publicly owned institution of higher education and the opportunity of similar action towards some of the technical colleges lay well within the power of Government.

Turning to the universities, Lord Eustace asked what change the Newcastle Division of the University of Durham, for example, would be compelled to make in its existing organization if it were to be converted into a technological university. This would be an ideal opportunity for those experimenters who had suggested that a technical university should be located near a general university. Presumably, Newcastle would transfer to the Durham Division all its Honours teaching for the B.A. degree. Yet, if it were to be modelled on a technical university like Zurich, Newcastle would not require fewer teachers of modern languages, history, economics or philosophy than it now had, though most of its best teachers would, no doubt, desire to teach the higher branches of their subjects and would follow their Arts students to Durham. Would Newcastle require to transfer its Honours and post-graduate schools of pure science to Durham also? He did not think that the pure scientists had sought to impose inappropriate standards or regulations on their colleagues in applied science and they had often provided a focus for the reconciliation of divergent views among applied scientists. Would Newcastle be required, too, to transfer the technologies of Medicine and Dentistry? The posing of these questions, he felt, was to provide an obvious answer. He believed that many of the provincial and Scottish universities had, in fact, succeeded in achieving a natural integration of all the sciences, an integration which was the life blood of all the technologies. And an institution which was seeking such an integration must offer much more favourable grounds for further experiment than could a new institution serving a vague ideal of segregation.

Lord Eustace ended his address by appealing for a deeper partnership between the universities and industry. He felt that the possible establishment of a new technological institution wholly for post-graduate studies might be well worth discussion, but its success would require a willingness on the part of industry to provide suitable conditions for its staffs to be detached for extended post-graduate courses. In this country, the professional training of an engineer was shorter than that of a first-class chemist or physicist, largely because of the established requirements of engineering apprenticeship and the membership conditions of professional associations. By changes of these traditional practices much could be done to provide any needed facilities for the training of technologists within the ambit of the present university system.

DISCUSSION

There had been no doubt of the strength of Lord Eustace Percy's conviction that the existing universities and a few specially developed Royal Colleges of Technology could and would meet the whole need of undergraduate training of technologists. The opposing view was no less firmly advanced by Lord Cherwell, who opened the general discussion on Lord Eustace's address. Lord Cherwell began by deploring the inferior status usually accorded to technological studies by those educated in the humanities. The common view that a man possessing little more than a highly specialized knowledge of the lives of figures of history was somehow better "educated" than a man highly trained in the intricacies of modern technological processes was a distortion which was not only deplorable but which retarded the proper development of technological education.

In Lord Cherwell's view, many of the difficulties which Lord Eustace Percy had felt to stand in the way of the establishment of a technical university were, in fact, quite imaginary. Moreover, Lord Eustace had not appreciated the scale of expansion of technological education which was required if the real needs of British industry were to be met; if the expansion were to be adequate, it could not be provided by the existing universities. He quoted the example of the technical university of Aix-le-Chapelle. There there were numerous full, part-time and extra-ordinary professors who gave lectures two or three times a week; the university was run at a high cost. To establish a similar system in this country, it would be necessary to provide about half a million pounds a year for running costs and a student population of not less than two or three thousand would be essential to make it a reasonable economic proposition. No existing institution in the country, with the possible exception of the Imperial College of Science and Technology, could conceivably envisage such a system.

Lord Cherwell stressed the serious position of British technological progress. In the nineteenth century, Britain had been very far ahead of all other countries in technological development, but, in the last fifty years or so, the United States and some continental countries had gone ahead. Industrial competition was, therefore, becoming increasingly difficult for this country to maintain. During the recent war, also, the Government had had difficulty in obtaining adequate numbers of highly trained engineers of the quality readily found in Germany.

Turning to Lord Eustace Percy's query concerning the position of the pure sciences if a technological university were to be established, Lord Cherwell felt that the pure sciences should remain in the old universities and that not the slightest difference to them would result from the founding of the new type of institution. He believed that the answer to Lord Eustace's hypothetical case of the conversion of the Newcastle Division to a technological university was that the Durham Division would carry on with the old fundamental studies and Newcastle would start on new, equally fundamental, work. In Lord Cherwell's opinion, the difference that would be noticeable in a technological university would be a rather smaller emphasis on languages and English literature.

As to the means of overcoming the practical difficulties of establishing a new organization, it might be that one way would be to expand and develop technical colleges, but the intention should be the eventual development to

the full technological university. This was badly needed if the country's industry was to be given an adequate supply of technologists of the right calibre.

Dr. P. Dunsheath, following Lord Cherwell, disagreed with the unfavourable contrasting of this country with Germany in the production of first-class engineers; he felt that our system was producing good engineers in good numbers. He made a plea for breadth of education of engineers and technologists. Dr. Dunsheath also supported strongly the remarks of Lord Eustace Percy about opportunities for further study for men who had spent some years in industry. It would be necessary to provide really adequate grants to enable these men, most of whom would have family responsibilities, to go back to college for post-graduate study and he wondered whether some help might be given by the University Grants Committee.

Lord Eustace's point concerning the need for longer initial training for engineers was supported by Dr. S. Whitehead, who, however, was doubtful whether most students could be attracted to carry on in the longer courses. The comparison of this country and Germany was again referred to by a member of the audience who stated that he had a good knowledge of the German technical high schools. In his opinion, the German technologist was better equipped with basic theoretical knowledge than was the British technologist, but Britain was considerably further ahead in the practical application of technology.

Replying to the discussion, Lord Eustace Percy stressed the danger of making superficial comparisons between our own and foreign systems. In Germany, for example, the normal university had, in the past, been largely an Arts college and the so-called technical university was broadly similar to the pure science departments of our universities. It might be misleading, too, to compare the size of the professorial staffs of these universities with our own; the title of professor had, on the Continent, a wider application than here and a staff of fifty professors in a German technical university might include many men of very similar ability and reputation to those on the non-professorial staff of a British university.

Lord Eustace said that there was no evidence to support the view that any necessary expansion of technological departments would be resisted by the universities. The universities had not been asked to agree to further expansion. Indeed, no real investigation into the extent of the need for technologists had been made since the Percy Committee had conducted a rough survey in 1944.

COMMENTS

The Board of the Institute of Physics has published its own proposals for changes in the higher technological educational system.[2,3] In those proposals, the establishment of a technical university was not specifically discussed but, by implication at least, the immediate creation of such an institution was considered undesirable. The essence of the Institute's views was contained in the passage in the Percy Committee Report which urged freedom for the (Royal) Colleges of Technology—"The surest way to raise the whole standard of technical education in this country is to devolve upon the Colleges of Technology . . . the responsibility of setting their own standards and making their own plans." In order to achieve this opportunity for responsibility on the part of a small number of selected technical colleges, the Board proposed for them a high degree of financial and academic autonomy and the removal of the control of the Ministry of Education and the local education authorities. In the later statement,[3] commenting upon the recently published proposals of the National Advisory Council for Education in Industry and Commerce,[4] the Board suggested that "perhaps, later, some form of regional grouping of these colleges might evolve which would lead to the creation of one or more technical universities of national standing. We believe, however, that developments should not be forced but should proceed at a pace which reflects the demands of industry."

Those who are in general agreement with the Institute of Physics' published statements will welcome most warmly Lord Eustace Percy's forthright expression of the urgent need to remove some of our major technological institutions from the control of local authorities and they will share his regret that no such clear proposal was contained in the Report of his Committee. On the other hand, Lord Eustace Percy probably takes a much more optimistic view of the extent to which the universities can meet the country's requirements for technological manpower than does the Institute's Report. On this point, Lord Cherwell's insistence upon the great scale of expansion that is needed is more in line with the Institute's opinion. The Institute, however, has taken the view, here rather similar to Lord Eustace's, that the way to achieve the most rapid and most satisfactory expansion of our facilities is by developing, as a matter of urgency, the best of the technological institutions that we now have, putting the main emphasis upon the rapid provision of finance, material facilities and teaching staff and leaving the problem of possible evolution to university status to be solved in the light of experience and industrial demand. There was much in the contributions of all those taking part in the present discussion that provided support for such a view.

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The scattered light method of exploration of stresses in two- and three-dimensional models

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An examination of the theory of the photoelastic effect and of the conditions affecting the appearance of interference fringes in scattered light shows that scattered light observations provide in many cases the quickest and the most accurate means of measuring principal stress differences in a three-dimensional model, and also of separating the stresses in a two-dimensional problem. A method is developed by which a high intensity may be obtained in the light scattered from a very narrow beam traversing the model, and this, combined with an improved measuring technique, permits of a relatively high degree of accuracy in the observations. Examples are given of the use of the method in some typical problems.

The appearance of interference fringes in the light scattered from a polarized beam passing through a stressed photoelastic model was first recorded by Weller[1] in 1939. He examined the theory of the formation of the fringes and suggested that observation of the effect might be used in the exploration of three-dimensional stresses in a loaded model. He and other investigators have in fact used this method successfully in the case of a prismatic bar subjected to pure torsion to which problem it yields a complete solution with much less labour than is involved in the use of any of the analogy methods. Owing, however, to difficulties of technique and also to difficulties in interpreting the observations in the general case, the method has been largely discarded as of no practical value except in the problem quoted above.

In this paper a fresh investigation of the phenomenon is presented, and new techniques are described by the use of which many of the difficulties previously encountered are overcome, and the scattered light method of observation becomes the most accurate, and frequently the quickest and most convenient, means of examining the stresses in certain important two- and three-dimensional cases.

1. THEORY OF FORMATION OF FRINGES

(*Introductory note.*—In the discussion of theory which follows, the characteristics of any plane polarized light wave will be taken as represented by a single vector in the plane of the wave-front, the direction of this vector being normal to the plane of polarization, and referred to as "the direction of the vibrations" of the wave. Also, in dealing with light passing through a stressed medium, any divergence between the paths of the oppositely polarized rays which may occur will be neglected, the two waves being assumed to have a common path and a common wave-front. This assumption is permissible in any practical photoelastic application, since the difference in the two refractive indices is exceedingly small.)

It has long been known that when a beam of plane polarized light passes through an isotropic "scattering" medium the secondary waves propagated in all directions from any point of the beam have different intensities in different directions. The magnitude of this variation in intensity depends upon the scattering properties of the medium. If the scattering is due to very large molecules, or to extremely fine particles in suspension, most of the scattered light will be plane polarized, and the amplitude of the vibrations of the secondary wave propagated in any given direction will be proportional to the resolute perpendicular to that direction of the amplitude of the vibrations of the primary wave. Thus if the plane of the paper in Fig. 1 is the plane of the wave-front of the primary wave, OA representing the amplitude of the primary vibrations, then the scattered wave in direction OP, making an angle α with OA, will have an amplitude $\sigma\, OA \sin\alpha$, where σ is a coefficient of scattering for the medium. If, however, the scattering is produced by particles in suspension in the medium, there will generally be, in addition to the true "scattered" light, a proportion of "reflected" light which is not completely polarized and the intensity of which does not vary with the direction of propagation. The effect of this reflected light is to mask, to some extent, any interference phenomena occurring in the true scattered light, but it does not otherwise affect the interference, and may be ignored in any consideration of the theory.

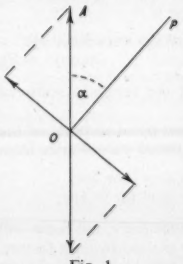


Fig. 1

If the medium through which the polarized wave travels is stressed, the wave is in general split into two plane polarized waves travelling with different velocities. The directions of vibration of these waves are the directions of the secondary principal stresses in the plane of the wavefront, and the difference of their velocities is proportional to the difference of these stresses. Let Fig. 2 represent the plane of the wave-front

at any point, OA and OB being the directions of the vibrations of the two polarized waves whose amplitudes are respectively a and b. Each of these waves scatters its own quota of light, and there will thus be propagated in any direction OP two waves whose amplitudes are respectively $\sigma a \sin\alpha$ and $\sigma b \cos\alpha$. The vibrations of these secondary waves will be parallel since both lie in the plane of the primary wave front, and normal to the direction of propagation OP. The displacements in the two waves may therefore be added algebraically to give the resultant wave in direction OP.

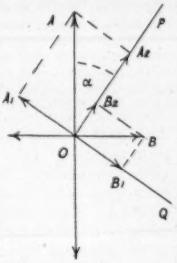


Fig. 2

The displacements in the two primary waves at any point of the beam may be represented by

$u = a \cos nt$ in direction OA

and

$v = b \cos(nt + \psi)$ in direction OB,

the phase-difference ψ changing from point to point of the beam as one wave is retarded relative to the other. The light scattered from this point in the direction OP will then be the sum of the two waves

$$q_1 = \sigma a \sin\alpha \cos nt$$

and

$$q_2 = \sigma b \cos\alpha \cos(nt + \psi),$$

both vibrations being in direction OQ, normal to OP.

These add to give

$$q = \sigma(a^2 \sin^2\alpha + b^2 \cos^2\alpha - 2ab \sin\alpha \cos\alpha \cos\psi)^{\frac{1}{2}} \cos(nt + \beta),$$

where q is the displacement in the resultant scattered wave.

As the value of ψ changes from point to point of the beam the amplitude of this scattered wave varies from

$$\sigma(a \sin\alpha - b \cos\alpha) \text{ when } \psi = 0 \text{ or } 2r\pi,$$

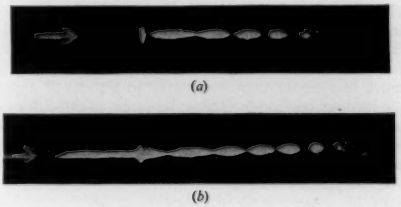
to

$$\sigma(a \sin\alpha + b \cos\alpha) \text{ when } \psi = (2r - 1)\pi,$$

and, if these amplitudes differ sufficiently, the beam will appear to an observer at P to be crossed by dark fringes. The bright points on the beam will have their greatest intensity when viewed from a direction for which $(a \sin\alpha + b \cos\alpha)$ is a maximum, i.e., when $\alpha = \tan^{-1} a/b$, while the dark points will have zero intensity when $(a \sin\alpha - b \cos\alpha)$ is zero, i.e., when $\alpha = \tan^{-1} b/a$. Thus, in general, interference fringes will be seen more or less distinctly for a considerable range of angles of observation, but if $a = b$, the fringes will be black and will have maximum brightness between them for the direction of observation $\alpha = 45°$.

It should be noted that at points where the two primary waves are in the same phase, the scattered waves will have maximum or minimum intensity according to the direction of viewing. If in Fig. 2 OA and OB represent simultaneous maximum displacements of the primary waves, the corresponding displacements in the waves scattered in direction OP will be OA_1, OB_1 of opposite signs, but the simultaneous maximum displacements in the waves propagated in direction OQ, perpendicular to OP, will be OA_2, OB_2 of the same sign.

Thus if the beam is observed from P interference fringes will be seen at points at which the primary waves are in the same phase, while from Q they will be seen at points at which these waves are in opposite phases. The most favourable conditions for observing the fringes are obtained when the direction of the vibrations in the primary beam are either parallel to or normal to the direction of observation, and the directions of the secondary principal stresses in the plane of the wave-front are at 45° to these directions. Figs. 3(*a*) and 3(*b*) show the two sets of fringes seen in the same beam with the two alternative positions of the polarizer. The fringes indicate points at which the relative retardation is (*a*) zero or a whole number of wavelengths, (*b*) an odd number of half wavelengths.


(*a*)

(*b*)

Fig. 3. Scattered light fringes in narrow pencil through eccentrically loaded tension bar
(*a*) Whole wave fringes, (*b*) half-wave fringes.

It should also be noted that, since the stresses in general change from point to point in all directions in the material, observation must be restricted to the scattered rays emanating from a very small area of the wave-front. This may be done by using as the primary beam a very narrow pencil of rays. The narrower this pencil the more nearly will the effects observed represent the stress conditions at points through which the pencil passes. For ease of observation of the fringes, however, it is frequently convenient to use a fine slit aperture giving a thin ribbon beam. Observations will normally be confined to the central narrow pencil of the beam, but identification of the fringes is facilitated by having this wider illuminated field.

2. INTERPRETATION OF THE FRINGES

Case 1. In attempting to establish the relations between the interference fringes observed in the scattered light and the stresses in the medium there are two cases to be considered. The first, and simplest, is that in

which the secondary principal stresses in the plane of the wave-front of the primary beam maintain constant directions for all points on the path of the beam. In this case, the two oppositely polarized waves maintain constant directions of vibration, and the rate of change of phase between them at any point as they traverse the stressed model is proportional to the difference of their velocities at that point, and therefore to the difference of the secondary principal stresses in the plane of the wave-front. If then we measure the positions of the interference fringes on the path of a narrow beam, and

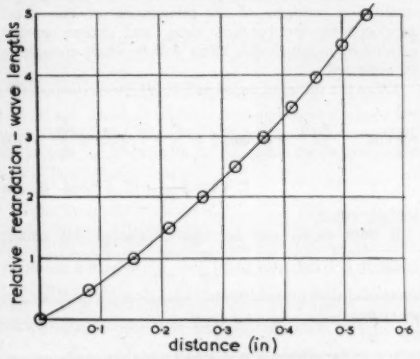


Fig. 4. Graph of the fringes of Fig. 3

plot a graph of phase-difference against distance, the difference of the secondary principal stresses in the plane of the wave-front at any point is given by the product of the slope of the graph at that point and the fringe value of the material. Fig. 4 is the graph plotted from the fringes shown in Fig. 3. If the phase-difference is measured in wavelengths of the monochromatic light used, the distance in inches and the fringe value in lb/in^2/wavelength/in, the stress-difference will be in lb/in^2.

The sign of the relative retardation in the primary beam is not indicated by the fringes. Frequently we shall know, from general considerations, what this sign is. If not, we may determine it by introducing, by means of a compensator, a small relative retardation of known sign into the beam before it enters the model. Observation of the direction of movement of the fringes produced by this additional retardation will then give the required information. If the stress-difference in the plane of the wave-front changes sign at any point, the fringes on opposite sides of this point will move in opposite directions. Alternatively, if no colour filter is used, a change in sign of the stress-difference may be detected by a reversal of the coloured edges of the fringes.

Case 2. The second case is that in which the secondary principal stresses in the plane of the wave-front are changing in direction from point to point in the path of the beam of light. In this case the two oppositely polarized waves do not retain their identities from point to point, but are being continually re-resolved into different directions, so that the two waves whose phase-difference is observed at one point are not the same two waves as are observed at another point. We cannot assume therefore that the changes in phase-difference indicated by the fringes are equal to the changes in relative-retardation due to the stresses, but must endeavour to separate the stress-effect from that due to the rotation of the axes. Using the simplified vibration theory employed here, we consider the changes in relative retardation due to stress and the changes in orientation of the axes to take place in alternate small steps. We then proceed to the limit and obtain the differential equation connecting the observed rate of change of phase-difference with the rates of change of these two quantities.

Let the displacements in the two polarized waves at time t at a point x of the path of beam be given by

$$u = a\cos nt, \text{ and } v = b\cos(nt + \psi).$$

Suppose the stresses to maintain their directions for a small distance δx along the path, and let the relative phase-retardation produced in this distance by the stress-difference be $\delta\phi$. Then at the point $x + \delta x$ the two waves will be represented by

$$u = a\cos nt,\ v = b\cos(nt + \psi + \delta\phi),$$

(neglecting changes in phase common to the two waves).

Now suppose the directions of the axes of stress in the plane of the wave-front to turn through a small angle $\delta\alpha$. The two waves will be re-resolved in these directions giving displacements

$$u' = a\cos\delta\alpha\cos nt + b\sin\delta\alpha\cos(nt + \psi + \delta\phi)$$

and

$$v' = b\cos\delta\alpha\cos(nt + \psi + \delta\phi) - a\sin\delta\alpha\cos nt,$$

or

$$u' = [a\cos\delta\alpha + b\sin\delta\alpha\cos(\psi + \delta\phi)]\cos nt - b\sin\delta\alpha\sin(\psi + \delta\phi)\sin nt,$$

$$v' = [b\cos\delta\alpha\cos\delta\phi - a\sin\delta\alpha\cos\psi]\cos nt + \psi) - [b\cos\delta\alpha\sin\delta\phi + a\sin\delta\alpha\sin\psi]\sin(nt + \psi).$$

If we neglect terms of the second order in the small quantities $\delta\alpha$, $\delta\phi$ these reduce to

$$u' = (a + b\delta\alpha\cos\psi)\cos nt - b\delta\alpha\sin\psi\sin nt,$$

$$v' = (b - a\delta\alpha\cos\psi)\cos(nt + \psi) - (b\delta\phi + a\delta\alpha\sin\psi)\sin(nt + \psi).$$

These may be written

$$u' = (a + \delta a)\cos(nt + \delta\psi_1) \quad (1)$$

and

$$v' = (b + \delta b)\cos(nt + \psi + \delta\psi_2), \quad (2)$$

where

$$a + \delta a = (a^2 + 2ab\delta\alpha\cos\psi)^{\frac{1}{2}} \simeq a + b\delta\alpha\cos\psi, \quad (3)$$

$$b + \delta b = (b^2 - 2ab\delta\alpha\cos\psi)^{\frac{1}{2}} \simeq b - a\delta\alpha\cos\psi, \quad (4)$$

$$\tan\delta\psi_1 \simeq \delta\psi_1 = \frac{b\delta\alpha\sin\psi}{a + b\delta\alpha\cos\psi} \simeq \frac{b}{a}\delta\alpha\sin\psi, \quad (5)$$

and $$\tan\delta\psi_2 \simeq \delta\psi_2 = \frac{b\delta\phi + a\delta\alpha\sin\psi}{b - a\delta\alpha\cos\psi}$$

$$\simeq \delta\phi + \frac{a}{b}\delta\alpha\sin\psi. \quad (6)$$

The phase difference between the two waves represented by equations (1) and (2) is $(\psi + \delta\psi_2 - \delta\psi_1)$. The increase in phase difference is thus given by $\delta\psi = (\delta\psi_2 - \delta\psi_1)$ or, from (5) and (6),

$$\delta\psi \simeq \delta\phi + \left(\frac{a}{b} - \frac{b}{a}\right)\sin\psi\,\delta\alpha \quad (7)$$

In the limit then we get the relation

$$\frac{d\psi}{dx} = \frac{d\phi}{dx} + \left(\frac{a}{b} - \frac{b}{a}\right)\sin\psi\frac{d\alpha}{dx}$$

or

$$\frac{d\psi}{dx} = \frac{d\phi}{dx}\left[1 + \left(\frac{a}{b} - \frac{b}{a}\right)\sin\psi\frac{d\alpha}{d\phi}\right] \quad (8)$$

Also, from equations (3) and (4) we get

$$\frac{da}{d\phi} = b\cos\psi\frac{d\alpha}{d\phi} \text{ and } \frac{db}{d\phi} = -a\cos\psi\frac{d\alpha}{d\phi} \quad (9)$$

Equation 8 is essentially Neumann's equation[2] as obtained by Mindlin and Goodman.[3] The factor $d\alpha/d\phi$ is the parameter R used by Mindlin and Goodman, and is the ratio of the rate of rotation of the secondary principal axes in the wave-front to the rate of phase-retardation which would obtain in the absence of rotation. Exact solutions of this equation have been obtained in two special cases; that in which the rate of rotation of the axes, $d\alpha/dx$, is constant,[4] and that in which it is a linear function of x[5]; but in any practical problem the form of $d\alpha/dx$ is unknown and no solution is possible.

From the point of view of practical application of scattered light observations, however, an examination of the equation gives useful information as to the relation between the quantity we wish to determine, $d\phi/dx$, and the quantity we observe, ψ.

First, we see that $d\phi/dx$ is equal to $d\psi/dx$ at points where either $a = b$, or $\sin\psi = 0$. The first condition can be satisfied at the point of entry of the beam by orienting the model so that the principal stresses at this point are inclined at 45° to the direction of the vibrations of the incident light, but it is not, in general, satisfied elsewhere, since equations (3) and (4) show that a and b do not retain their initial values. The second condition is satisfied at points where whole or half-wave fringes appear, but the value of $d\psi/dx$ at these points cannot be obtained from a smooth curve plotted through the observations. Equation (8) shows that there is a periodic variation in the difference between the slopes of the ψ and ϕ graphs. Such a variation cannot occur in the value of $d\phi/dx$, for if it did it would mean that a change in magnitude of the load on the model (which would alter the positions of the fringes), would result in a change in distribution of the stress. We deduce therefore that the slope of the ψ, x graph is modified by small periodic changes between the points representing the observed fringes, and that without knowing these changes we cannot determine this slope at any point. We can, however, form a qualitative estimate of the magnitude of the effect produced by the rotation term, and deduce certain conditions in which this effect will be small enough to be neglected.

Using the values of $da/d\phi$ and $db/d\phi$ from equation (9),

we find $$\frac{d}{d\phi}\left(\frac{a}{b} - \frac{b}{a}\right) = \left(2 + \frac{a^2}{b^2} + \frac{b^2}{a^2}\right)\cos\psi\frac{d\alpha}{d\phi} \quad (10)$$

$$\simeq 4\cos\psi\frac{d\alpha}{d\phi}, \text{ if } a \text{ and } b \text{ are initially equal.}$$

If both $d\alpha/d\phi$ and its rate of change are small, equation (10) indicates that $\left(\frac{a}{b} - \frac{b}{a}\right)$ will have a modified sinusoidal form (approximately $4d\alpha/d\psi\sin\psi$), and the term $\left(\frac{a}{b} - \frac{b}{a}\right)\sin\psi\,d\alpha/d\phi$ will then represent a quantity of type $\mu\sin^2\psi$, where μ is a small variable coefficient of the order $(d\alpha/d\phi)^2$. We can obtain some indication of the order of magnitude of $d\alpha/d\phi$ in any particular case by observing the sharpness of the fringes in the beam. Observation has shown that a marked diminution in sharpness of the fringes is produced by a rotation of the model (and therefore of the axes of stress) through about 15°. Thus if all the fringes observed in any length x of the beam have sensibly their maximum sharpness at the same orientation of the model, the total change in α over the distance x will be less than 30°. If, then, this range covers n wavelengths relative retardation, the mean value of $d\alpha/d\phi$ cannot exceed $1/12n$. We may therefore deduce that if the fringes are sharply defined over a range of two or more wavelengths, the error in $d\psi/dx$ introduced by drawing the ψ, x graph as a smooth curve through the points obtained by measurement of the positions of the fringes will not exceed something of the order of 1%, and this will generally be less than the possible error of measurement.

It may be noted that the values of $d\alpha/dx$ and its derivatives at any point in a stressed model are independent of the magnitude of the load applied, but they decrease as the dimensions of the model are increased, while the value of $d\phi/dx$ is limited only by the maximum permissible stress in the model. The rotation effect, therefore, will be minimized by using as large a model as possible.

APPLICATION OF THE METHOD

There are four types of problem in which the scattered light method of observation may be employed with advantage.

These are:

(*a*) The separation of the principal stresses in a two-dimensional model;

(*b*) The determination of the second principal stress-difference at points in a plane of symmetry of a three-dimensional model;

(*c*) The complete determination of the stresses in a prismatic bar of any section under pure torsion;

(*d*) The determination of the principal stress-difference, and of the approximate directions of the principal stresses, at any point on the "free" surface of a three-dimensional model.

Case (*a*). Consider a two-dimensional model, in which the stress-distribution is symmetrical about an axis AB (Fig. 5). Then AB will be one line of principal stress (say the P stress), and the second principal stress Q will be normal to it at all points. If we pass a beam of light through the model along the axis AB, the only stress in the plane of the wave-front will be the principal stress Q, and this will retain its direction throughout the length of the beam. The fringes observed will thus indicate relative retardation due to the single stress Q, and the slope of the retardation graph will give the value of Q at all points on AB.

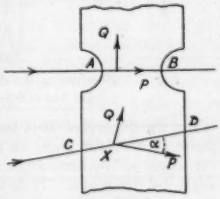


Fig. 5

If the beam of light is passed along any straight line such as CD in the plane of the model, the slope of the relative-retardation graph at any point X gives the value of the secondary principal stress $P\sin^2\alpha + Q\cos^2\alpha$, where α is the inclination of the P stress to CD. If to this quantity we add the value of $(P - Q)\cos^2\alpha$, $(P - Q$ and α being obtained from the normal two-dimensional observations), we obtain the value of the stress P at the point. This is thus a simple and accurate method of separating the stresses along any straight line in the model, and since it is independent of boundary conditions it can be used in regions near to load points.

Case (*b*). Scattered light observations in this case may be made either on the complete model or on a thin slice cut from it containing the plane of symmetry. The primary light beam must lie in the plane of symmetry, and the procedure is precisely the same as in Case (*a*), but in this case it yields the difference of the principal stresses Q and R, where R is the stress normal to the plane of symmetry.

**Case* (*c*). Consider a normal cross-section of a prismatic bar under pure torsion. The stress at any point 0 (Fig. 6) will be a shear-stress S acting across the section. If a light beam passes through 0 in the plane of the section in any direction x making an angle θ with the direction of S, the stress in the plane of the wave-front will be a pure shear-stress $S\sin\theta$ across the section. This is equivalent to principal stresses $S\sin\theta$ and $-S\sin\theta$ in directions at 45° to the plane of the section. Thus the secondary principal stresses in the plane of the wave-front will everywhere have the same orientation, and the slope of the relative retardation graph will be given by $d\phi/dx = 2S\sin\theta/f$, where f is the fringe-value of the material. The shear stress S at any point will thus be given by $\frac{1}{2}f\operatorname{cosec}\theta\, d\phi/dx = \frac{1}{2}f\, d\phi/dn$, where n is the direction normal to the fringes at that point. Thus if we observe a thin flat beam of light which illuminates a whole cross-section of the bar (the direction of viewing being parallel to the axis of the bar), we get a fringe-pattern over the whole section in which the tangent to the fringe at any point represents the direction of the shear-stress, while the normal gradient of the fringes represents the principal stress-difference at that point. This is a unique case in two respects: it is the only practical problem in which observations on a single beam yield the principal stresses over an area of the model, and it is the only case in which the illuminating beam may be made quite thick without introducing error.

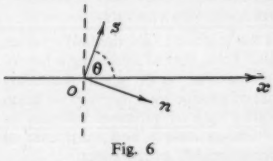


Fig. 6

Case (*d*). This is the only one of the four cases in which the question of rotation of axes occurs. If a narrow beam is incident normally on the surface of any stressed model at a point at which no external load has been applied, the stresses in the plane of the wave-front at the point of entry are the principal stresses at that point. As the beam penetrates into the model, however, the directions of the principal stresses in the plane of the wave-front will, in general, change. The effect of this rotation of axes has already been discussed and we have seen that in cases where several fringes can be seen the principal stress-difference at the surface will be given without serious error by the slope at that point of the graph of the fringes observed. If the model is viewed from a direction parallel to the vibration axis of the polarizer, the zero fringe appears at the point of entry of the beam. When the model is rotated about an axis coincident with the axis of the beam, this fringe appears at its sharpest when the principal axes of stress at the surface are inclined at 45° to the direction of viewing. This permits of the determination of the directions of the axes to within a few degrees. If a slice is now cut from the model in a plane containing the normal to the surface at the point and one of these indicated directions,

* For a more detailed account of this problem the reader is referred to Drucker and Frocht.(6)

it may be examined in the ordinary polariscope, using a simplified form of the tilting stage technique,[7] to obtain the direction of this axis more accurately, and to measure one of the principal stresses.

The method thus offers a means of exploring the stresses in the surface of a model, finding the points at which the stress-differences are greatest, and measuring the stresses accurately, without cutting and examining a number of slices.

METHOD AND APPARATUS

The practical use of the scattered light method involves three distinct problems. First, that of obtaining a homogeneous photoelastic material with good scattering properties; second, that of producing a very narrow beam of approximately parallel light of sufficient intensity to show the interference fringes clearly, and, third, that of obtaining accurate measurements of the effect.

Materials. The scattering property of any piece of unstressed photoelastic material may easily be tested by passing a beam of plane polarized light through it and observing the variation of intensity of the scattered light as the polarizer is rotated. For a material to give satisfactory results the path of the beam should appear brightly illuminated when the direction of the vibrations in the polarized light is normal to the direction of viewing, and should be almost invisible when the vibrations are parallel to the direction of viewing. Some photoelastic materials possess good natural scattering properties, but the degree of these appears to vary from one specimen to another. The scattering from Bakelite BT61/893 is generally poor, while that from such specimens of Fosterite as the author has examined has been fairly good. Experiments in the casting of Marco resin (S.C. 26*b*) carried out by C. Snell have so far failed to identify the casting conditions which produce this natural scattering. Of a number of specimens cast under as nearly as possible identical conditions, a few showed excellent scattering, while in others the property was entirely absent.

It has been found, however, that a satisfactory degree of scattering can be ensured either by putting into the styrene, before mixing with the resin, a small quantity of very finely divided silica, or by introducing colloidal platinum by obtaining sparks between two platinum electrodes immersed in the liquid. The models used in the illustrative examples in this paper were all machined from Marco resin containing finely divided silica.

The second property required in the material, homogeneity, is very difficult to achieve especially in large models. The possibility of lack of homogeneity is especially serious if the model under examination has no symmetry, for in that case conclusions must be drawn from observations at a single point. Scattered light observations, in fact, give considerable help in detecting such defects in the material, frequently revealing in the fringes irregularities which are not apparent in the total effect upon light which has traversed the whole thickness of the model. This lack of homogeneity, producing "mottle" in the photoelastic pattern, has been found to a greater or less extent in nearly all the specimens of Fosterite examined, as well as in the Marco resin cast in the laboratory. Experiments are still in progress to try to find a means of preventing its occurrence.

The illuminating system. The three requirements in the illuminating system are, strictly speaking, impossible of attainment simultaneously. If we adopt the usual method of obtaining a "parallel" beam by using a small intense source at the principal focus of a collimating lens, the intensity in the collimated beam (assuming the use of a source of given intrinsic intensity), is approximately proportional to the solid angle subtended at the principal plane of the lens by the area of the light source. This is the solid angle *AOB* in Fig. 7.

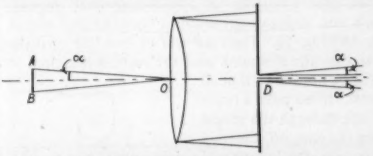


Fig. 7. Illustrating the divergence in a collimated light beam from a source *AB*.

But the departure from parallelism in the beam in any plane such as the plane of the diagram is measured by the angle α, the semi-angle of the section of this solid angle by the plane. Thus the greater the intensity the greater the departure from parallelism. Moreover, if the collimated beam is cut down by a slit in a diaphragm D to give a thin ribbon of light, the divergence α will persist in the beam, resulting in a progressive widening of the ribbon after passing the slit. This second effect is, in practice, much more serious than the first, for while a considerable degree of divergence may be introduced into a narrow beam without appreciable effect upon the sharpness or accuracy of the fringes, the width of the beam must, in general, be kept exceedingly small if the fringes are to be well defined and are to indicate the stresses with any degree of accuracy.

The necessary conditions of reasonably high intensity in a very narrow beam were achieved by use of the illuminating system shown in Fig. 8.

The arc of a 250 W. Sieray type mercury lamp by Siemens Bros. Ltd. provided an intense compact source. This was brought to a focus on an adjustable slit aperture D by a lens L_1, and a second lens L_2 formed an image of the illuminated slit at a point 0 in the model. The width of the beam of light at 0 was approximately the same as that of the slit, while the intensity in the beam and the degree of divergence were both dependent upon the aperture of the adjustable stop B. By moving the lens L_2 the focus 0 could be moved to any desired point on the axis of the beam so that, where necessary, all observations could be made in the immediate vicinity of

the narrowest part of the beam. The aperture permissible in stop B for a given focal length of L_2 depends upon the effect of obliquity of rays upon the difference of principal stresses in the plane of the wave-front. The normal stress in a direction inclined at an angle θ to a principal stress P involves the term $P\cos^2\theta$. Since the inclination to the axis of rays in the conical beam varies from zero on the axis of the beam to a maximum on the boundary, while the azimuth of the inclination also changes from point to point around the axis, the factor

Fig. 8. Optical system for obtaining an intense narrow beam of light at a point O of a photoelastic model.

$\cos^2\theta$ will vary in magnitude, and will operate upon the two principal stresses in varying ratio, as we go from ray to ray. The main effect therefore of using such a conical beam will be to make the fringes more diffuse. There will also be, in general, a small displacement of the centres of the fringes, the magnitude and sign of which will depend upon the relative magnitudes and signs of the principal stresses and upon their directions.

A preliminary estimate indicated that a maximum obliquity of about 4°, ($\cos^2 4° = 0{\cdot}995$), might be permissible in many cases, and this was allowed for by using a lens L_2 of diameter 1 in and focal length 4 in. Whether such an obliquity is permissible in any particular case may be determined by observing the effect upon the fringes of reducing the aperture of the stop B. If such reduction produces no measurable shift of the fringes, then the full aperture may be used without introducing sensible error. In none of the illustrations which follow was it found necessary to reduce this aperture.

Measurement of the fringes. The fringes seen in scattered light do not photograph well. With a monochromatic filter and a narrow beam the intensity of the light is feeble and a long exposure is necessary. (The photographs of Fig. 3 were obtained with a pinhole aperture approximately 0·02 in. in diameter and an exposure of 1 h.) If the width of the slit is increased, the fringes become less sharply defined and reflection from the surface of the model (which can rarely be completely eliminated), produces broad bright bands of light which make accurate measurement impossible. This effect is very marked in some of the photographs illustrating the examples which follow. These photographs were all taken with a much wider slit than was used for the actual measurements. The most accurate method of measuring the effect is to use a travelling microscope, and it was found that the best results were obtained with an instrument giving very low magnification. The microscope was provided with illuminated cross-wires and could be traversed parallel to the axis of the light beam by a micrometer screw. With a mercury green filter in the beam the whole and half-wave fringes seen with the two alternative positions of the polarizer may be measured, and in some cases these may be so closely spaced as to enable the relative retardation graph to be drawn with sufficient accuracy. In other cases it is necessary to obtain intermediate points on the curve, and this may be done by inserting a uniform field compensator in the path of the polarized beam. If the axes of the compensator are adjusted to be parallel to the axes of principal stress in the model (i.e. at 45° to the polarizer axis), a relative retardation of a fraction of a wavelength introduced into the beam before it enters the model shifts each fringe through a distance corresponding to this fraction, and allows of the determination of intermediate points on the graph.

The use of the compensator introduced a fresh difficulty, for owing to the convergence in the beam of light the passage through the quartz wedges led to a serious loss of definition in the fringes. The optical system was therefore modified to provide a region of much more nearly parallel light in which the compensator could be placed. The final arrangement was as shown in Fig. 9. The single lens L_2 was replaced by two lenses L_2 and L_3 each of 8 in focal length. The slit was at the principal focus of L_2, giving an approximately parallel beam between the two lenses in which the polarizer P and the compensator C were placed. Also it was found necessary to further reduce the obliquity of rays in this region by cutting down the length of the slit by means of a Vee slide which crossed it. Most of the actual measurements were made with a slit 0·005 to 0·01 in wide and about 0·1 in long. A further, and most valuable advantage accrued from the use of the compensator. If the relative retardation introduced by it is of opposite sign to that in the model, the black zero fringe may be moved from

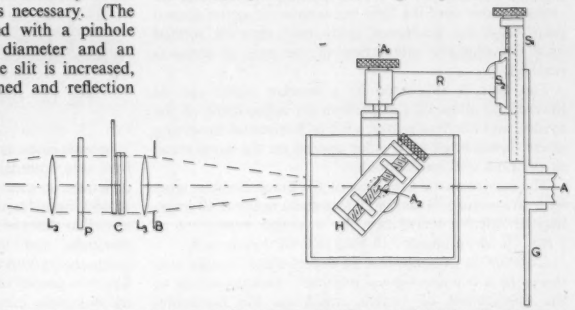


Fig. 9. Model holder and modified lens system for scattered light observations on "frozen" models

the boundary into the model, and all measurements may be made on this fringe. This does away with the need for a colour filter thereby greatly increasing the intensity of the light available and permitting the use of a much narrower slit.

Apparatus. The model, or the section of it to be examined, had to be held immersed in a liquid of the same refractive index, and a model-holder was required which could be adjusted for easy examination of the stresses along any path in the model. The final design adopted for this part of the apparatus is shown in Fig. 9. The model-holder assembly is mounted on a graduated circle G which can be rotated about an axis A fixed to coincide with the axis of the light beam. A pair of crossed slides S_1 and S_2 fitted with micrometer screws and attached to the circle carry the rod R, which in turn carries the model-holder H. This is a screw clamp capable of rotation about two perpendicular axes A_1

Fig. 10. Scattered light fringes in beam along horizontal diameter of disk under vertical thrust

and A_2. Preliminary setting about these two axes brings the model into such a position that the path along which it is desired to pass the beam is parallel to the axis of the optical system. The model is then moved in the plane of the wave-front by operation of the crossed slides until the light beam enters it at the desired point, and the graduated circle may then be rotated until the required orientation of the axes of stress is reached.

The tank is supported by a bracket which can be lowered to allow of the preliminary adjustment of the model, and the microscope, with its horizontal traversing screw, is mounted on a pillar carried on the same stand as the tank and model-holder.

The compensator used was of the Soleil-Babinet type. with an aperture 20 mm by 20 mm and a range of 10 wavelengths relative retardation for a wedge movement of 1 cm. It was graduated to read to 0·01 wavelength.

Example 1. *Comparison of experimental results with theory in a two-dimensional problem.* Determination of the distribution of vertical stress on the horizontal diameter of a disk under vertical load.

Fig. 10 is a photograph of the scattered light fringes in a polarized beam travelling along the horizontal diameter of the disk. Fig. 11 shows the graph plotted from measurements of these fringes, and Fig. 12 shows the distribution of the vertical stress as derived from the slope of the graph of Fig. 11. (In these examples the stresses have not been reduced to lb/in^2, but have been left in terms of "rate of change of relative retardation" for ease of comparison with the relative retardation graphs.) The value obtained for the vertical stress at the centre of the disk differs from the theoretical value by less than 3%.

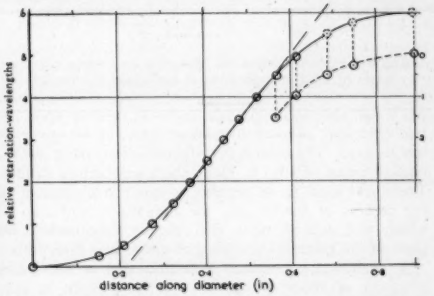


Fig. 11. Graph of fringes along horizontal diameter of disk

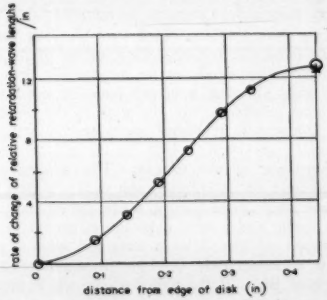


Fig. 12. Graph of vertical stress for one-half of the diameter of the disk

In such cases as this, in which the stresses are required from one boundary to another across a section, there is difficulty in measuring the relative retardation at the second boundary. This is overcome by reversing the model so that the light travels through it in the opposite direction, and taking another set of measurements commencing with the zero fringe on this second boundary. The two graphs obtained may then be combined to give the complete graph. This procedure was followed in examples 1, 2 and 3, the dotted curves in Figs. 11, 15 and 19 being drawn from the second set of observations.

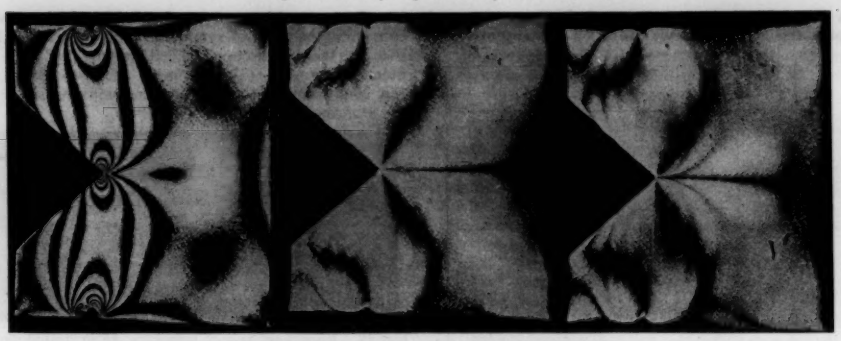


Fig. 13. "Two-dimensional" fringes in notched plate
(*a*) Isochromatics, (*b*) zero isoclinics, (*c*) 5° isoclinics.

Example 2. *Separation of the stresses in a two-dimensional problem.* The model chosen (Fig. 13) was a rectangular plate 1 in × 0·75 in × 0·24 in thick having a deep rectangular notch with a radius at its base of approximately 0·007 in. The plate was loaded under compression through two cylindrical pins resting in small notches in its top and bottom ends. The scattered light method was used to separate the stresses on the axis of symmetry through the notch.

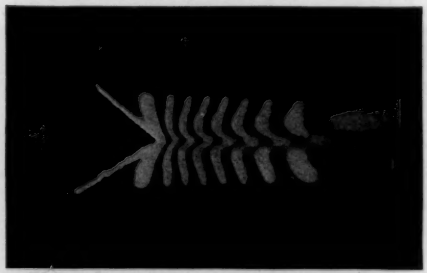


Fig. 14. Scattered light fringes in beam passing along axis of symmetry in notched plate

Fig. 13 shows the fringe-pattern obtained in the ordinary polariscope, and the zero and 5° isoclinic fringes. An inspection of these shows that in the neighbourhood of the notch both magnitudes and directions of the stresses are changing very rapidly from point to point, and the separation of the stresses in this region by any of the usual methods would be a laborious and difficult task. The scattered light method gives the solution with comparative ease, and with an accuracy which probably could not be achieved by any other method.

The scattering properties of this model were poor, and while the fringes were sufficiently clear for accurate measurement with the microscope no photograph of them could be obtained. Fig. 14 shows the fringes in a similar model and gives an indication of the rapid change in the stresses near the apex of the notch as we move to either side of the axis of symmetry.

Fig. 15 shows the graph of relative retardation plotted against distance obtained from these observations. In such a two-dimensional case we have some help in the interpolation between the boundary and the first fringe observed, for the magnitude of the stress at the boundary

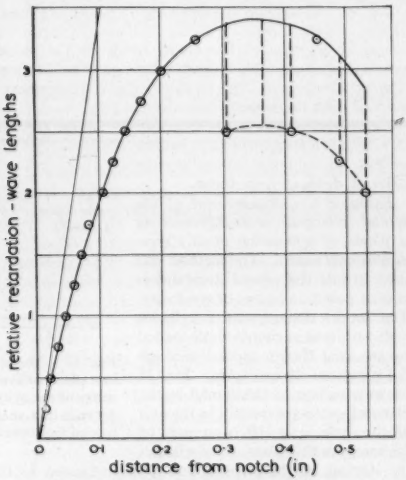


Fig. 15. Graph of fringes in notched plate

is given by the two-dimensional fringe-pattern, and using this information we can draw the tangent to the

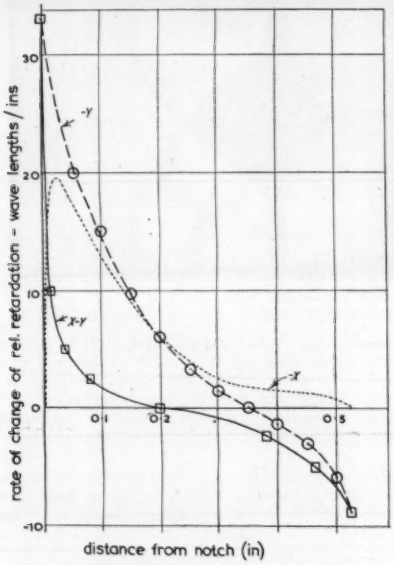


Fig. 16. Stresses on axis of symmetry in notched plate

graph at the origin. The curve which fits the plotted points and touches this tangent is thus quite well defined. The broken line in Fig. 16 shows the vertical stress Y derived from the slope of this curve, the continuous line is the graph of $X - Y$ taken from the fringe-pattern of Fig. 13, and the dotted line the stress X deduced from these.

Example 3. *Measurement of the second principal stress-difference in a plane of symmetry of a three-dimensional model.* The method was used to find the second stress difference at points on an axis of symmetry. The model chosen was a cylinder with a transverse circular hole loaded by an axial thrust applied through flat plates on its end faces. Fig. 17 shows a section of this model by the plane of symmetry normal to the axis of the hole, and AB is an axis of symmetry in this plane. We will call the stresses in the plane P and Q as indicated in the diagram, and the normal stress R. The stresses having

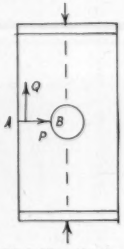


Fig. 17. Section in a plane of symmetry of the cylinder with transverse hole of Example 3.

been frozen into the model, a slice 0·05 in thick was cut containing the plane of symmetry. Observation of this slice in the polariscope yielded values of $P - Q$ at points along AB. (The points on the P–Q graph in Fig. 20 correspond to relative retardations measured by the Sénarmont method.)

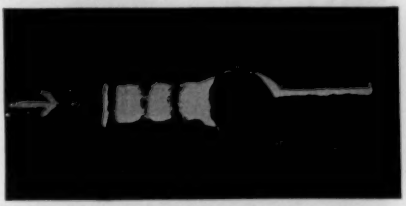


Fig. 18. Fringes on axis of symmetry AB in central plane of cylinder with hole

Fig. 18 shows the fringes in the scattered light. (Part of the slice has been cut away to allow of the second observation mentioned in Example 1.)

Fig. 19 is the graph of the scattered light observations, and Fig. 20 that of the stress-differences R–Q and P–Q, from which the values of Q and R at both boundaries may be deduced.

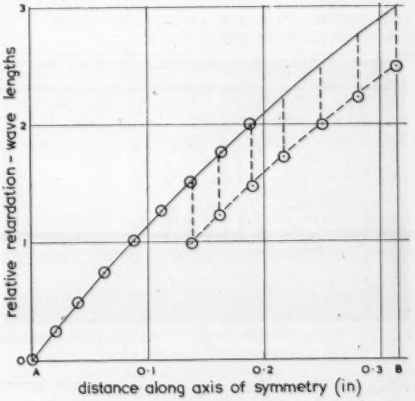


Fig. 19. Graph of fringes in cylinder with hole

Example 4. *Exploration of the stresses in the fillet of a shouldered shaft under torsion.* Fig. 21 shows the model used in this example. The shaft was subjected to torsion and the stresses frozen into it, and a preliminary inspection was made in the scattered light apparatus. This showed that in two small regions near the boundary there were patches of mottle which caused marked irregularities in the fringes. The distribution of the fringes in the rest of the model, however, was quite

symmetrical, and all observations could be made at points well removed from the faulty regions.

Fig. 22 shows the fringes in a beam passing through the central section *AB* (Fig. 21), indicating a symmetrical distribution of pure shear across the section.

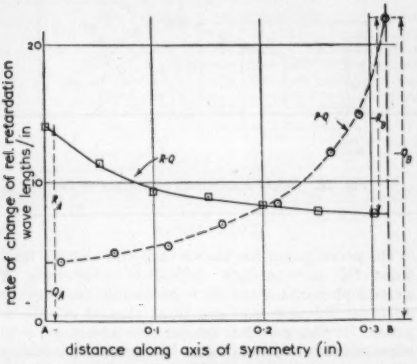


Fig. 20. Principal stress differences along *AB* in cylinder with hole

The important region of the model from the point of view of stresses is the surface of the fillets. The procedure for exploring the stresses in the fillets was as follows. The model was first adjusted so that the axis of the shaft was inclined at any desired angle θ to the axis of the incident beam. It was then moved to such a position that the axis of the beam was normal to the surface of the fillet at the point of incidence, the latter operation being facilitated by observing through the microscope and adjusting until the vertical and horizontal cross-wires appeared respectively tangential and normal to the boundary. The model was then rotated about the axis of the beam until maximum sharpness of the fringes

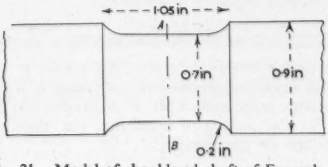


Fig. 21. Model of shouldered shaft of Example 4.

was observed. When the graph of relative retardation against distance along the normal was plotted, its slope at the origin gave the principal stress-difference at the corresponding point on the surface of the fillet. Fig. 23 shows the fringes in this model when set in position to measure the stresses at a point in the fillet where the inclination of the tangent to the shaft axis is 20°. (Lines representing the microscope cross-wires have been drawn on the photograph.)

In these observations it was found that a very small difference between the refractive indices of the model and the immersion fluid caused serious errors in the

Fig. 22. Fringes in mid-section of shaft under torsion

Fig. 23. Fringes along normal to surface of fillet at the point where the tangent is inclined at 20° to the axis

readings of fringe-positions near the point of entry of the beam (Fig. 24). Evaporation of benzene from the liquid made it impossible to keep the refractive index constant, and it was found necessary to cut away the front part of the model (the shaded portion in the figure), so that the scattered rays emerged normal to the surface.

Fig. 25 shows the graphs of observations at a number of points, and Fig. 26 the stress-distribution for the critical region. The results indicate a stress-concentration factor of about 1·7 at a point just beyond the beginning of the fillet. The directions of the principal stresses, indicated by the position for maximum sharpness of the fringes, were everywhere sensibly at 45° to the generator of the surface of the shaft.

The stress-concentration factor is considerably higher than that quoted by Timoshenko[8] for shafts of similar proportions investigated by an electrical analogy method. The figure he gives is about 1·3, but it is worth noting that the present observations would give that figure if the surface stress were deduced from the mean slope of the graph over the first 0·01 in instead of the slope of the tangent at the origin.

It is not easy to assess the accuracy of the results obtained by this method. In regions close to points of stress-concentration where the stresses are changing rapidly from point to point, settings on the fringes could be repeated to $\pm$ 0·001 in, though the possible error increases as the fringes become broader and more diffuse. Most of the fringes, moreover, show a certain amount of variation in width and density due to lack of homogeneity in the material, and this is probably responsible for some of the divergence of points from smooth curves. Also it is rarely possible to obtain accurate settings on a fringe of order lower than about 0·25 wavelength, as the fringe then begins to overlap the boundary. The point of entry of the light beam into the model is, however, very well defined, and the graphs obtained indicate that even with the errors of fitting curves and drawing tangents, this method gives a much better value for a point of concentration on the surface of a model than would be obtained by a more accurate measurement of the total relative retardation through the thinnest practicable slice cut from the model by a plane parallel to the tangent plane. The slope of the chord *OA* in the first graph of Fig. 25, for example, gives the mean rate of change through a thickness of 0·03 in, and this differs from the slope of the tangent at 0 by more than 15%. In the other graphs the difference is still more marked.

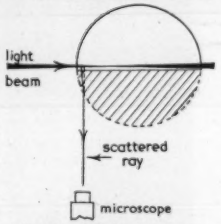


Fig. 24. Illustrating the production of large errors, due to difference of refractive indices of model and immersion fluid, in regions where the angle of incidence approaches 90°.

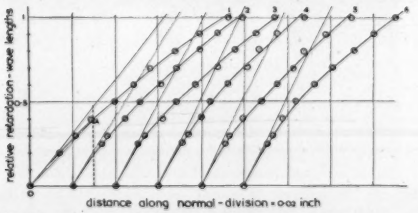


Fig. 25. Graphs of fringes along normals at points on the surface of shaft

(1) Mid-section 0·35 in from fillet. (2) 0·1 in from fillet. (3) Beginning of fillet. (4) 5° in fillet. (5) 10° in fillet. (6) 20° in fillet

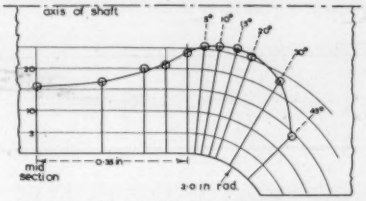


Fig. 26. Stress distribution on surface of shaft

CONCLUSION

The investigation has shown that with suitable technique the scattered-light method of observation of stressed photoelastic models is practicable and presents no more difficulty than any other method of investigation. It also possesses considerable advantages over other methods in certain cases. By sensibly eliminating errors due to averaging the observations over an area of the model, the method removes one of the main obstacles to accurate determination of stresses in three dimensions. Also it can be used with accuracy on quite small models; a matter of some importance in view of the difficulty and expense of obtaining large blocks of stress-free, homogeneous photoelastic material.

Although in all the examples given here the investigations have been carried out on "frozen" models, the method could be used equally well with models under load. This procedure would present certain technical difficulties, but these would not be insurmountable in most cases, and in certain problems such as those involving contact stresses in three dimensions, for which the frozen-stress process is unsuitable on account of the large strains introduced, scattered light observations offer the only means of investigation.

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Photoelastic determination of free boundary stresses on "frozen stress" models by an oblique incidence method

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The methods of determining the stress at the boundary of a three-dimensional photoelastic model are reviewed and the "oblique incidence" method as applied to boundary points is investigated in some detail. This method permits the complete determination of the boundary stress in a predetermined area of the boundary but is particularly susceptible to errors in observations. Tests have been carried out on a grooved cylinder in which a comparison has been made between oblique incidence determinations of the boundary stresses and determinations from slices cut in the directions of the boundary principal stresses. To obtain reasonable accuracy, extrapolation to the boundary and compensation methods of determining relative retardations are obligatory.

1. INTRODUCTION

The photoelastic technique known as the "frozen stress" method which was developed by Oppel[1] and Hetenyi,[2] enables the stress distribution in three-dimensional engineering components to be studied. A scale model of the component is machined from a suitable plastic and loads are applied to simulate the load distribution on the component. It is subjected to a heating and cooling cycle through which it acquires permanent birefringence persisting after the load is removed; this birefringence will give a quantitative estimate of the principal shear stresses at any point. Measurements of the birefringence may be made by slicing the model and viewing the slices in a polariscope. From the engineering point of view the usual requirement is firstly to locate the points where the maximum stresses occur and secondly to measure the stresses. The distribution of stress over the rest of the component is of secondary importance except in problems of a special nature which arise in the course of research. These stress concentrations nearly always occur at the surface of the component, usually at a concavity in the surface. The possible methods of locating the stress concentrations and measuring the stress at these points, using a "frozen stress" model, are noted below.

Location. (*a*) Symmetrical models, symmetrically loaded. In these cases there is generally no difficulty in deciding by inspection where the maximum stresses occur, likewise their directions are usually obvious. For example, a cylindrical bar with an annular groove loaded in tension would have a stress concentration at the base of the groove, and the higher principal stress would be parallel with the axis of the bar.

(*b*) Unsymmetrical models, or symmetrical models unsymmetrically loaded. Here again it is frequently possible to guess the positions of the maximum stresses, especially if the surface includes small radii. There is always an element of doubt, however, and in any case the principal stress directions would not necessarily be known. A procedure which has been recommended for these uncertain cases is to cut a thin slice parallel to the surface of the model in the region where the stress concentration is suspected to be, and to view it in a polariscope. The position of the greatest maximum shear stress in the surface can be obtained from relative retardation measurements, and the directions of the principal stresses follow from the plane polarized light isoclinic pattern. As an alternative, some workers recommend a separate test on the engineering part using brittle lacquer.

Measurement of the stress. Dependent on the criterion of failure which is adopted, one requires to know the maximum shear stress, the greater principal stress, or both principal stresses at the stress concentration. According to recent theories it may also be required to find the rate of change of stress in its vicinity. It is generally agreed that the most convenient and accurate way of getting the full information is to cut slices with their planes normal to the boundary in the directions of the surface principal stresses and to measure the relative retardations at the edges of the slices, the incident light being normal to the slices. A difficulty immediately presents itself here, since it is not possible to cut two such slices at right angles through the same point. With axially symmetrical models, there is no problem, since the necessary slices may be cut from different parts of the model which have the same stress system. In unsymmetrical cases we have the following possible methods. (In (*a*) and (*b*) it is assumed that the directions of the principal stresses are known.)

(*a*) Use two models, and cut slices as above in the directions of the maximum principal stresses in one case, and of the minimum in the other.

(*b*) With one model, cut slices in the directions of the maximum principal stresses and measure retardations. "Thaw" the slices and measure the lateral strain as described by Prigorovskii and Preiss,[3] which will give a measure of the second principal stress. A simple calculation of the amount of strain which may be expected under the most favourable conditions, using the values of effective Young's modulus and Poisson's ratio of known "frozen stress" plastics, will show that low accuracy must be expected for a slice of average thickness.

(*c*) If the maximum surface shear stress only is required, a surface slice may be cut from the region of the surface under study. Leven[4] has suggested using

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such a slice, and progressively thinning it down, taking relative retardation measurements at each step. The results are then extrapolated to an infinitesimally small thickness of slice. The cutting process is obviously not at all easy, and Leven has pointed out that the precise measurement of thickness of the slice, which is necessary for the conversion of the observations to stress, is difficult.

(*d*) For the complete exploration of the stress in the vicinity of a stress concentration, employing one model only, it is apparent that oblique transmission in some form must be used, i.e. the light must pass for some retardation measurements at an angle other than 90° to the slice. For example, if we have prior knowledge of the stress directions, slices could be cut following the directions of one set of the surface maximum shear stress trajectories, observations being taken parallel to the directions of the surface principal stresses, i.e. at 45° to the slice normal. For the general case of unknown location of stress concentration and unknown stress direction it appears that there is one technique which has promise of being convenient to apply and of giving the full state of the boundary stress with reasonable accuracy, and that is an adaptation of the "oblique incidence" method of Drucker.[5] Drucker first used the method to separate principal stresses in a plane stress model and drew attention to its possible application in three dimensional work. The general principles were first put forward in a paper by Drucker and Mindlin[6] and Frocht[7] has applied the method to a "frozen stress" model representing a torsion problem. It is the purpose of this paper to look into the method as a possible means of exploring the stress at the free boundaries of three-dimensional models, paying particular regard to stress concentrations where the most adverse conditions will apply.

2. "OBLIQUE INCIDENCE" METHOD

The "oblique incidence" method devised by Drucker consists in taking relative retardation observations at three different angles of incidence at each point in the model (in this case a slice from the model) where it is required to find the stress. From these observations it is possible to calculate the three principal stress differences and the principal stress directions. It is necessary to have prior knowledge of the direction of one principal stress. This condition applies at the free boundary of a model where the principal stress which is normal to the boundary is zero. The derivation of the equations using only the fundamental properties of the index ellipsoid is given below. (Throughout the paper the terms "ray" and "wave normal" are taken as synonymous since they are practically identical for stress-birefringence.)

The birefringence at any point in the model may be defined by the index ellipsoid which is an ellipsoid with principal semi-axes proportional to the principal refractive indices. The principal stresses p, q, r are in the directions of the principal axes and are related to the refractive indices by the following equations:

$$\left.\begin{aligned}\mu_p - \mu_o &= C_1 p + C_2(q + r)\\ \mu_q - \mu_o &= C_1 q + C_2(p + r)\\ \mu_r - \mu_o &= C_1 r + C_2(p + q)\end{aligned}\right\}$$

μ_o = refractive index of unstressed material

μ_p, μ_q, μ_r = refractive indices for magnetic vectors coinciding with the directions of p, q and r

C_1, C_2 are secondary stress optical coefficients.

The above is the general expression for the stress-optic law.

It is only possible by normal photoelastic methods to measure the difference of two refractive indices, and the stress-optic law takes Brewster's form

$$\mu_p - \mu_q = C(p - q), \text{ etc. } [C = C_1 - C_2]$$

where C is the stress-optical coefficient for the material. The index ellipsoid is useful in that refractive indices and vibration directions may be predicted for an arbitrary direction of the incident ray from its geometry.

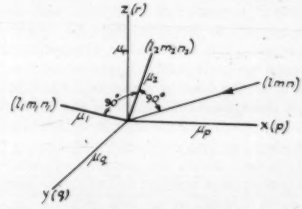

Fig. 1. Incident and propagated rays in a birefringent element

Let the co-ordinates x, y, z of the system be in the directions of the principal axes of the ellipsoid (Fig. 1). A ray passing in a direction $\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n}$ will be split into two components polarized in mutually perpendicular planes; the direction cosines of the vibrations being $\boldsymbol{l_1 m_1 n_1}$ and $\boldsymbol{l_2 m_2 n_2}$ respectively. The three directions $\boldsymbol{lmn}$, $\boldsymbol{l_1 m_1 n_1}$, $\boldsymbol{l_2 m_2 n_2}$ are orthogonal. The components have refractive indices μ_1, μ_2, proportional to the major and minor semi-axes of the elliptic central section of the ellipsoid to which the incident ray is normal, and the vibration directions are parallel to these axes. From the geometry of the ellipsoid we have

$$1/\mu_1^2 = \boldsymbol{l}_1^2/\mu_p^2 + \boldsymbol{m}_1^2/\mu_q^2 + \boldsymbol{n}_1^2/\mu_r^2$$

and

$$1/\mu_2^2 = \boldsymbol{l}_2^2/\mu_p^2 + \boldsymbol{m}_2^2/\mu_q^2 + \boldsymbol{n}_2^2/\mu_r^2$$

Hence

$$1/\mu_1^2 - 1/\mu_2^2 = (\boldsymbol{l}_1^2 - \boldsymbol{l}_2^2)(1/\mu_p^2 - 1/\mu_r^2) + (\boldsymbol{m}_1^2 - \boldsymbol{m}_2^2)(1/\mu_q^2 - 1/\mu_r^2)$$

Since all refractive index differences are small we may write the above expression:

$$\begin{aligned}\mu_1 - \mu_2 &= (\boldsymbol{l}_1^2 - \boldsymbol{l}_2^2)(\mu_p - \mu_r) + (\boldsymbol{m}_1^2 - \boldsymbol{m}_2^2)(\mu_q - \mu_r)\\ &= C[(\boldsymbol{l}_1^2 - \boldsymbol{l}_2^2)(p - r) + (\boldsymbol{m}_1^2 - \boldsymbol{m}_2^2)(q - r)] \quad (1)\end{aligned}$$

The relative retardation in wavelengths for unit thickness of material is given by $n = (\mu_1 - \mu_2)/\lambda$. If unit thickness

is taken as one inch $\lambda/C = F$ (the material fringe value, i.e. the stress difference/fringe). Hence

$$n = 1/F(p - r)(l_1^2 - l_2^2) + (q - r)(m_1^2 - m_2^2) \quad (2)$$

This is the fundamental equation for oblique transmission retardation measurements.

In the oblique incidence method as generally used, the light travels in a direction perpendicular to one of the principal stresses. Taking this "pivot axis" as the r stress direction equation (2) becomes

$$n = 1/F\,[(p - r)\sin^2\phi + (q - r)\cos^2\phi]$$

where ϕ is the angle made by the incident ray with the p axis. As shown in Fig. 2 three observations are taken at each point of a slice where it is required to find the stress, with angles of incidence $\pm\alpha$ and normal. From the three measurements, $p - r$, $q - r$, and the orientations of p and q with respect to the slice may be calculated.

Let the values of n corresponding to angles of incidence $+\alpha$, normal and $-\alpha$ be n_1, n_2 and n_3 respectively, and let θ be the angle made by the normal to the slice with the p axis. A positive value of θ is assumed when it is traced by a counter clockwise rotation from the p axis to the normal, likewise a positive value of α is taken for a counter clockwise rotation from the normal to the direction of incidence.

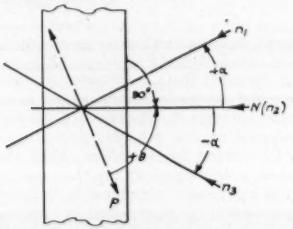

Fig. 2. Angle and relative retardation nomenclature in "oblique incidence" observations. View looking inwards at the surface of the model

We have

$$Fn_1 = (p - r)\sin^2(\theta + \alpha) + (q - r)\cos^2(\theta + \alpha)$$
$$Fn_2 = (p - r)\sin^2\theta + (q - r)\cos^2\theta$$
$$Fn_3 = (p - r)\sin^2(\theta - \alpha) + (q - r)\cos^2(\theta - \alpha)$$

By putting the equations in terms of double angles it may be shown that

$$\tan 2\theta = \frac{n_1 - n_3}{n_1 - 2n_2 + n_3}\tan\alpha$$

$$p - q = F\left[\left(\frac{n_1 - n_3}{\sin 2\alpha}\right)^2 + \left(\frac{n_1 - 2n_2 + n_3}{1 - \cos 2\alpha}\right)^2\right]^{\frac{1}{2}}$$

$$p + q - 2r = F\left(\frac{n_1 - 2n_2\cos 2\alpha + n_3}{1 - \cos 2\alpha}\right)$$

from which we may derive $p - r$ and $q - r$ since

$$p - r = \tfrac{1}{2}[(p + q - 2r) + (p - q)]$$
$$q - r = \tfrac{1}{2}[(p + q - 2r) - (p - q)]$$

It is better from the point of view of simplifying computation to keep to one particular value of α for all sets of observations. Obvious values one might choose are 30°, 45° or 60°.

The final accuracy depends on the errors in the three observations, the effect of these errors being greater or less dependent on the value of α selected. For particular errors in n_1, n_2 and n_3 a value of α of 60° will give the least errors in $p - r$, $q - r$, and θ; 30° will result in the greatest error. From this aspect 60° is therefore the best value for α, but there are other considerations. For example, this high angle of incidence increases the effective thickness of the slice and difficulties due to rate of change of stress are intensified. Moreover, it is found in practice that scatter of light at the surface of the slice reduces sensitivity of compensation to such a degree that individual measurements are reduced in accuracy. Taking all factors into account it was decided that $\alpha = 45°$ was the best compromise, and this is the angle which has been used in the investigation. The formulae may be simplified by substituting as below.

Summary of formulae to be used in calculation.

angle of incidence	*relative retardation in wavelengths for 1 in thickness*
$+45°$	n_1
Normal	n_2
$-45°$	n_3

Let $U = n_1 - n_2$, $V = n_3 - n_2$, $W = n_1 + n_3$

$$\theta = \tfrac{1}{2}\tan^{-1}\left(\frac{U - V}{U + V}\right) \quad (3)$$

$$p - q = S = F\,[2(U^2 + V^2)]^{\frac{1}{2}} \quad (4)$$

$$p + q - 2r = R = FW$$
$$p - r = \tfrac{1}{2}(R + S)$$
$$q - r = \tfrac{1}{2}(R - S)$$

At the free boundary of a model r is zero, hence the principal stresses in the surface of the model are determinable.

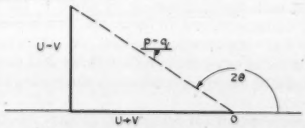

Fig. 3. Graphical construction for θ and $(p - q)$

The apparent ambiguity of 90° in the value of θ may be resolved by a graphical construction analogous to Mohr's circle. A right-angled triangle is drawn with the base line proportional to $U + V$ and the vertical proportional to $U - V$ as shown in Fig. 3. $U + V$ is drawn right or left from O depending on whether it is positive or negative; likewise $U - V$ is drawn up or down. The hypotenuse is proportional to $(p - q)/F$ and the angle between the base line and the hypotenuse is 2θ. The quadrant in which the hypotenuse is located decides whether 90° must be added to the calculated value of θ.

3. MAGNIFICATION OF OBSERVATIONAL ERRORS DUE TO THE NATURE OF THE FORMULAE

Differentiate equations (3) and (4). If the errors are relatively small compared with U and V the differentials may be replaced by errors ϵ_θ, ϵ_S, ϵ_V. Thus

$$\epsilon_\theta = \frac{V\epsilon_U - U\epsilon_V}{U^2 + V^2}$$

$$\epsilon_s = \frac{\sqrt{2}F(U\epsilon_U + V\epsilon_V)}{(U^2 + V^2)^{\frac{1}{2}}} \qquad (5)$$

It will be seen that when circumstances are such that the effect of observational errors on θ is a maximum, the effect on S will be a minimum, and vice versa. As an example of an unfavourable case for S let $\epsilon_U = \epsilon_V$. The error ϵ_S is then approximately equal to $2\epsilon_U$ over a wide range of values of U and V. Let the true values of n be n_1, n_2 and n_3 and the observed values n_1', n_2', n_3'. Let

$$n_1' = n_1 - \epsilon \qquad \epsilon_U = 2\epsilon$$
$$n_2' = n_2 + \epsilon \qquad \epsilon_V = 2\epsilon$$
$$n_3' = n_3 - \epsilon \qquad \epsilon_W = -2\epsilon$$

and it is found that $\epsilon_S \simeq 4F\epsilon$

But
$$p - r = \tfrac{1}{2}[R + S]$$
$$q - r = \tfrac{1}{2}[R - S]$$

Thus the error in $p - r$ is $F\epsilon$ and in $q - r$ is $3F\epsilon$. As a rough guide one may state that the possible error in the evaluation of $p - r$ or $q - r$ is of the order of the sum of the errors in the three observations. The error in the determination of the angle θ may be great if $p \simeq q$, i.e. if the stress in the pq plane approximates to biaxial tension or compression.

It is appropriate here to remark on the possible shear stress error as indicated by formula (5). Let the observational errors be assumed to have about the same values at all points in the model, then the percentage errors calculated on the actual maximum shear stresses at points where the maximum shear stress is low may be very high. Elastic problems, however, are generally concerned with the distribution of stress in the vicinity of stress concentrations or over the component as a whole. The important criterion is, therefore, the absolute error at any point, and it follows that percentage errors would be more fairly based on the maximum stress in the model.

It is apparent that for reasonable accuracy in the final estimation of stress the individual observations must be made to high accuracy. In Section VI there is a discussion on observational errors with indications of methods for reducing them.

4. THE TECHNIQUE APPLIED TO BOUNDARY POINTS

It is suggested that a number of parallel slots evenly spaced should be milled in the region of the model to be investigated (Fig. 4). The milling cutter would be as thin as possible and would be taken into the model to a depth of, say, $\frac{1}{4}$ in from the lowest point of a concavity. The cuts would be as far as possible normal to the surface, and in any event their angle with respect to the tangent plane at the surface where measurements are required should not be more than 30°. The material left between the slots constitutes the slices, and can be removed by cutting away with a hand fretsaw. The slices after light lapping would be clamped in a swivel head and held in an immersion cell containing liquid of matched refractive index in the polariscope field. The swivel head should enable the slice to be adjusted so that the light passes parallel to the tangent plane at the boundary point under study, and also the turning of the slice about the r stress to $\pm$ 45° when required.

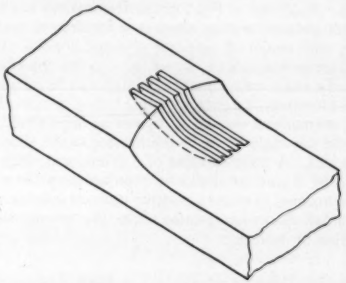

Fig. 4. Sketch showing slots cut across stress concentration

It should be noted that the formulae are based on the assumption that the stresses vary neither in magnitude nor direction through the thickness of the slice. At a stress concentration one can only approximate to this condition by making the slice thin. Since the relative retardation is proportional to the thickness, it follows that sensitive compensation methods must be used. The maximum sensitivity to be expected in unassisted visual compensation is of the order of 0·01 wavelength. Unless impractically large and expensive models are used, this is not good enough, and it is necessary to improve the sensitivity of the compensation by some method such as the photoelectric photometer described by Brown and Hickson.[8] In this, a small aperture is placed at the point under study on the projected image, and when compensation is applied the photometer is used to detect the setting for minimum illumination.

It is not possible to take an observation actually at the boundary of the model, i.e. the edge of the slice, as the photometer aperture has finite dimensions and the boundary itself is generally curved. It is, therefore, necessary to resort to extrapolation of observations taken within the boundary, and since the rate of change of stress can be very high at a stress concentration, this involves taking several observations very close to the boundary at each angle of incidence. Practical details of the procedure are given in the next section which deals with a test which was devised to investigate the feasibility of the method.

5. EXPERIMENT TO CHECK THE METHOD

An experiment was carried out in which a comparison could be made between results calculated from oblique incidence observations and from normal incidence observations on slices cut with two principal stresses in the plane of the slice. A cylinder 3 in long by $1\frac{1}{2}$ in diameter was machined from Fosterite. A semi-circular cross-section groove of radius $\frac{1}{4}$ in was cut in the cylindrical surface as shown in Fig. 5. The ends of

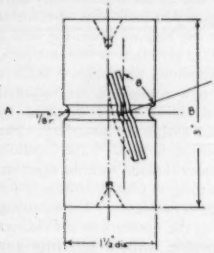

Fig. 5. Diagram showing cylindrical model with typical pair of saw cuts

the model were provided with centres through which compressive load was applied. The intention was to provide a symmetrical model loaded along its axis, the semicircular groove producing a stress concentration. The load itself was applied by a spring and was transmitted through accurate line-up centres in a jig as illustrated in Fig. 7. Its value was measured approximately by the compression of the spring and amounted to 135 lb at room temperature, being less, of course, at the critical temperature due to the distortion of the model. The model, mounted in the jig, was placed in an electric oven and the temperature was raised from room temperature to 70° C in $\frac{1}{4}$ h; it was maintained at 70° C for $2\frac{1}{2}$ h, and then lowered over a period of five hours.

Slicing. After removal from the jig, slots were cut in the model as indicated in Fig. 5. The slots were machined with a $\frac{1}{16}$ in side-and-face milling cutter and were in pairs, the material between a pair constituting the slice of thickness about 0·04 in. Data on cutting conditions is given below.

Cutter	Side-and-face 4 in diameter × $\frac{1}{16}$ in. Pitch $\frac{3}{8}$ in.
Speed	364 r.p.m.
Feed	0·5 in p.m.

A large volume of standard machine coolant was used. All slices were cut with their planes at right angles to the tangent planes at the base of the groove; the saw cuts were taken in to a depth of $\frac{3}{8}$ in from the cylindrical surface, and the central portions of the slices were removed with a hand fretsaw. Slices were cut at the following orientations with respect to the axis of the model as shown in Fig. 6.

(*a*) Three axial slices $90°_1$, $90°_2$, $90°_3$ disposed symmetrically at 120° spacing round the model.

(*b*) Three transverse slices at the base of the groove $0°_1$, $0°_2$, $0°_3$ also at 120° spacing.

(*c*) Other slices at 60°, − 75° and − 30°.

The angles above refer to the angle of the normal to the slice with respect to a line in the surface parallel to the axis of the model, i.e. the *p* stress direction. (*a*) and (*b*) were the standard slices mentioned previously with two principal stresses in the plane of the slice.

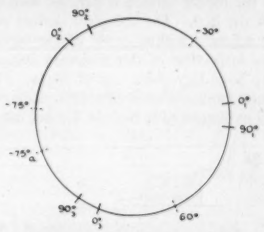

Fig. 6. Relative positions with respect to the model of slice reference points

This figure is the section *AB* of Fig. 5 at right angles to the axis. The points marked round the periphery of the circle are where the central planes of the slices meet the bottom of the groove; the angles marked at the points are slice inclinations θ, Fig. 5.

Later another slice was cut at − 75° and designated − 75°a.

The slices, after removal from the model, were rubbed gently on a cast iron lap with water and Aloxite optical smoothing powder No. 95 by Carborundum Co. Ltd.: this treatment producing a fine grain matt surface, removing machine marks and edge burrs. The slices were not polished after lapping since rounding at the edges almost certainly would have occurred; sufficient transparency is given to the surfaces by the use of an immersion liquid.

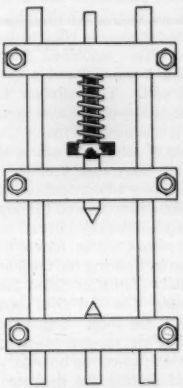

Fig. 7. Loading jig

Mounting of slices and immersion arrangements. The slices were clamped in turn in a swivel head shown in Fig. 8. The device was designed to facilitate setting a slice to the correct orientation in the polariscope. A spring *A* with 3-pronged end presses the slice against a flat surface on the central plane of a square section rod *B*. This rod is clamped in a brass plate *C* which rests on a graduated Perspex dial *D* machined with a spherical upper surface of radius $3\frac{1}{4}$ in; the plate and dial were lapped together to give a

good fit. The dial has a $1\frac{1}{2}$ in diameter hole at the centre and may rotate about a vertical axis. The Perspex frame *E* rests on the lower bars of the straining frame of the optical bench and straddles the immersion cell. The slice is set so that the boundary point under study is at the centre of curvature of the spherical surface of the dial, and the brass plate *C*, which retains its position by friction, is moved until the normal to the model surface is parallel with the axis of rotation of the dial. The spherical seating permits the slice to be adjusted within $\pm$ 10° of the vertical in any azimuth by inspection of the projected image, without moving the boundary point out of focus. (The device would require modification for the exploration of a portion of a model as suggested in Section 4, since adjustment in inclination up to 30° would be necessary to allow for cases where the normal to the surface does not lie in the plane of the slice.) The straining frame is fitted with accurate guides and lifting screw which permit the precise adjustment in a vertical direction of the swivel head—immersion cell assembly. It is only necessary to clamp a dial gauge to the base with its anvil pressing on one of the bars of the straining frame to allow vertical movements of the slice to be made and measured to 0·0001 in.

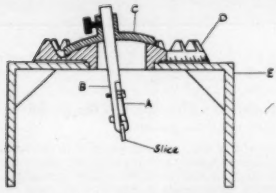

Fig. 8. Swivel head

The glass immersion cell is filled with a mixture of α-brom-naphthalene and medicinal paraffin of the same refractive index as Fosterite (for mercury green illumination). The matching of refractive indices is effected by immersing a small right-angled prism machined from Fosterite and viewing through it a wire placed a few inches behind the immersion cell. The mixture is accurately matched when no deviation of the wire seen through the prism is detectable, it is considered that when this method is used the difference of refractive indices of the liquid and prism would be not more than 0·0002.

Observation procedure. Preliminary adjustment of the slice is made by viewing the projected image, and turning the dial until the light passes approximately normal to the plane of the slice. The brass plate *C* is then moved to tilt the slice about this normal so as to bring the tangent at the boundary point under study (hereafter called the reference point) to the horizontal. The dial *D* is then rotated to give an end-on view of the slice. The brass plate may now be moved to bring the tangent plane at the reference point to the horizontal; when the boundary is concave the immersion liquid permits the projected view of the reference point region to be seen through the thickness of the slice. Turning the dial back through 90° will set the slice to the normal incidence position, and the rotation axis of the dial will be parallel with the normal to the boundary at the reference point.

The compensation technique is as described by Brown and Hickson.[8] The photometer aperture, which is in an adjustable screen, is moved until the image of the reference point is at its centre. This operation is performed most easily when the slice is turned so that the incident light is parallel to one of the lines of curvature of the boundary. It is necessary to make this adjustment as exactly as possible; with the help of a viewing lens it should be possible to make the setting to 0·005 in which, since the magnification is about 10, represents 0·0005 in on the slice. The dial is now rotated to the required incidence angle and the straining frame dropped by 0·005 in to bring the aperture within the boundary. The Tardy method of compensation is used[9]; the isoclinic setting of polarizer and analyser is made with the aperture in this position, and this setting is usually sufficient for all observations at one angle of incidence. The straining frame is moved back to bring the aperture to the nearest point to the boundary possible without allowing any part of the boundary to cut across the aperture. Apertures of diameter $\frac{1}{24}$ in and $\frac{1}{12}$ in were used in the tests; in these cases it was found that displacements from the original setting of 0·003 in and 0·0025 in respectively were suitable. Tardy compensation is now applied and observations taken at displacements of 0·002 in, e.g. 0·003 in, 0·005 in, 0·007 in, 0·009 in, 0·011 in, 0·013 in from the reference point. In the tests each angle of incidence was repeated with the rays passing in the opposite direction through the slice, resetting to the boundary. It is not suggested that the number of readings specified above is obligatory; they were taken with a view to estimating repeatability and curvature of the relative retardation—distance graphs, also to verifying the absence of spurious surface stress. Slices were not left in the immersion cell for more than two hours as there is a suspicion that the immersion liquid may cause some relaxation of stress in course of time.

Observations. (*a*) Thickness. The thickness of each slice near the reference point, i.e. at the base of the groove, was measured by means of a Cambridge travelling microscope focused on the edge. There was no significant rounding of the edge and this thickness measurement was, therefore, taken as applying to all points near the boundary.

(*b*) Relative retardation measurement. In general six retardation measurements were made at each setting as described above. Measurements were made at normal incidence only on the standard slices and at $\pm$ 45° and normal incidence on the oblique slices. A set of retardation measurements was also made in the direction of the principal stress nearest to the normal on each of the oblique slices. Attempts were made to measure the retardation in the direction of the other principal stress but reduction in light intensity due to surface scatter

reduced sensitivity of compensation to such a degree that results were not considered worth recording. This effect is one of the chief reasons for restricting angles of incidence to about 50°. (Note.—The effective angle of incidence, if the boundary normal is at 30° to the plane of the slice and the nominal angle of incidence is 45°, would be 52°.)

Two special sets of observations were taken on slice 90°, at normal incidence. One was a continuation of the edge retardation measurements to 0·03 in from the edge, the purpose being to verify lack of "rind" effect. The other consisted of two series of six observations on lines parallel with the normal at the reference point, the intention being to estimate the rate of change of stress in the lateral direction.

After all the above measurements had been made, it was decided that the change of stress through the thickness of the slice was having a significant effect on the results. The oblique slices and the 0° slices were, therefore, lapped down to about 0·02 in and the observations were repeated. Unfortunately, it soon became apparent that prolonged lapping had affected the stress in the slices. There was sufficient material left to cut one more slice at − 75° obliquity and this was machined with a 0·03 in milling cutter to a thickness of 0·02 in, lapped for a short time, and the stress was measured as above.

6. DISCUSSION ON ERRORS

We have seen in Section 3 that the oblique incidence method is very susceptible to error in the three observations. These errors are of various types and most are subject to reduction by the use of good measurement technique, or by the application of corrections. They have been considered in detail and it is hoped to publish shortly a survey of errors in observations near the boundary of a model.

Disregarding errors due to the difference in mechanical properties of the model and the engineering component it represents, the errors may be divided into three main types:

(1) limitations of the material, (2) measurement errors, (3) errors due to finite thickness of slice.

Although no attempt is made here to give a complete account of the errors, some general comments and conclusions are recorded.

With regard to type 1, the errors which may be introduced by the use of material which is subject to "time-edge" or "rind" effect may be so great as to invalidate the results. Fosterite has been found to be suitable since the material is virtually free from these effects and machining stress may also be avoided, both in machining the model and slicing, by adhering to the recommendations in Section 5. This plastic as supplied cannot be said to be completely homogeneous and annealing will not remove all the initial birefringence. The initial stress may be allowed for by "thawing" the slices and subtracting the residual stress vectorially from the stresses calculated from the oblique incidence observations.

Errors of type 2 may be minimized by careful measurement technique. Measurements of thickness may be made to 2 μ using the travelling microscope; this accuracy is adequate for 0·02 in thick slices, which is about the minimum thickness for convenient handling. If the swivel head is well constructed angle settings may be regarded as free from error, and as an error due to imperfect location of the aperture with respect to the reference point is added to all three measurements, it will only result in an error of about the same magnitude in the computed stress. It is necessary that error in location should be as low as possible, since the rate of change of stress near the boundary may be very high. The most important polariscope error which may be encountered is that due to error in the relative retardations of the quarter-wave plates. An expression has been calculated for the true retardation in terms of Tardy compensation reading when the errors of the quarter-wave plates are known. An error due to the finite size of photometer aperture has been investigated and amounts to a very small quantity which is negligible in the experimental example.

Errors of type 3 are due to variations of the stresses through the thickness of the slice and to the curvature of the boundary. Unless a preliminary investigation has been made which will give an approximate idea of the rate of change of stress in the boundary no correction can be made for the variation of magnitude or direction of the stress through the thickness of the slice. It is, however, possible to make a correction for the Tardy compensation error which will result from variation of direction ("rotation") and this may amount to a significant quantity when dealing with low magnitudes and high rates of rotation. Curvature of the boundary results in uncertainty as to the exact position, with respect to the reference point, to be assigned to the boundary. Although exact correction for this effect is not feasible, a correction based on the assumption that the stress is dependent only on the distance of the ray from the boundary in a direction parallel with the normal at the reference point, and that the relative retardations due to elements of the path are additive, will provide a much closer approximation to the true stress at the reference point than a simple extrapolation to it.

7. RESULTS

Typical curves of relative retardation/in thickness versus distance from reference point are shown in Fig. 9. The plotted points were calculated from the average of two observations at each aperture location. Actual relative retardations range from 2·2 fringes to 0·2 fringe and compensation readings were taken to 0·002 fringe. Agreement between two observations at each point was generally better than 0·005 fringe. Observations on the 90° slices at normal incidence were made with the two sizes of aperture 0·031 in and 0·047 in; a fortnight had elapsed between the two sets of observations which demonstrate the absence of time-edge effect, and also the

insignificance of the effect of the size of aperture within this range. It will be noticed that the curves in Fig. 9 are remarkably free from scatter, which constitutes a general check on the location of the aperture and compensation technique.

Fig. 10 give the values of $(p - r)/F$ and $(q - r)/F$ for the four oblique slices. Although it is the purpose of this paper to investigate the stress actually at the boundary; to avoid the uncertainty which would arise if an excessive number of extrapolations are made, the values of $p - r$ and $q - r$ have been calculated by the oblique incidence formulae for each location, and these results have been extrapolated to the boundary. The values interpolated from the axial and transverse slices, and the relative retardations per inch calculated from observations on the oblique slices in the direction of one of the principal stresses, are also shown. The relative retardations corresponding to p and q at the boundary are given by the intersections of the curves with the zero distance ordinate.

In this test, the errors of types 2 and 3 are negligibly small with the exception of that due to change of magnitude of stress through the thickness, the Tardy "rotation" error, and the error due to curvature of the boundary. As mentioned in the discussion on errors, the change of stress error requires previous knowledge of the stress distribution for correction, and as this had not been determined fully it was not possible to apply a correction. Correction for the "rotation" error would require facilities for independent turning of polarizer and analyser, for which the polariscope at present is not adapted. It was therefore only possible to apply the corrections for boundary curvature.

The table gives a comparison between the actual boundary stresses derived by the three methods, a figure of 3·85 being taken for the effective fringe value of the material.

A—oblique incidence computation.

B—interpolated from standard slices.

C—observations on slice in direction of principal stress.

Comparison between boundary stresses derived by three methods

slice orientation	p stress (lb/in²) A	B	C	q stress (lb/in²) A	B	C
60°	179	191	191	58	55	—
−30°	177	189	—	60	57	56
−75°	184	186	182	50	54	—
−75°a	179	187	182	63	55	—

All stresses tabulated above are compressive.

Since the load after deformation of the model had taken place was 111 lb, the mean axial stress across the section at the base of the groove was 90 lb/in². The stress concentration factor was therefore 2·1.

Some general comments may be made on the accuracy of the estimation of boundary stress.

(1) For all oblique slices the calculated values of p and q are within 6% of the maximum principal stress as measured on the standard slices.

(2) The 0·02 in thick slice — 75°a gave results which are slightly worse than the 0·035 in thick slice — 75°. As the errors, apart from material errors, must be less in general for the 0·02 in thick slice, it appears likely that there is more error due to machining stress or stress relaxation.

(3) The boundary stresses extrapolated from observations in the direction of one principal stress agree within 3% of the interpolated values.

(4) The agreement between the calculated angles of the principal stresses with respect to the axis is good. The discrepancy amounts to not more than 3½° for any set of three observations on any slice.

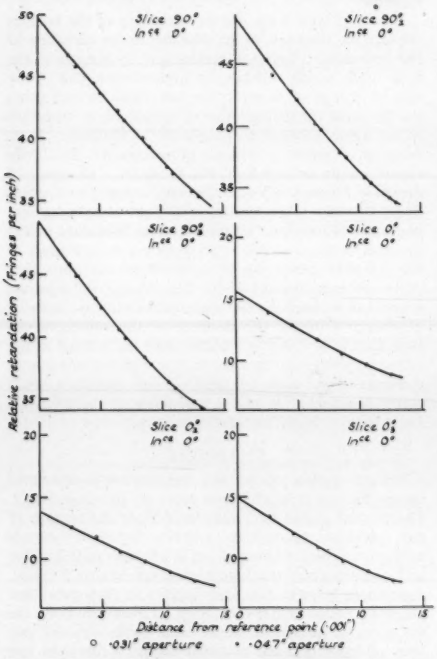

Fig. 9. Typical observations at one incident angle reduced to relative retardation per inch path

8. CONCLUSION

The oblique incidence method applied to the determination of boundary stresses can give results which are within the accuracy normally required in engineering problems. An investigation of the possible errors which

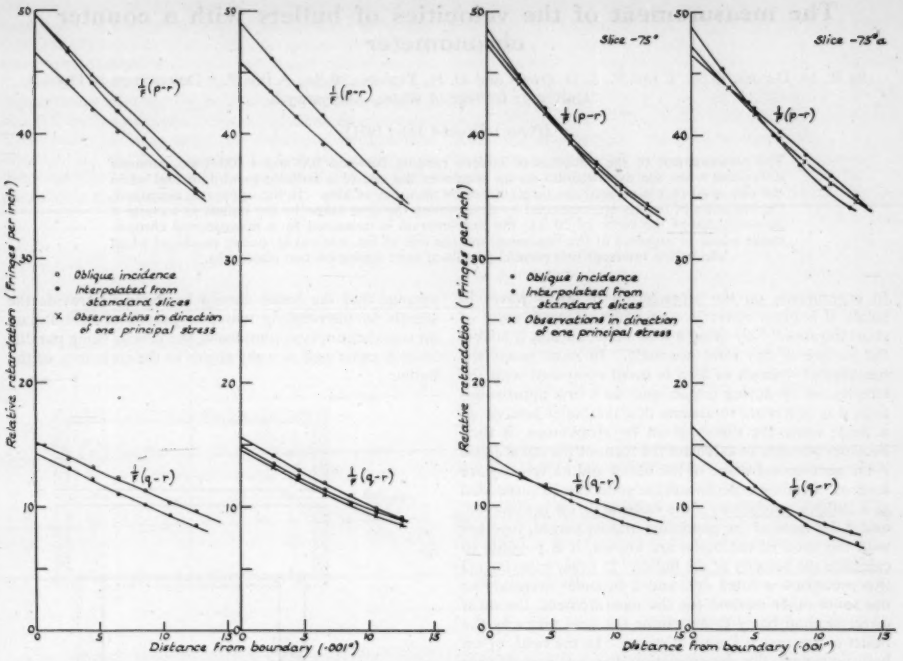

Fig. 10. Comparison of results calculated by oblique incidence formulae with interpolated values and direct observations

may arise in the photoelastic determination of the stress at a stress concentration shows that it is theoretically possible to attain an accuracy of 3% of the maximum principal stress without applying corrections, if the slice thickness does not exceed one-sixth of the minimum surface principal radius. An experiment on a slice of this thickness gave a value 4% different from the nominal maximum principal stress showing that limitations of the photoelastic material must have some effect. The practical lower limit of slice thickness at present is 0·02 in which implies that the minimum surface radius which can be dealt with is $\frac{1}{8}$ in for the specified accuracy.

For a higher thickness: principal radius ratio part correction may be made, but the over-riding factor which restricts the ratio for a specified accuracy is the change of magnitude of stress through the thickness. Observations taken on one particular type of stress concentration show that an accuracy of 6% of the maximum principal stress may be attained with a ratio of one-quarter.

ACKNOWLEDGMENT

The work described above has been carried out as part of the research programme of the National Physical Laboratory, and this paper is published by permission of the Director of the Laboratory.

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The measurement of the velocities of bullets with a counter chronometer

By R. M. Davies, D.Sc., F.Inst.P., J. D. Owen and D. H. Trevena, B.Sc., A.Inst.P.,* Department of Physics, University College of Wales, Aberystwyth

[*Paper received* 4 *May*, 1951]

The measurement of the velocities of bullets, ranging between 700 and 1 400 ft/sec, presents difficulties when the experimental set-up precludes the use of a ballistic pendulum and when the rifle must be placed near the target to secure accuracy of aim. In the apparatus described, the velocities of bullets are measured by determining the time taken by the bullets to traverse a base-distance of the order of 20 cm; the time-interval is measured by a microsecond chronometer which is triggered at the beginning and the end of the interval by pulses produced when the bullets intercept two parallel pencils of light falling on two photocells.

In experiments on the propagation of stress waves in solids, it is often convenient to initiate a stress pulse of short duration[1,2] by firing a lead bullet so that it strikes the surface of the solid normally. In most cases, the mechanical strength of lead is small compared with the stresses set up during impact and, to a first approximation, it is legitimate to assume that the bullet behaves as a fluid; using the theory given by Hopkinson, it then becomes possible to calculate the form of the stress pulse if the geometrical shape of the bullet and its velocity are known. In some experiments the solid can be suspended as a ballistic pendulum; if the ballistic throw is observed, and if the mass of the pendulum and its period, together with the mass of the bullet are known, it is possible to calculate the velocity of the bullet. In other experiments this procedure is ruled out, and it becomes necessary to use some other method for the measurement, the most direct method being to determine the time taken by the bullet to traverse a known distance. In the field, or on ballistic ranges, this determination does not present very great difficulty, because the base-distance, over which the bullet is timed, can be made sufficiently long to give a reasonably large time interval; in stress wave experiments, however, accuracy of aim is important and this factor limits the maximum base-distance to the order of 20 cm; the fastest bullets (0·22 in calibre) available have a velocity of about 1 400 ft/sec so that they will traverse a base of 20 cm in about 465 μsec. It is clear that a chronometer which will measure a time-interval with a maximum error of a few μsec is necessary in order to give an accuracy of the order of $\pm$ 1% in the velocity of the bullet.

One chronometer, available commercially, which satisfies this condition is the Microsecond Counter Chronometer by Cinema-Television Ltd. For its operation, this instrument requires two positive pulses of 10 to 50 V, each of which must reach its peak in less than 1 μsec; the first pulse is used to start the chronometer, the second to stop it. Various methods of deriving these pulses from a fast-moving bullet can be envisaged and, after some preliminary experiments, it was decided to arrange that the bullet should be made to provide the signals by interrupting two thin pencils of light falling on two vacuum-type photocells, the pencils being parallel to each other and at right angles to the trajectory of the bullet.

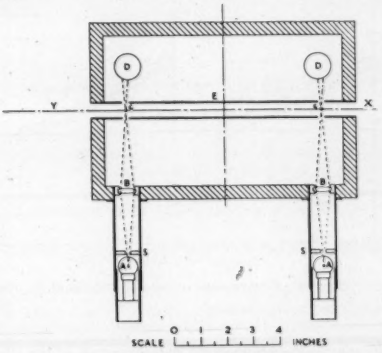

Fig. 1. Section of photocell box

A horizontal section of the optical apparatus is shown in Fig. 1, in which the line XY represents the path of the bullet and A, A are two 8 V 4 W lamps mounted so that their filaments are vertical. The lenses B, B serve to form images of the filaments at two points C, C which lie on XY; the images are focused by adjusting the telescopic tubes in which the lamps are mounted, stray light being reduced by the stops S, S. After diverging from C, the two pencils of light fall on the photocells D, D. The apparatus is protected from damage by flying fragments of lead, etc., by a brass tube with its axis lying along XY; small circular holes are drilled at appropriate positions in this tube to allow the passage of the pencils of light.

The form of the pulse obtained from the photocells when the pencils are interrupted is determined, on the one hand, by the peak value of the output p.d. of the

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photocell, and, on the other, by the finite width of the pencils at *C*, *C* and the velocity of the bullet. The output from the photocells was too small to operate the chronometer and, furthermore, since the length of the images *C*, *C* in the direction *XY* was about one millimetre, the time taken to interrupt the pencils with a bullet travelling at 1 400 ft/sec was about 2·5 μsec, whereas the rise-time of the pulses for triggering the chronometer must be less than 1 μsec. For these reasons the unit whose circuit is shown in Fig. 2 was designed to amplify and sharpen the pulses from the photocells.

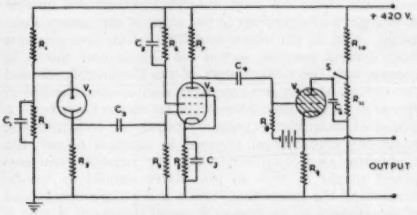

Fig. 2. Triggering circuit for microsecond timer

Component values

Resistors		*Capacitors*		*Valves*	
R_1	= 68 kΩ	R_9	= 33 kΩ	C_4	= 0·002 μF
R_2	= 33 kΩ	R_{10}	= 68 kΩ	C_5	= 0·1 μF
R_3	= 1 MΩ	R_{11}	= 7 kΩ	C_6	= 0·1 μF
R_4	= 3 MΩ				
R_5	= 120 Ω				
R_6	= 150 kΩ	C_1	= 0·1 μF	V_1	= V.A. 16
R_7	= 13·5 kΩ	C_2	= 0·002 μF	V_2	= E.F. 54
R_8	= 68 kΩ	C_3	= 0·1 μF	V_3	= G.T.I.C.

A power-pack supplies the h.t. voltage of 420 V d.c., and the anode voltage of the photocell V_1 is adjusted so that its value is about 135 V by the potential divider R_1R_2. When the pencil of light falling on the photocell is momentarily interrupted by the bullet the potential of the cathode of the photocell is lowered, giving rise to a negative pulse at the control grid of the pentode V_2, and an amplified positive pulse at its anode. In turn, this positive pulse is applied to the grid of the thyratron V_3 and, if the peak value of the pulse is greater than the standing negative bias on the valve, the thyratron fires. Thus the potential of the cathode of V_3 rises rapidly and a positive pulse appears across the cathode resistor R_9. The thyratron can be re-set by momentarily breaking its anode circuit by opening the switch *S*. In order to obtain a sharp output pulse, it is essential that the time-constants of the grid circuit of V_2 and of the cathode circuit of V_3 should be kept as small as possible. To satisfy the first condition, since $R_3 \simeq 1$ MΩ, the pentodes must be mounted in the photocell box, as close as possible to the appropriate photocell. The thyratrons, together with the power-pack, are contained in a separate screened box. The time-constants of the cathode circuits of the thyratrons are kept low by making the length of screened lead between the output terminals and the input terminals of the chronometer as short as possible.

When these precautions were taken, the output pulses were suitable for triggering the chronometer. Since the circuit of Fig. 2 will serve to supply either the "start" or the "stop" signal, two identical units supplied by the same power-pack were used, the first to start the chronometer and the second to stop it.

In order to check the overall accuracy of the measurement, the velocity of bullets, after passing through the photocell box, was measured ballistically, the ballistic pendulum consisted of a suitably-suspended steel bar, 1 in diameter, 18 in long, on which was fitted a brass extension tube filled with Plasticine into which the bullet was fired. On the whole, there was good agreement between the values of the velocity given by the chronometer and by the ballistic experiment. Occasionally, however, there would appear a large discrepancy; this was traced to the photocell circuits being triggered by the smoke and dust projected ahead of the bullet, and the trouble was eliminated by covering the hole in the photocell box nearest the rifle with a sheet of thin paper. When this precaution had been taken, the difference between the two independent values of the velocity never exceeded 2%, which is satisfactory in view of the uncertainty in the ballistic experiment and the variation in the masses of bullets taken from the same container.

ACKNOWLEDGMENTS

The authors are indebted to the Department of Scientific and Industrial Research, first for a grant for the development of special researches to one of the authors (R.M.D.) for the purchase of the apparatus, and secondly for maintenance allowances to two of the authors (J. D. O. and D. H. T.); the work was carried out during the tenure of these allowances.

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Notes and Comments

Elections to The Institute of Physics

Fellows: I. I. Berenblut, J. B. Birks, R. L. Brown, H. W. French, J. H. Fewkes, W. R. Lang.

Associates: D. K. Bewley, D. S. Box, P. R. Brett, J. W. Christian, H. J. Cull, T. A. Heddle, J. Holden, G. F. Lewin, D. J. R. Martin, E. N. Martin, K. Swaminathan, D. F. Urquhart, A. F. Watson, K. Westerman, E. Williams.

Nineteen Graduates, nine Subscribers and five Students were also elected.

Courses and examinations for instrument maintenance mechanics. The City and Guilds of London Institute announces the adoption of a new scheme of courses and examinations in instrument maintenance.

The scheme is intended to meet the needs of mechanics and technicians concerned in the maintenance, repair and installation of the instruments used for process and production control in industrial plants and in H.M. Forces. Particular attention is given to the requirements of the chemical, iron and steel, and petroleum industries, to the fuel economy and the needs of the Services, but the structure of the scheme is such that it will be found to have wide application.

The course of part-time study is of five years' duration, with an Intermediate examination at the end of the third year, and a Final examination at the end of the fifth year. There is provision for students holding an appropriate Ordinary National Certificate to enter the course at the beginning of the fourth year.

A pamphlet containing the regulations and syllabuses for the City and Guilds examinations will be available shortly, price 6d. from the City and Guilds of London Institute, Department of Technology, 31 Brechin Place, London, S.W.7. Further information about this scheme (subject No. 79) may be obtained from the same address.

The Report of the Committee of Weights and Measures Legislation.

We have received a copy of the Report of a committee which was appointed in October 1948 by the President of the Board of Trade "to review the existing weights and measures legislation containing provisions affecting Weights and Measures and the administration thereof, and to make recommendations for bringing these into line with present-day requirements."

The Committee's main conclusion is that the existing law is soundly based, but it recommends a considerable number of changes and extensions to the scope of the present legislation. One of the recommendations which diverges most from existing practices is that the Government should, in concert with the Governments of the Commonwealth and the U.S.A., abolish the imperial system of measurement in favour of the complete adoption of the metric system over a period of about twenty years. Whether this long-term and admittedly controversial proposal be accepted or not, the Committee also recommends that the Imperial yard and pound should as soon as possible be defined as specific fractions of the International metre and kilogramme instead of by reference to the present Imperial standards, in such a way as to bring, if possible, the values of the United States' and the British yard and pound into exact agreement. It further recommends that a permanent Commission of Units and Standards of Measurement, to be composed mainly of persons distinguished in the science of measurement, should be appointed by the President of the Board of Trade to advise on matters relating to the units and standards of all forms of measurement.

Copies of the Report may be obtained from H.M. Stationery Office, price 3s. 6d. net.

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A combined mechanical stage and focusing control for photomicrographic work. By D. G. Stevens.

A note on an automatic control intensity of a light source. By K. C. Wight.

Calibration of ionization gauges for various gases at low pressures. By S. Wagener.

A modification of Sucksmith's method for the measurement of susceptibilities of para- and ferromagnetic materials at temperatures between 180 and 1 100 C. By W. P. van Oort.

A image converter to extend the useful range of photomultipliers to longer wavelengths. By E. R. Holiday and W. Wild.

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ORIGINAL CONTRIBUTIONS

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Fluid flow in ducts with a uniformly distributed leakage. By J. F. Holdsworth, F. W. Pritchard and W. H. Walton.

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On the problem of focusing diffracted X-rays on an oblique part. By F. Buckens.

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SPECIAL ARTICLE

The reinforcement of rubber by carbon black

By D. PARKINSON, D.Sc., Ph.D., F.Inst.P., F.I.R.I., Research Centre, Dunlop Rubber Co. Ltd., Birmingham

Recent research on the carbon reinforcement of rubber is reviewed and existing theories examined. The stiffening effect of carbon black is thought to be little influenced by the size of the individual spherical particles, but values of tensile strength, tear strength, and abrasion resistance increase with decreasing particle size of the black. Departures from this generalization are considered and possible factors discussed. The nature of the attachments between carbon particles and rubber molecules forms the subject of much current research, the results of some of which indicate the possibility that chemical cross linkages supplement physical adsorption at the carbon-rubber interface.

Carbon black or colloidal carbon exists in a variety of forms which resemble each other in being composed of spherical or spheroidal particles, but differ from each other mainly in the size of the particles which range from 5 to 500 mμ in diameter. About 90% of the world's production of carbon black, which now amounts to upwards of 700 000 tons per annum, is used as a reinforcing agent for rubber. Reinforcement of vulcanized rubber implies increase in stiffness and tensile strength, together with resistance to tearing and abrasion. Within the range of particle sizes of the carbon blacks employed by the rubber industry the degree of reinforcement in general increases as the particle diameter decreases, the practical low limit being reached at about 20 mμ.

The remarkable wear-resisting properties of tyre treads depend primarily on the incorporation into the rubber of about one-fourth of its volume of reinforcing carbon, which until recently was all of the type known as "channel black." This substance is produced by an impingement process in which natural gas is burnt in small flames in a limited supply of air, the carbon resulting from the partial combustion being deposited on relatively cool iron "channels" which move to and fro over the flames. Rubber-grade channel blacks consist of spherical particles of average diameter 25–30 mμ.

Within recent years there has been a tendency towards the replacement of channel blacks by the so-called furnace blacks which are produced either from natural gas or from oil by incomplete combustion in large furnaces in a controlled supply of air. The resulting particles of carbon are carried along in the spent reaction gases and collected by a combination of electrostatic and centrifugal precipitation. The particle size-range for furnace black is 25–85 mμ, the fully reinforcing types averaging 30 mμ.

Other blacks that are used to a limited extent in rubber are lamp-black, formed by incomplete combustion of oils and tars, thermal black, made by thermal decomposition of natural gas and similar hydrocarbons, and acetylene black, produced by thermal decomposition of acetylene.

An account of carbon black manufacture and properties by W. R. Smith[1] can be recommended as probably the most up-to-date general contribution to the subject. Earlier articles by Wiegand,[2] Cadman,[3] Cohan,[4] Drogin and Bishop,[5] and Speedy[6] contain much useful information.

The United States is the world's principal producer of the various types of carbon black. Lamp-black has long been made in the United Kingdom. A small quantity of thermal black is now being manufactured in Britain and, more important, two plants have recently started production of fully reinforcing furnace black.

Although the mechanism of carbon reinforcement of rubber is as yet very imperfectly understood, the literature on the subject is considerable and is increasing rapidly. A critical review by the author of research published up to 1944 appeared in 1946.[7] Some of the views expressed in that contribution are now in need of revision.

PROPERTIES OF CARBON BLACK

As revealed by the electron microscope, carbon black particles, with rare exceptions, are essentially spherical or spheroidal in shape. To varying degrees, according to the method of manufacture, they aline themselves into chain-like structures, the tendency to form such structures being greater for acetylene black than for other types. These chains of particles form the basis of electrically conductive rubber.

The intensity of blackness of the carbon depends primarily on particle size, the smallest particle blacks (5–20 mμ) produced by the channel process being used as pigments in the paint and ink industries. The rubber-grade channel blacks are rather greyer and coarser and the furnace and thermal blacks are coarser still in that order. Any given sample shows a size distribution of particles, and in this respect the furnace blacks are less uniform than the channel blacks. Furnace blacks, moreover, exhibit more chain structure than do channel blacks.

The mean particle diameters and specific surfaces of a

Diameter and surface area of carbon blacks

Type	*Symbol*	*Mean particle diameter, mμ*	*Surface area from size distribution, m²/g*	*Surface area from nitrogen adsorption, m²/g*
Medium thermal	MT	300–500	—	8
Fine thermal	FT	120–200	—	15
Semi-reinforcing furnace	SRF	80	26	22
High modulus furnace	HMF	60	38	34
Fine furnace	FF	39	59	75
High abrasion furnace	HAF	27	89	86
Medium processing channel	MPC	25	106	120
Hard processing channel	HPC	24	105	142

selection from the large number of types of carbon black used in the rubber industry are listed in the table.

Surface area determinations by low-temperature adsorption isotherms indicate that most carbon blacks are non-porous. In the case of the finest of the channel blacks the measured surface is three to four times that computed from the diameter as seen in the electron microscope, indicating in the view of W. R. Smith and others[1, 8, 9] a considerable degree of porosity.

X-ray diffraction studies have established the presence in carbon black of turbostratic[10] structure, the two-dimensional graphitic layers being parallel, but otherwise in random orientation. Heating to very high temperatures causes the crystallites to grow and to assume a more orderly arrangement, as in graphite. Carbon black treated in this way largely loses its capacity to reinforce rubber.

Chemically carbon black consists of carbon with hydrogen and oxygen varying in quantity according to the method of manufacture. The hydrogen seems to be distributed throughout the particle, but the oxygen is chemisorbed on the particle surface supposedly in the form of carbon-oxygen complexes. Furnace blacks contain less than ½% of oxygen, but in channel blacks the quantity ranges from 3 to more than 10%. The pH of the water sludge of carbon black decreases with increasing chemisorbed oxygen. Channel blacks, with a low pH value, have a retarding effect on the rate of vulcanization of rubber.

STIFFENING EFFECT OF FILLERS

The most obvious effect of the incorporation of finely divided powders or fillers in rubber is the increase in stiffness imparted to the vulcanizate. The rubber technologist defines stiffness or "modulus" as the stress at a given extension or compression. The stiffness increases progressively with increasing quantity of filler in the rubber and some of the increase is due merely to a dilution of the rubber phase by replacement with filler. Probably all the stiffening can be accounted for in this way in the case of the coarse fillers which merely act as diluents and stiffen only to a minor degree.

The stiffening effect under tension of a reinforcing material having spherical particles such as the various carbon blacks has been shown[7, 11, 12, 13, 14] to have relatively little dependence upon particle diameter, and in this respect stiffness differs from other reinforcing properties such as tensile strength and abrasion resistance which are highly dependent on particle diameter. A factor influencing stiffness is the degree of chain structure exhibited by the black when dispersed in rubber; the greater the structure the higher the modulus. To account for the major effects of stiffening it seems necessary to postulate some kind of attachment of the filler particles to the rubber molecules. The possible nature of such association is considered later.

Gehman[15] has considered the problem by regarding the filler particles as providing restraining surfaces between films of rubber. If instead of filler particles a series of parallel plates are passed through a block of rubber there will be no increase in stiffness under compression if the plates are lubricated; otherwise the plates will act as restraining surfaces and cause an apparent stiffening of the rubber. From consideration of the packing of spheres between films of rubber and of "shape factors" Gehman shows that the thickness of the rubber film between particles for a given volume concentration is proportional to the particle diameter, and deduces that if the type of packing does not change "the stiffness of the film should depend on the volume loading" (i.e. concentration) "but not on the particle size." This statement is approximately true in the case of tension stiffness. Under compression smaller particles tend to give greater stiffness than larger ones, which is explained by Gehman on the supposition that compression stiffness depends not only on shape factor but to some extent on the actual thickness of the film.

Theoretical approaches to the problem of the stiffening action of fillers have been made by Rehner[16] and Smallwood[17] from considerations of stresses in the rubber in the region of spherical filler particles. Rehner deduces that these stresses are independent of particle size and depend only on concentration of filler. Smallwood points out that because of the similarity between the field equations of the theory of elasticity and those of hydrodynamics his approach is substantially the same as that of Einstein,[18] who considered the increase in viscosity caused by suspension of spherical particles in the liquid, and the result is the same except that viscosity

is replaced by Young's modulus E in Einstein's equation

$$\eta^* = \eta(1 + 2{\cdot}5c) \qquad (1)$$

where η^* and η are the viscosities of the emulsion and solvent respectively and c is the volume concentration. Smallwood found therefore that the relative modulus E^*/E is directly proportional to the volume concentration and independent of the particle size of the filler.

Carefully controlled experiments were carried out by Smallwood on a series of rubber vulcanizates containing different fillers in varying proportions. Close agreement with theory was shown in the case of the thermal blacks which consist of relatively coarse spherical particles that disperse easily in rubber, but a large departure from theory in the case of the much finer channel blacks which are more difficult to disperse. The high moduli found were attributed primarily to flocculation of the black and the general conclusion was reached that the Einstein equation applies under the idealized conditions of a relatively small concentration of spherical particles, completely dispersed and with complete adhesion between the particles and the rubber matrix when the rubber is stretched to small elongations.

Guth[19] also regarded the problem as analogous to that presented by the theory of viscosity and pointed out that if a rubber-black compound is stretched the suspended particles perturb the stresses and strains, which leads to an increase in the elastic constants or, in other words, to a stiffening of the vulcanizate. Guth and Gold[20] had previously added a second term to the Einstein equation by computing the hydrodynamic interactions of pairs of particles. This equation, with Young's modulus E replacing the viscosity, may be written

$$E^* = E(1 + 2{\cdot}5c + 14{\cdot}1c^2) \qquad (2)$$

and Guth showed that it agreed with experiment in the case of relatively low concentrations of semi-reinforcing furnace black in GR-S synthetic rubber.

Because of the presence of chain-like aggregates as well as fully dispersed particles of carbon black in the rubber the stiffness at concentrations greater than 10% by volume increases more rapidly than is predicted from theory, and Guth, by considering rod-like particles of filler embedded in a continuous matrix, derived the expression

$$E^* = E(1 + 0{\cdot}67fc + 1{\cdot}62f^2c^2) \qquad (3)$$

in which f is a shape factor of the rod.

Cohan[14] in work designed to test the validity of the above equations, pointed out that although they were derived for Young's modulus they should also hold for modulus as ordinarily understood by the rubber technologist (i.e. stress at a given strain) "provided the stress-strain curves are such that the ratios of the modulus at one loading" (i.e. concentration of filler) "to that at another is the same at all elongations up to the one at which modulus is measured; that is the curves belong to a single parameter family." He found that in the case of a finely divided calcium carbonate up to a concentration of 40% by volume close agreement of the "modulus" at 300% extension with the theoretical curve of Guth and Gold. The data for a high-modulus furnace black indicated a more rapid increase in stiffness than that for calcium carbonate and the modulus-concentration curve up to 28% agreed approximately with Guth's equation for rod-shaped particles with the shape factor f equal to six. This black consists of spherical particles in which chain structure is pronounced. Cohan's results, like those of other workers and in agreement with existing theories, showed no appreciable dependence of tension stiffness on particle size.

THE INFLUENCE OF CARBON BLACKS ON SOME PHYSICAL PROPERTIES OF RUBBER

Tensile strength. There are two important differences between the stiffening action of reinforcing fillers and their effects on the strength properties of rubber. In the first place the stiffening continues up to the highest practical concentration of filler (Fig. 1) whereas the

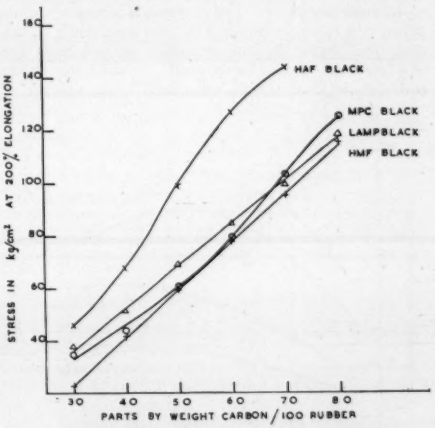

Fig. 1. Relationship between type and concentration of carbon black and stiffness of natural rubber

tensile strength (and also the tear strength and abrasion resistance) passes through a maximum at a moderate concentration (Figs. 2 and 3). Secondly, although the stiffness is little influenced by particle size of filler the tensile strength in general increases as the particle size decreases. In natural rubber the increase in tensile strength is of the order of 40% with a fully reinforcing black. Non-black vulcanizates of the butadiene-styrene

copolymer GR-S (the standard general-purpose synthetic rubber) have extremely low tensile strength and satisfactory physical properties are only obtainable by the incorporation of reinforcing agents. A fine particle size carbon black can, in fact, increase tensile strength of GR-S by as much as 1 000%.[13]

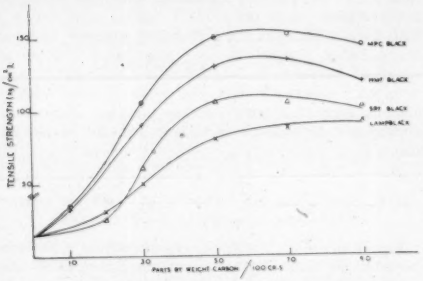

Fig. 2. Relationship between type and concentration of carbon black and tensile strength of GR-S synthetic rubber

Abrasion resistance. This property is a measure of the ability of a rubber compound to resist wear under service conditions, by far the most important instance being that of a tyre tread running on the road. Without reinforcing materials in their treads, pneumatic tyres, whether of

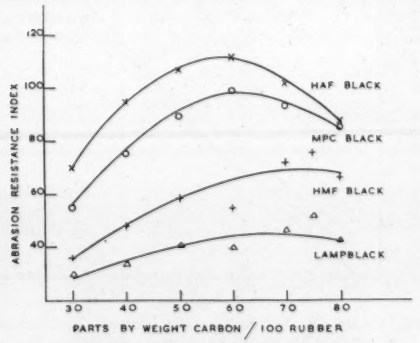

Fig. 3. Relationship between type and concentration of carbon black and abrasion resistance of natural rubber

natural or synthetic rubber, would wear out very rapidly. A four- to five-fold increase in wear-resistance of both natural rubber and GR-S is effected by the employment of a fully reinforcing black. Fig. 3 shows the relation between abrasion resistance as judged by a laboratory test specially designed to line up with tread-wear in service and the type and concentration of black added to the rubber. A general dependence of abrasion resistance upon particle diameter is indicated, but HAF black is shown to be superior to channel black although its particle size is no smaller. This point is considered later.

Rebound resilience. This property progressively decreases with increased concentration of any given carbon

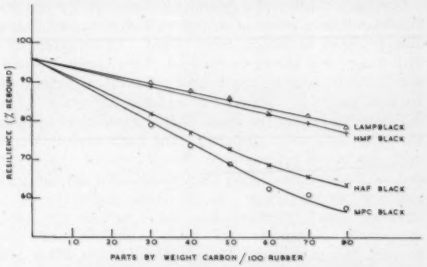

Fig. 4. Relationship between type and concentration of carbon black and resilience of natural rubber

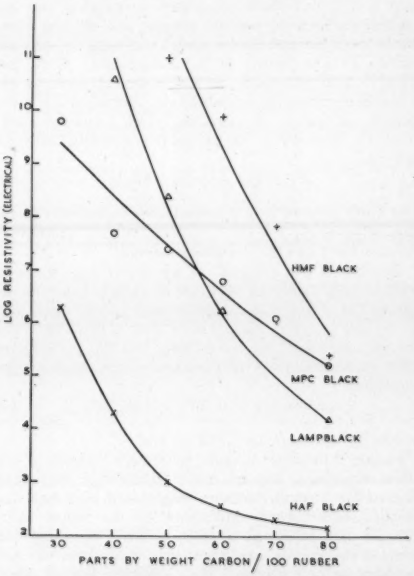

Fig. 5. Relationship between type and concentration of carbon black and specific resistance of natural rubber

black, the relationship being approximately linear up to fairly high loadings of black.[12, 13] It also decreases in general with particle diameter of black (Fig. 4), but the effect of particle size is modified somewhat according to the extent of chain structure retained by the black after incorporation in the rubber.

Electrical resistance. In Fig. 5 are plotted the changes in the logarithm of resistivity with the concentration of the same four blacks as those considered above. The curves illustrate the principle that specific resistance decreases with volume loading of all blacks and also with both particle size and chain structure of black. The requirement for an electrically conductive rubber, therefore, is a high concentration of a fine particle black which retains a high chain structure in the rubber.

ADSORPTION OF RUBBER BY CARBON BLACK

It seems necessary to postulate some kind or kinds of attachment of carbon particles to rubber molecules in order to account for the facts of reinforcement, and much recent work has had for its objective the nature of such associations which might conceivably range from physical (van der Waals) attachments to primary valence linkages.

Experiments of various kinds have been designed to show conclusively that carbon black can adsorb rubber, but in much of the work the results are capable of more than one explanation. Within recent years, however, research has thrown more light on the problem, and it is now usually conceded that adsorption of rubber by carbon black is a factor in reinforcement. The work of Goldfinger[21, 22] suggests that there is preferential adsorption of the synthetic rubber GR-S on the particles of carbon black, the fraction of the rubber left between the particles being tougher than the average of the original material. From a study of the viscosity of GR-S solutions containing channel black and the application of the Einstein viscosity equation Amborski and Goldfinger[23] inferred that under the conditions of the experiment the suspended carbon particles are surrounded by a rubber layer 150 to 200 A thick.

Work by Sweitzer, Goodrich and Burgess[24] seems to confirm Goldfinger's conclusion that carbon black selectively adsorbs the lower molecular weight fractions of rubber from dilute solution. These authors and other members of the Columbian Carbon Company[25] have developed the concept that insoluble rubber in association with carbon black or "bound rubber" is a factor in rubber reinforcement. Bound rubber or "carbon gel"[24] is formed when moderately high concentrations of finely divided carbon black are mixed into rubber, a portion of the rubber being insolubilized and held to the carbon, as has been known for many years, and can be shown when mixed unvulcanized rubber is immersed in a solvent such as benzene. The proportion of insoluble rubber increases not only with carbon concentration, but also with mixing time and temperature and time of resting before extraction. The studies of Sweitzer and others indicate that under their conditions of experimentation less than one-tenth of the insolubilized rubber is actually adsorbed on the carbon black surface and these authors postulate a "three dimensional lattice structure with carbon-rubber units joined together by additional rubber through primary and/or secondary valence forces." They also give evidence suggesting that small amounts of oxygen help to promote the development of "carbon gel." Finally they suggest that tyre tread-wear performance is improved by processing techniques which have the effect of increasing the bound rubber in the vulcanizate.

It would appear that if the insolubilization of rubber by carbon blacks is promoted under similar processing conditions it is a function of particle size. This was first suggested by Fielding in 1937[26] and has received support from work by Dannenberg and Collyer,[8] whose data show a linear relationship between bound rubber values and surface areas as measured by the electron microscope. This suggests, according to the authors, that the rubber molecules can be almost totally adsorbed on the surfaces of the carbon particles. It should be pointed out that Dannenberg and Collyer's results were obtained in rubber mixed under different conditions—e.g. at much lower temperatures—than those employed by Sweitzer and others and the bound rubber values are consequently much lower. The data of Sperberg and colleagues[27] indicate a linear relationship between bound rubber and abrasion resistance. Recent work by Stearns and Johnson[28] has led them to suggest that bound rubber is due to covalent bonding between the black and the polymer in solution.

On the whole it seems that the phenomenon of insolubilization of rubber by carbon black supports the view that molecules of rubber are adsorbed on the carbon surface but that other forces of a stronger nature than van der Waals attachments are also involved.

HEATS OF ADSORPTION STUDIES

It is inferred that reinforcement by small spherical particles is concerned in some way with extent of the carbon-rubber interface since surface area is inversely proportional to particle diameter for a given volume of filler. The question arises as to whether reinforcement is also a function of the nature or activity of the carbon surface.

The nature of the surface of carbon particles depends upon the method of manufacture. Thus furnace blacks, which contain little volatile matter, apparently have particle surfaces consisting mainly of carbon atoms, whereas channel blacks have a chemisorbed layer consisting largely of oxygen in various kinds of combination with carbon, but some nitrogen and hydrogen are also present.[29] This difference influences the value of pH and hence the rate of vulcanization of the rubber incorporating the different blacks, the channel blacks

promoting the slowest rates of vulcanization. The difference in the nature of the surface may also have an influence on reinforcing ability since furnace blacks for equivalent particle size reinforce a little more than channel blacks. A further point of interest is the fact that channel black, on partial graphitization at 3 000° C *in vacuo*, largely loses its capacity to reinforce without a significant change in the extent of surface.

In order to throw more light on the activity of carbon black surfaces and their bearing on rubber reinforcement, W. R. Smith of the Cabot Carbon laboratories and co-workers[9, 29, 30, 31, 32] have studied the heats of adsorption of various types of carbon black for a number of substances, comprising elementary gases and a series of saturated and unsaturated hydrocarbons, including dihydromyrcene, which is the simplest diolefin possessing the same unsaturation pattern as that of natural rubber. It was thought that such measurements would reflect the magnitude of the binding forces operative at carbon black surfaces and might also reveal their nature. In the case of nitrogen, oxygen and argon, and the lower molecular weight unsaturated hydrocarbons the adsorption was shown to be entirely of the van der Waals type, but with dihydromyrcene it was found that something other than physical adsorption is involved and it was suggested[9] that "the nature of association between rubber and carbon may involve the unsaturation of the elastomer and the volatile matter normally present on the carbon black." They further suggest the possibility that "a few points of exceptionally strong cross-linkage may play an important role in reinforcement, and that the small fractions of the carbon surface involved may be highly specific." The present author had previously put forward the view that in order to account for certain of the facts of reinforcement a limited number of strong bonds might be operative at the rubber-carbon interface.[7, 13]

From a study of the thermodynamical changes accompanying the deformation of reinforced rubber Stearns and Johnson[28] conclude that physical adsorption of the van der Waals type is inadequate to account for the experimental results. They endeavour to show by a calorimetric method that the surface of a carbon black particle contains sites that react with bromine to liberate the same amount of heat as low molecular weight olefins and suggest that the rubber and black are combined chemically through carbon-rubber-sulphur bonds into a continuous three-dimensional cross-linked matrix.

MOBILITY OF CARBON PARTICLES IN RUBBER

The extent to which carbon black particles can migrate in rubber has been and is still a subject of much discussion.[7, 33, 34, 35] Flocculation of particles has been invoked to explain some of the changes, and particularly changes in electrical resistance, that take place in both vulcanized and unvulcanized rubber-black compounds following different treatments. As a result of considerable research the author and Blanchard[35] reached the conclusion that although there is limited mobility of carbon particles in rubber there is no convincing evidence of extensive flocculation of discrete carbon particles into groups or a "catenation" into chains within the rubber itself. The chains of particles that undoubtedly do exist in carbon-reinforced rubber are interpreted as portions of original structures which have survived the stresses set up in the rubber during the mixing operation, though it is admitted that there is some increase in chain structure when the rubber is heated. Remilling the rubber following heat treatment under certain conditions can break down the structure permanently as is shown by the enormous increase in electrical resistivity which does not change on further heating.

Mullins and Whorlow have recently[36] discussed the changes that take place in masticated unvulcanized rubber containing fillers. Before vulcanization the rubber is in a plastic condition and it softens on shearing and stiffens on standing. Such thixotropic behaviour is small unless the rubber contains reinforcing fillers, when it is much more pronounced. Mullins and Whorlow argue that the changes can be explained by a breaking down and building up of structure and that at least two processes are involved: "(1) the orientation and/or disentanglement of both the long molecular chains" (of rubber) "and anisotropic filler particles or chains of filler particles; (2) breaking down of interactions either between neighbouring filler particles or between the filler and the rubber, or both." These authors consider that the bonds will have a range of strengths, the weak bonds breaking more readily. On standing after shearing the structures are supposed to re-form at rates depending on the temperature.

EFFECT OF STRESSING ON STIFFNESS OF CARBON-REINFORCED RUBBER

The normal tensile stress-strain test for rubber involves only the first extension, though it is well known that the shape of the stress-strain curve changes with successive deformations. This change is small for pure vulcanized rubber, but in the case of vulcanizates containing substantial concentrations of reinforcing fillers it is considerable, the greatest effect being caused by the first extension; after about six stretch-cycles a more or less steady state is reached.

The changes under consideration have been studied in detail by Mullins,[37, 38] who showed that a large part of the increased stiffness imparted to natural rubber by reinforcing agents is destroyed by stretching. The degree of softening is greater the greater the stretch and the greater the stiffening effect of the filler. Mullins found that on stretching the test-pieces after the first extension substantially all the softening occurred at elongations less than the previous extension and that at elongations in excess of the previous extension the rubber behaved as though it had not previously been stretched.

Hence the tensile strength is uninfluenced by previous stressing. Mullins considers that the softening results mainly from a "breakdown of filler structure."

Blanchard and the author[39] have argued that except perhaps at small stresses the stiffening bonds which are broken during stressing are largely carbon-rubber bonds and give reasons for the view that a substantial part of carbon chain structure survives no matter how severely the rubber has been stressed. Their work is consistent with the view that most of the carbon-rubber bonds are of low energy (van der Waals) type, but that a small number of strong bonds stiffen the rubber substantially at high stresses and are largely responsible for increasing the tensile strength.

Recent work by Waring[40] relates to the changes taking place in reinforced rubber resulting from high-frequency compression at amplitudes of the order of 0·05% of the length of the test-piece in the temperature-range of 30 to 70° C. Waring found a decrease in dynamic stiffness with increasing temperature and a decrease in electrical resistance with increasing temperature when no vibration was present, but that the application of mechanical vibration increased the resistivity. He postulates the existence of "two states of the carbon black structure," one of which has a higher and the other a lower conductivity, and visualizes vibration as promoting a thixotropic change from one to the other by producing a decatenation of the carbon structure. Temperature is supposed to have a mixed effect, on the one hand catenation of carbon particles "and at the same time a breakdown of the cohesive bonds between carbon and rubber which do not affect the conductivity but contribute a considerable effect to dynamic modulus."

COMPARISON OF FURNACE AND CHANNEL BLACKS IN RUBBER

It is now generally recognized that the finer of the furnace blacks are at least equal to channel black, both in natural and synthetic rubber.[8, 27, 41, 42] In their influence on abrasion resistance the high abrasion furnace blacks are even better than the rubber-grade channel blacks, but they have some disadvantages in that they are not quite so easily processed in the rubber factory.

In view of the increasing tendency of the high-abrasion furnace blacks to replace the channel blacks in tyre-tread and similar compounds, research and development work involving comparisons of the two types of black is of paramount importance at the present time. In this country the matter is of particular interest because furnace blacks made in Great Britain are beginning to replace imported channel blacks. Some recent work on this aspect of the problem is briefly considered below.

Comparisons have been made above of natural rubber vulcanizates incorporating four types of carbon black including the two types now under consideration, namely medium processing channel (MPC) and high abrasion furnace (HAF). Among the features to be noted are the higher "modulus" of the furnace-black compound due presumably to a greater development of chain structure in the HAF than in the MPC black and also its higher abrasion resistance. The question arises whether the difference in abrasion resistance is also a consequence of a difference in structure. Abrasion resistance, as we have seen, is, unlike stiffness, primarily a function of particle size. The table shows that the specific surface of HAF black is less than that of MPC black, and electron micrographs indicate that the former has a wider size distribution and greater proportion of larger particles than the latter. It is evident that the furnace black gives a higher abrasion resistance than is accounted for by its particle size and it is tempting to attribute this to an increased chain structure. However, breaking down the structure by severe remilling does not decrease abrasion resistance,[43] and it seems to be true that the acetylene black, with the greatest chain structure of all, imparts poor abrasion resistance in relation to particle size.[7, 12] Chain structure, therefore, does not in itself seem to be a major factor in abrasion resistance and the beneficial effect of furnace black may depend, as suggested earlier in this review, on a difference in the nature of the surface. Furnace blacks differ from channel blacks in having relatively little chemisorbed oxygen on the surface of the particles. The surface oxygen is largely responsible for the retarding effect of channel blacks on the rate of vulcanization and it is possible that it influences physical properties. Dannenberg and Collyer[8] have shown that the results of calcining channel black to remove the surface oxygen are an increase in the rate of vulcanization and an improved laboratory abrasion resistance by about 10%. They conclude that calcined MPC black is about equal to HAF black in laboratory abrasion resistance and suggest that "a factor responsible for high abrasion furnace black superiority over a channel black of equivalent particle size is its relative freedom from chemisorbed surface oxygen." Dannenberg and Collyer also considered "bound rubber" as a possible factor, but since under their conditions of experimentation bound rubber was found to be linearly related to surface area it did not help to explain the increased capacity of HAF blacks over channel blacks to reinforce rubber.

CONCLUSIONS

It will be evident from the foregoing considerations that existing theories of rubber reinforcement are incomplete and that many of the facts await a satisfactory explanation. For a fuller understanding of the mechanism of the process it is necessary to determine the nature of the association between carbon particles and rubber molecules. It is generally agreed that van der Waals forces, which are of low energy, are operative, but these do not account for all the experimental observations. Recent work of various kinds[9, 24, 28, 39] indicates the possibility that chemical cross-linkages may

play an important part in the carbon reinforcement of rubber. Here, then, are opportunities for research in which the special methods developed by physicists can be of the greatest value to industry.

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Aerial prospecting for radioactive minerals

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The γ-ray flux over a radioactive mineral deposit has been calculated and expressions derived for the variation with operational flying conditions. An airborne equipment comprising 49 Geiger-Müller tubes was designed to investigate the overall sensitivity and the practical problems arising from the use of such a method of prospecting. Thus a surface mineral deposit of concentration factor (% U_3O_8 by weight) × area (sq yd) of about 100 is just detectable above "background" fluctuations at an altitude of 500 ft overhead and speed of 120 m.p.h. The effect of deposit and background activity upon the comparative performance of Geiger and scintillation counters is such that a practical improvement of three times may be expected from the scintillation counter.

1. INTRODUCTION

The possibility of conducting a survey of radioactivity by air over areas of virgin country has attracted the interest of geologists. Not only does such a method of survey increase the convenience of a systematic search, but the speed of covering a given area, possibly barely accessible on foot, is greatly increased. The use of aircraft for similar purposes has been successfully demonstrated in magnetometer and photographic surveys. Provided that an accurate means of navigation is available, the area may be traversed according to a simple flight plan and with the assurance that all obtrusive radioactive deposits above a certain minimum strength, dependent upon the sensitivity of the detecting apparatus, will be located. A deposit discovered in this manner can be referred to a ground party for fuller investigation. In aerial prospecting the sensitivity and response of the detecting apparatus are in part dependent upon the altitude and speed of flight, and hence upon the choice. or availability, of different aircraft. For a given sensitivity, a lower limit to detection will be set by the fluctuations in response to background activity. The background effect arises from cosmic rays and from local natural radioactivity. The local component is due mainly to the active content of the earth's surface (generally equivalent to about 0·001% concentration of U_3O_8) together with possible effects due to atmospheric activity, and to active contamination of the walls of equipment and aircraft. A variation of background with altitude may be expected. In order to establish the optimum conditions of design and adjustment in the detecting apparatus, it is desirable to calculate, with reasonable accuracy, the inter-dependence of the several operational variables, i.e. the relations between sensitivity and speed of response ("time constant") and altitude, distance and aircraft speed.

There are three methods of detection suitable for aerial prospecting. These are based on the use, with appropriate circuits and recorders, of (i) ionization chambers, (ii) Geiger counters, and (iii) scintillation counters. Geiger counters were adopted for this present work because of their greater discrimination against cosmic background as compared with ionization chambers. Potentially, the scintillation counter provides greater advantages than the Geiger counter but the technique was considered insufficiently developed to permit reliable and consistent operation in an airborne equipment.

2. γ-RAY FLUX AND DETECTOR RESPONSE

2.1. *Statement of problem.*—It is required to calculate the γ-ray flux at any altitude and range due to a deposit of given surface area. An ideal flat earth's surface is assumed. The expression for γ-ray flux is determined by an inverse square factor together with exponential decay terms due to absorption within the deposit and absorption in the atmosphere. The flux expression, integrated over the whole deposit, is introduced in the differential equation controlling the time response of the detecting apparatus. The peak detector signal is then derived as a function of altitude, speed, range, background deviation and other factors.

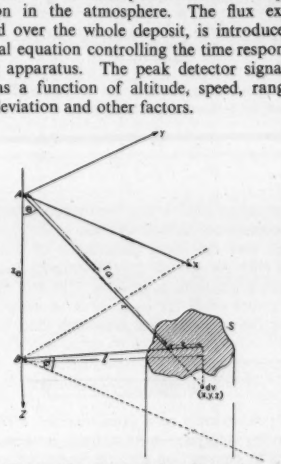

Fig. 1. Geometry of aircraft and deposit

2.2. *γ-ray flux.*—With reference to Fig. 1, consider a detector of area A situated at the origin of the co-ordinates and a radioactive mineral "deposit" of infinite

depth with surface S at ground-level in the z_a plane. The solid angle subtended by A at an elementary volume dV situated at x, y, z within the deposit

$$= \frac{A}{r^2} \text{ where } r^2 = x^2 + y^2 + z^2$$
$$= \frac{A}{l^2 + z^2}$$

Then, taking account of absorption within the deposit and absorption in the atmosphere, the γ-flux (dI) from dV incident upon A is given by

$$dI = \frac{A}{4\pi r^2} \cdot N\rho_s dV \cdot \epsilon^{-\mu_s(r-r_a)} \epsilon^{-\mu_a r_a}$$

where $N = \gamma$ activity of deposit/unit mass/unit time.
r_a = distance in air between A and dV.
ρ_s = density of the deposit and also of the surrounding earth.
$\mu_s = \gamma$ absorption coefficient of the deposit, and also of the surrounding earth.
$\mu_a = \gamma$ absorption coefficient of air.

μ_s and μ_a represent values of the absorption coefficients weighted according to the intensity and energy distribution of the γ-radiation. Values for μ_a have been tabulated by Heitler:[1] the value chosen in the following computations is $\mu_a = 0{\cdot}0001\ \text{cm}^{-1}$. This may be in error by 30%.

Assuming A has spherical symmetry, the total γ-flux through A due to the whole deposit

$$I = \frac{NA\rho_s}{4\pi} \int_V \frac{1}{r^2} \epsilon^{-\mu_s(r-r_a)} \epsilon^{-\mu_a r_a} dV$$

This integration is carried out in Appendix 1: it is shown that, to a close approximation,

$$I = \frac{NA\rho_s}{4\pi} \cdot \frac{1}{\mu_s z_a} \int_S \frac{\epsilon^{-\mu_a \sqrt{(l^2 + z_a^2)}}}{(l^2 + z_a^2)^{\frac{1}{2}}} dS \qquad (1)$$

where l, equal to $\sqrt{(x^2 + y^2)}$, represents the range of A from the elementary surface area dS.

Provided that the linear dimensions of S are small compared with the altitude z_a, the quantity l and hence the integral, dependent upon $(l^2 + z_a^2)^{\frac{1}{2}}$, will vary little over the extent of S. It is possible to select a mean range $\bar{l}$ for the whole surface area such that, with good accuracy,

$$I = \frac{NA\rho_s}{4\pi} \cdot \frac{S}{\mu_s} \cdot \frac{\epsilon^{-\mu_a \sqrt{(\bar{l}^2 + z_a^2)}}}{z_a(\bar{l}^2 + z_a^2)^{\frac{1}{2}}} \qquad (2)$$

Strictly (2) refers only to a point deposit. The errors introduced by applying it to a finite area have been determined by comparison with the numerical computation of the integral (1), showing that, for a circular deposit area of diameter $z_a/5$ the error is within 1% over the range $\bar{l}/z_a = 0$ to 3. For the particular (and simpler) case where the aircraft is immediately overhead (i.e. $\bar{l} = 0$), an expression is easier to derive, and since this may, in practice, find useful application, it is given in Appendix 1.

In (2) the quantity S/μ_s represents the volume and hence $1/\mu_s$ represents the depth of radioactive material which in the *absence of internal absorption* would be equivalent to the deposit of infinite depth with absorption. Similarly, $\rho_s S/\mu_s$ is the equivalent mass of material: this is independent of the nature of the deposit since μ_s/ρ_s ($\sim 0{\cdot}06$ for 1 MeV γ energy), the mass absorption coefficient, is practically a constant. The deposit may be defined therefore by the product of N the activity and S the area.

2.3. *Detector response.*—If the flux of a stationary γ-ray field through the detector A is I_s, the detector indicator or signal i may be made proportional to I_s ($= kI_s$). The signal may be registered as a γ-ray count or as a deflexion on a recorder following a d.c. amplifier or ratemeter. When the flux I is varying as in the case of an aircraft moving in the vicinity of a radioactive deposit, the recorded signal will be modified by the time of response of the detector. The differential equation determining the variation of i with time is

$$\frac{di}{dt} + \frac{i}{\tau} = \frac{k}{\tau} \cdot I \qquad (3)$$

where the time constant τ measures the time of response of the detector.

Generally, during aerial prospecting, the aircraft will not pass directly above any deposit. If the nearest approach or minimum range is $\bar{l}_0$ (Fig. 2) then

$$l = (\bar{l}_0^2 + v^2 t^2)^{\frac{1}{2}} \qquad (4)$$

where v = speed of aircraft
and $t = 0$ when $\bar{l} = \bar{l}_0$

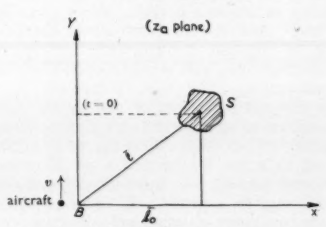

Fig. 2. Minimum range

The peak signal i_{max} is derived in Appendix 2 by substituting from (2) and (4) in (3), integrating and maximizing for t. Thus

$$i_{max} = kI_0 \frac{e^{-\mu_a \sqrt{(z_a^2 + \bar{l}_0^2)}}}{z_a(z_a^2 + \bar{l}_0^2)^{\frac{1}{2}}} \left\{ 1 - \frac{v^2\tau^2}{z_a^2 + \bar{l}_0^2} \frac{[1 + \mu_a\sqrt{(z_a^2 + \bar{l}_0^2)}]}{2} \right\} \qquad (5)$$

where $$I_0 = \frac{NA\rho_s S}{4\pi\mu_s}$$

The expression in brackets is the factor introduced by the movement of the detector, modifying equation (2).

At $z_a = 500$ ft, $I_0 = 0$, $\tau = 1$ sec and $v = 120$ m.p.h. the bracketed expression becomes $1 - 0{\cdot}16$, i.e. the peak signal is reduced to 84% of the stationary value.

2.4. *Performance factor.*—In addition to transient signals due to radioactive anomalies, the detecting apparatus will register continuously an indication, i_b, due to background radioactivity which may have a masking effect on the real signal. The performance of a particular detecting apparatus may be measured in terms of i_{max}/i_b or more appropriately by the ratio R of i_{max} to the amplitude of the statistical variation of i_b, i_{max} and i_b being expressed as counts/unit time. Assuming the background flux to be of random origin the standard deviation of i_b is $\sqrt{(i_b/2\tau)}$. This sets a lower limit to the detectability of i_{max}.

Let $R = i_{max}\sqrt{(2\tau/i_b)}$ and $i_b = \Sigma\,(kI)_b$
$= k_b I_b$

where k_b and I_b refer to total background. Therefore

$$R = i_{max}\sqrt{\frac{2\tau}{k_b I_b}}$$

$$= \frac{k}{\sqrt{k_b}}\sqrt{\frac{2}{I_b}} \cdot I_0 \frac{\epsilon^{-\mu_a\sqrt{(z_a^2+I_0^2)}}}{z_a(z_a^2+I_0^2)^{\frac{1}{2}}} \left\{1 - \frac{v^2\tau^2}{z_a^2+I_0^2}\,\frac{[1+\mu_a\sqrt{(z_a^2+I_0^2)}]}{2}\right\}\sqrt{\tau} \qquad (6)$$

from (5).

It will be profitable to consider the nature of the various quantities that determine the magnitude and behaviour of the performance factor R.

The nature and relative importance of the several components of I_b are discussed in Section 5. For the purposes of the present calculation I_b will be assumed constant in (6).

The quantities k and k_b determine the respective detection efficiencies for signal and total background. k_b is generally not equal to k owing to the diverse origins of I_b, and to the possible dissimilar energy distribution of signal and background γ rays. Choice of the type of detector will be influenced by $k/\sqrt{k_b}$, remembering that k_b may be decreased by auxiliary devices that discriminate against background.

The detector area A (cross-sectional area of counter or chamber) determines with k the overall sensitivity of the apparatus. In practice increase of A is limited only by considerations of space and weight and, indirectly, by serviceability and maintenance problems.

It is seen in (6) that R is dependent upon a function of τ

$$f(\tau) = \left\{1 - \frac{v^2\tau^2}{z_a + I_0^2}\,\frac{[1+\mu_a\sqrt{(z_a^2+I_0^2)}]}{2}\right\}\sqrt{\tau}$$

The value of τ can be chosen to suit the condition of the experiment. Differentiating with respect to τ gives an optimum value

$$\tau_0 = \frac{(z_a^2+I_0^2)^{\frac{1}{2}}}{v}\left\{\frac{2}{5\,[1+\mu_a\sqrt{(z_a^2+I_0^2)}]}\right\}^{\frac{1}{2}}.$$

This gives $f(\tau_0) = 0{\cdot}64\,\dfrac{(z_a^2+I_0^2)^{\frac{1}{4}}}{v^{\frac{1}{2}}}\,\dfrac{1}{[1+\mu_a\sqrt{(z_a^2+I_0^2)}]^{\frac{1}{2}}}$

and hence from (6)

$$R_{max} = \left\{0{\cdot}90 \cdot \frac{kI_0}{\sqrt{k_b}\sqrt{I_b}}\right\} \frac{\epsilon^{-\mu_a\sqrt{(z_a^2+I_0^2)}}}{v^{\frac{1}{2}}z_a(z_a^2+I_0^2)^{\frac{1}{4}}}\,\frac{1}{[1+\mu_a\sqrt{(z_a^2+I_0^2)}]^{\frac{1}{2}}} \qquad (7)$$

The optimum time constant τ_0 is plotted against altitude in Fig. 3 for different aircraft speeds. The effect of air absorption is demonstrated by the broken curves for

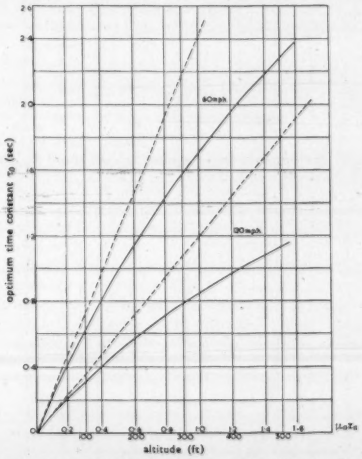

Fig. 3. Optimum time constant against altitude ($I_0 = 0$)

$$\tau_0 = \frac{(z_a^2+I_0^2)^{\frac{1}{2}}}{v}\left\{\frac{2}{5[1+\mu_a\sqrt{(z_a^2+I_0^2)}]}\right\}^{\frac{1}{2}}$$

Minimum range $I_0 = 0$ —— $\mu_a = 0{\cdot}0001$ cm^{-1} - - - - $\mu_a = 0$

$\mu_a = 0$. The variation of R_{max} with speed is illustrated in Fig. 4. Since each speed has its appropriate time constant to give R_{max}, this leads to $\tau_0 \to \infty$ as $v \to 0$ and hence $R_{max} \to \infty$ as $v \to 0$. Fig. 5 shows R_{max} plotted against altitude. In Figs. 4 and 5 the ordinates are independently normalized, and have no practical significance.

2.5. *Conditions of search for minimum deposit.*—The preceding calculations and figures relate to detector response over a known radioactive deposit. To prepare a scheme of aerial prospecting it is necessary to proceed

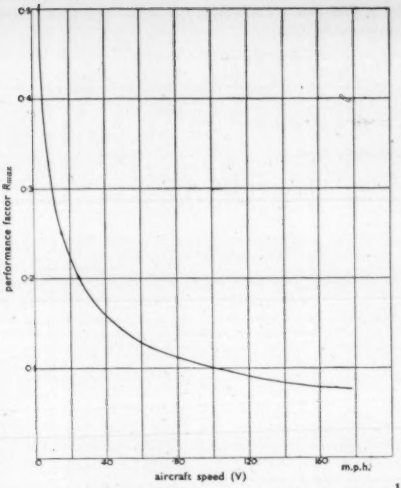

Fig. 4. Performance factor against aircraft speed $R_{max} \propto \frac{1}{v^{\frac{1}{2}}}$

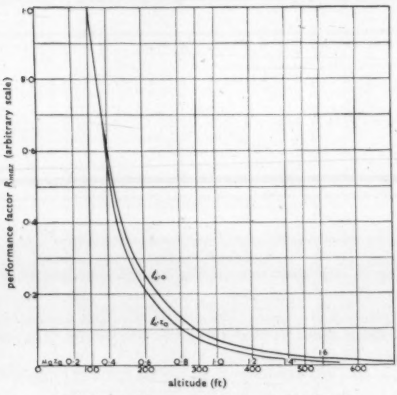

Fig. 5. Performance factor against altitude

$$R_{max} \propto \frac{\epsilon^{-\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}}}{z_a(z_a^2+\bar{l}_0^2)^{\frac{1}{4}}[1+\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}]^{\frac{1}{2}}}$$

$\mu_a = 0{\cdot}0001\ \text{cm}^{-1}$

inversely and investigate the operational conditions that ensure detection of all radioactive deposits above a certain minimum strength. This minimum will be set by geological and economic considerations. The factors influencing the choice of aircraft are discussed in Section 3.2. The plan of flight will depend upon the navigation system employed but will generally consist of traversing the territory by a series of parallel equidistant lanes. Since for a given speed, the time taken to traverse the territory is inversely proportional to the lane width, it is desirable to derive a relation between minimum deposit strength, lane width and performance factor.

Substituting for I_0 from (5) in (7), R_{max} is plotted in Fig. 6 against NS for various values of $\bar{l}_0/z_a$ with $z_a = 500$ ft; $\bar{l}_0$ now represents the half-lane width and NS the strength of a deposit (N = specific activity, S = area of deposit). To give this graph practical value

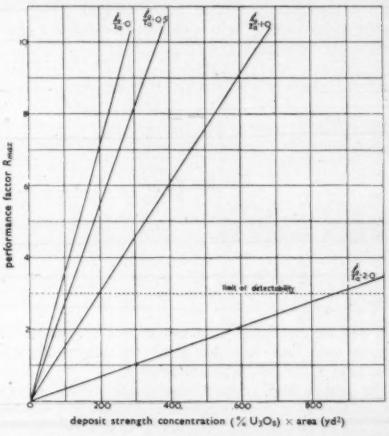

Fig. 6. Performance factor against deposit strength (500 ft)

$$R_{max} \propto \frac{NS\epsilon^{-\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}}}{z_a(z_a^2+\bar{l}_0^2)^{\frac{1}{4}}[1+\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}]^{\frac{1}{2}}}$$

$\mu_a = 0{\cdot}0001\ \text{cm}^{-1}$ $v = 120$ m.p.h.

$2\bar{l}_0$ = lane width

the results of Section 4 are anticipated by drawing in a boundary line defining the limit of detectability as $R_{max} = 3$. Thus the intersection of $R_{max} = 3$ with any $\bar{l}_0/z_a$ curve defines the half-lane width and the deposit strength that is just detectable in mid-lane. The deposit strength is measured practically by the product of concentration, in equivalent percentage by weight of U_3O_8, and area, in square yards. The deposit strengths and lane widths derived in this way represent limiting conditions: points lying above $R_{max} = 3$ refer to conditions of greater detectability.

From statistical reasoning[2] the probabilities of a signal deviation greater than 1, 2 and 3 times the standard

deviation are respectively, and approximately $\frac{1}{3}$, $\frac{1}{11}$ and $\frac{1}{110}$. Consequently a lower limit of detectability chosen at $R_{max} = 3$ would appear pessimistically high. Practical experience has shown however that this figure is satisfactory and that it allows for the effect of small local variations in background and removes uncertainty in interpretation of records.

Throughout this mathematical assessment of the aerial prospecting problem no attempt is made to account for rapid changes in topography or for the presence of inactive superficial drift masking the radioactive deposit. A few feet of "overburden" will cause severe attenuation of γ intensity. Some slight modification may be caused by the effect of scattering within the deposit and in the atmosphere. Deposit strengths have been regarded in terms of equivalent U_3O_8 concentration: these may be converted to a ThO_2 scale for thorium deposits. The choice of an effective value for μ_a is admittedly controversial, attended as it is by an attempted estimation of several imponderables such as atmospheric scattering, density variations and the mean energy representative of the radioactive γ spectrum. Further information may be derived from data provided by Evans and Evans.[3]

3. APPARATUS

3.1. *Choice of detector.*—Following the argument developed in Section 2.4, it is clear that γ *signal* sensitivity is an incomplete criterion of the efficacy of a detecting instrument. Instead a factor $k/\sqrt{k_b}$ is introduced to account also for background sensitivity. Ionization chambers give undesirable prominence to the intensely ionizing particles and showers forming the cosmic component of background. Geiger counters register cosmic and γ events with equal prominence and further, it is possible to inter-connect a bundle of counters so that most of the penetrating cosmic component is cancelled (Section 5.1). The γ signal intensity and remaining γ component of background are counted with a Geiger counter efficiency, of $\sim 0{\cdot}003$, compared with ~ 1 for cosmic rays. Scintillation counters seem capable of an increase of as much as 50 times in efficiency if γ counting, thereby providing virtual elimination of the penetrating cosmic component. The necessity for the maintenance of high stability in the amplifier and discriminator circuits associated with a scintillation counter detracts somewhat from this improvement in efficiency.

The order of merit, measured in terms of $k/\sqrt{k_b}$ is then, scintillation counter, Geiger counter, ionization chamber. For the work described in this paper the Geiger counter was preferred to the then incompletely developed scintillation counter.

3.2. *Choice of Aircraft.*—Consideration of (7) or Figs. 4 and 5, when attempting to obtain the best performance, leads to the choice of an aircraft having the lowest possible cruising speed and safe operating height. Such a specification is probably best satisfied by the helicopter and this may well be the most suitable choice under certain conditions. The desirable minima of speed and altitude must be reconciled with the more practical consideration of:—

(i) time spent over survey increases with decreasing cruising speed;

(ii) local topography—rapidly varying ground level raises safe operating altitude;

(iii) space for installation of detecting equipment and possible navigational aids;

(iv) endurance—avoidance of frequent stops for refuelling favours large petrol capacity, hence large aircraft;

(v) aircraft maintenance and servicing;

and (vi) availability.

Availability will have over-riding importance when Service aircraft or aircraft attached to the Geological Department, engaged on transport, magnetometer or photographic surveys already operate in or near the area to be surveyed.

3.3. *Aerial survey equipment type* 1073*A*.—The equipment consists of an assembly of 49 Geiger tubes with quenching, anti-coincidence and ratemeter circuits, pen recorder and a power pack drawing current from a 24 V supply. Fig. 7 is a schematic diagram of the circuit.

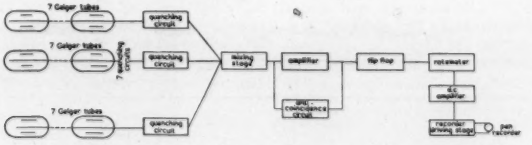

Fig. 7. Aerial survey equipment type 1073A. Schematic diagram

Geiger tubes: The Geiger tubes (type GM1B) have graphite cathodes of length 63 cm and diameter 3·4 cm and are filled with an argon-ethyl bromide low temperature mixture at pressures of 9 cm and 1 cm respectively. The tubes are arranged horizontally in 7 rows of 7, the anodes, spaced at 4·3 cm, being connected in staggered columns of 7, each column connected to a quenching circuit.

Quenching circuits: The quenching circuits have quench periods of 500 μS which is sufficient to

suppress any possible spurious counting from the Geiger tubes.

Mixing circuit: The mixing circuit combines pulses from the 7 quenching circuits. Any pulses occurring simultaneously (within 6 μS), i.e. coincidence pulses due to penetrating cosmic rays initiating tubes in 2 or more columns, combine and produce additive pulses of magnitude greater than single amplitude. Pulses of single amplitude are due to normal γ initiation. The rate of accidental coincidence is small at the γ count rates experienced.

Anti-coincidence circuit: The anti-coincidence circuit discriminates against pulses of greater than single amplitude and hence against cosmic rays.

Ratemeter circuit: The ratemeter circuit converts the Geiger pulse rate into a rate voltage. Sensitivity may be adjusted over a nominal range of 1 : 10 : 100 by selection of appropriate condensers. Alternative time constants of 1·5 and 0·75 sec are available. Time constant and sensitivity adjustments are made by relays switched externally. Feed-back introduces non-linearity into the recorder deflexion-count relation to prevent overloading at high γ intensities.

Recorder: A robust recorder of the single channel type by Kelvin-Hughes Ltd. uses electrical marking of facsimile paper at a paper speed of 0·25 cm/sec. The centre-tapped coil has a resistance of 5 000 Ω, sensitivity of 0·72 mm/V and resonant frequency of 30 c/s. Time marking is introduced through an auxiliary fixed pen from a half-second clock. On the most sensitive range the background is automatically offset by a pre-set amount to allow a full recorder scale for signal deflection. On the other ranges the background indication is included in the recorder deflexion and forms a useful overall calibration.

Power supply: 20 A are required at 24 V d.c. This is converted by motor generator to 80 V at 1 600 c/s.

Size and weight: 4 ft 6 in × 1 ft × 1 ft: 150 lb.

Aircraft: The aircraft used for this work was a standard R.A.F. Anson—a twin-engined monoplane, crew capacity 6, cruising speed 120 m.p.h. and endurance 4½ h.

4. FLIGHT RESULTS

The aerial prospecting equipment has been tested over a high grade artificial "deposit" of small area and over more extensive low grade natural deposits. The experimental flying programme was planned as a comprehensive check of the preceding theoretical results. Although flying was discontinued before the programme was completed, sufficient reliable data were obtained to set a practical scale for the calculations and graphs derived in Section 2.

4.1. *Artificial deposit.*—Ten cylindrical containers of pitch-blende (60% U_3O_8) each half-filled and containing 1 cwt of ore were placed horizontally on the aerodrome to form an approximately square ore surface at ground level of effectively infinite depth (≮ 10 cm) over an area of 1·4 yd². Some specimen records are reproduced in Fig. 8 which demonstrates the variation, with time constant, of signal and background deviation at constant altitude and speed. The detectability level of Fig. 6 is fixed by reference to results obtained over the artificial deposit (concentration × area = 84). Altitudes were

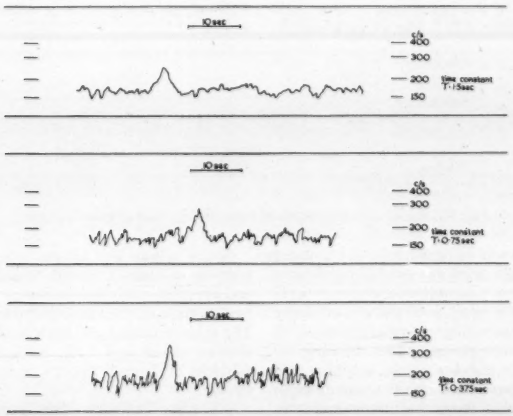

Fig. 8. Flight records—artificial deposit. Altitude 300 ft above ground. Speed 115 m.p.h.

measured by altimeter, checked at ground level at the deposit before and after each series of flights; speeds measured by the aircraft instrument were corrected for wind velocity to give true ground speed; flight path was determined visually or with the Decca Navigator.

4.2. *Natural deposits.*—A series of flights was made over natural deposits of uranium ore. Peak recorder deflexions over a typical natural deposit are plotted against altitude in Fig. 9 for signal and *mean* total

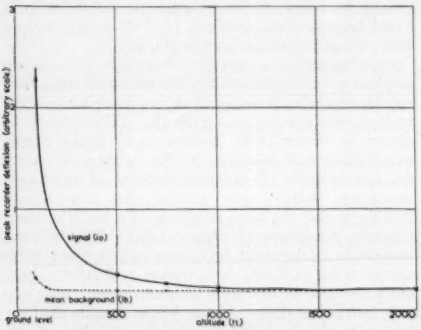

Fig. 9. Measured recorder deflexion against altitude—typical natural deposit. Speed 138 m.p.h. time constant 3 sec

background. It is interesting to note the steep rise of signal with decreasing altitude and the constancy of background deflexion from 200 to 2 000 ft. The rise in background below 200 ft is abnormal and due to extensive contamination of the country surrounding the deposit.

Fair agreement was shown between the results for these deposits of large linear dimensions and the theory of Section 2, which applies strictly only to deposits of small area. Some typical measurements over the artificial deposit are shown in Table 1, together with calculated figures, based on these measurements, and derived from Figs 4, 5 and 6

5. BACKGROUND ACTIVITY

It was shown in Section 2.4 how the performance factor R is influenced by the background flux I_b and the total background detection efficiency k_b. It is of interest to consider the nature of I_b and how R may be increased by reducing k_b. The total background consists of four components: that due to cosmic activity I_c; γ activity of the earth's surface I_e; γ activity of the atmosphere I_a; and γ active contamination of equipment and aircraft I_x. Thus

$$i_b = k_b I_b$$
$$= k_c I_c + k_e I_e + k_a I_a + k_x I_x$$

Representative values of the various Geiger detection efficiencies are

$$k_c \sim 1$$
$$k_e, k_a, k_x \sim 0{\cdot}003$$

5.1. *Cosmic activity.*—Cosmic activity is due to a complex radiation composed of a penetrating or *hard* part consisting mainly of mesons and possibly some very energetic electrons and photons together with a comparatively absorbable or *soft* part of electrons and photons. The total omni-directional cosmic intensity/cm^2 horizontal cross-sectional area/min is about 1·2.[4] This applies at sea level in geomagnetic latitudes greater than 45°. It is estimated that the hard part represents $\frac{2}{3}$ of the total intensity under these circumstances, the separation of the hard from the soft being defined by the arbitrary absorption criterion of 10 cm of lead.

Unlike normal γ radiation, the majority of the cosmic rays are each capable of initiating several of a bundle of Geiger tubes. Then if the average number of tubes operated, effectively simultaneously, by a single cosmic particle is n, the detection efficiency per unit area for the whole bundle is reduced to k_c/n. n is a function of the total number and closeness of tubes in the bundle. In addition, k_c may be further reduced by an anti-coincidence arrangement as described in Section 3.3. In this case all cosmic particles registering on more than

Table 1. *Flight results*

		area sq yd	concentration % U_3O_8	altitude ft	speed m.p.h.	R_{max}	lane width ($2\bar{l}_0$) ft	remarks
		measured						
1.	*Artificial deposit*	1·4	60	500	120	3	overhead	just detectable
2.		1·4	60	300	120	12	overhead	
		calculated						
3.		420	0·2	500	120	3	overhead	just detectable
4.		420	0·2	300	120	12	overhead	
5.		3·3	60	500	120	3	1 000	just detectable
6.		1 000	0·2	500	120	3	1 000	just detectable
7.		0·9	60	300	60	3	600	just detectable
8.		270	0·2	300	60	3	600	just detectable
9.		7·5	0·2	100	60	3	overhead	just detectable

one tube are eliminated and only those registering singly or by grazing incidence at the corners of the bundle are counted. The total "cosmic elimination efficiency," taking into account the soft part, of the 49 tube assembly has been estimated with some uncertainty as 60% including some 7% due to the anti-coincidence circuit alone.

The cosmic intensity will increase by approximately 9%[5] from ground level to 1 000 ft altitude, i.e. for a decrease in barometric pressure of about 1 inch of mercury.

5.2. *Earth activity.*—The background flux I_e is due to γ-activity arising from uranium, thorium and potassium present generally in the earth's surface. The content varies with geological conditions, 0·001% equivalent U_3O_8 being a typical concentration. The ground level γ-intensity corresponding to this concentration would be of the order of 200/cm² cross-sectional area/min and produces 0·6 counts/cm² cross-sectional area/min in a Geiger tube of efficiency 0·003.

The theoretical variation with altitude is calculated in Appendix 3 and shown graphically in Fig. 10. The striking feature of this theoretical curve is the rapid decrease exhibited with increase of altitude. At 500 ft the flux is less than $\frac{1}{10}$ of the ground level value.

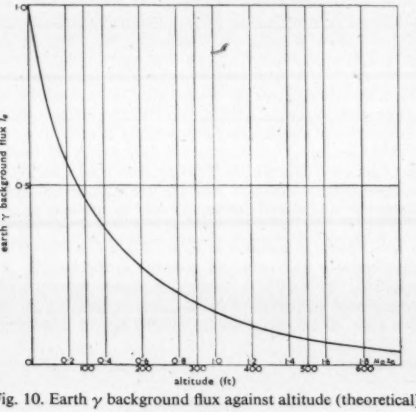

Fig. 10. Earth γ background flux against altitude (theoretical)

$$I_e = \frac{N_e A \rho_s}{2\mu_s} \int_1^\infty \frac{1}{q^2} \epsilon^{-\mu_a z_a q}\, dq$$

$$\mu_a = 0{\cdot}0001 \text{ cm}^{-1}$$

5.3. *Atmospheric activity.*—A component of background intensity may be tentatively attributed to the activity of the emanations diffusing from the pervading radioactive content of the earth's surface. The mechanism of this diffusion effect has been described by Hess:[6] it is reasonable to assume a fairly uniform atmospheric distribution of active material, mainly of radon and its products, over the operational range of aerial prospecting altitudes. By calculation similar to that of Appendix 3 it is possible to demonstrate a variation with altitude of atmospheric γ-flux I_a complementary to that for I_e. The value of I_a at 500 ft is nearly double that at ground level, owing to the contribution from below as well as above. A typical value for the mean radioactivity of the atmosphere is 3×10^{-13} c/l. The corresponding γ-ray intensity at ground level due to this atmospheric activity is only 10/cm² cross-sectional area/min, i.e. 0·03 counts/cm²/min with a Geiger counter efficiency of 0·003.

5.4. *Contamination activity.*—Although for normal purposes γ-activity due to contamination of equipment may be neglected, these aerial survey measurements are made at such low intensity levels that it is important to ensure no contact with radioactive materials during manufacture and assembly of the equipment. As a practical example of the undesirability of such contamination, activity was located after considerable difficulty within the glass envelopes of a batch of commercially manufactured Geiger tubes. The spurious increase in background indication was in some cases equivalent to an earth concentration of 0·002% U_3O_8. There was also trouble from luminous paint on the aircraft instrument dials. In the Anson aircraft used for this work the background intensity was appreciably enhanced in the vicinity of the pilot's seat. However the equipment could be suitably sited and where necessary screened by lead from any undesirable external influence of this type.

5.5. *Discussion of background factors.*—From the preceding estimates of background intensities and variations with altitude, it is possible to compare the total background count rate for the 49 Geiger tube equipment at ground level and at a typical operating altitude. Taking account of the increase with altitude of the residual uneliminated cosmic component, decrease of the earth's surface component, increase of a possible atmospheric component and ignoring the effect of any contamination activity, the total background count rate i_b at 500 ft over a surface containing 0·001% equivalent U_3O_8 should be approximately $\frac{2}{3}$ of the ground level value, and is predominantly cosmic in composition. Despite uncertainty over the response of a scintillation counter to the soft part of the cosmic component, it is safe to conclude that for this type of detector the γ background components form the major part of i_b at ground level and 500 ft. Assuming an improvement in γ-ray detection efficiency of 50 times that of a Geiger counter, the improvement in performance for a given bulk, having regard to the ratio $k/\sqrt{k_b}$, should be about 7 times. However, owing to the difficulty of producing large efficient volumes of a scintillation material such as napthalene, anthracene, sodium iodide, etc., this increase in performance may be limited in practice to about 3 times.

6. CONCLUSIONS

It has been demonstrated that, in the absence of overburden masking the radiation, it is possible to detect a radioactive deposit of concentration (% U_3O_8) times area (sq yd) factor of 84 with a 49 Geiger tube equipment flying at an altitude of 500 ft and speed of 120 m.p.h. By calculation it is possible to extend the measured performance to cover the full operational range likely to be encountered in a programme of aerial prospecting. The equipment has proved highly reliable under flight conditions of normal severity. Automatic operation with pilot control and a signal warning system could be introduced to simplify use of the equipment.

Scintillation counter: The potential importance as a γ-ray detector of the scintillation counter has been indicated. It is expected that compared with a Geiger equipment an improvement in performance of about 3 times could be achieved.

Navigation: Some form of navigational aid is desirable in order to ensure both an accurate fix of any discovery and complete coverage, without overlapping, of the whole survey area. Simple methods such as map reading and photography may suffice: the improved accuracy and reliability of radio aids such as "Gee" or "Decca" may be offset by additional expense and complexity.

7. ACKNOWLEDGMENTS

Acknowledgments are due to Mr. F. H. Wells and Mr. W. H. Rann for contributions in design and construction of equipment, to Dr. J. Howlett for suggesting the treatment of Appendix I, to Mr. C. Campbell (Geological Survey) for geological advice and to the Royal Air Force who provided flying facilities. The paper is published by permission of the Director of the Atomic Energy Research Establishment, Harwell.

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APPENDIX I

CALCULATION OF γ-RAY FLUX

The essence of the calculation is the computation of the integral

$$J = \int_V \frac{1}{r^2} \epsilon^{-\mu_s(r-r_a)} \epsilon^{-\mu_a r_a} dV$$

over the whole deposit defined in Section 2.2 and Fig. 1.

i.e.
$$J = \int_S \int_{z=z_a}^{\infty} \epsilon^{(\mu_s-\mu_a)r_a} \cdot \frac{\epsilon^{-\mu_s r}}{r^2} dS \,.\, dz \qquad (1)$$

Since $z - z_a$, the operative depth of the deposit, is of the order of $1/\mu_s$ and can be shown to be at least 100 times smaller than z for normal operational altitudes, e may be neglected in comparison with l. Then without appreciable error

$$r_a^2 = z_a^2 + l^2$$

Substituting in (1)

$$J = \int_S \epsilon^{(\mu_s-\mu_a)\sqrt{(l^2+z_a^2)}} \int_{z_a}^{\infty} \frac{\epsilon^{-\mu_s\sqrt{(l^2+z^2)}}}{l_2 + z_2} dz \,.\, dS \qquad (2)$$

Considering the integration with respect to z, putting $l^2 + z^2 = p^2$, and expanding,

$$J_z = \int_{p=\sqrt{(l^2+z_a^2)}}^{\infty} \frac{\epsilon^{-\mu_s p}}{p^2}\left(1 + \tfrac{1}{2}\frac{l^2}{p^2} + \tfrac{3}{8}\frac{l^4}{p^4} + \ldots\right)dp$$

As $p^2 > l^2$, the expansion converges and the general term is

$$\int_{p=\sqrt{(l^2+z_a^2)}}^{\infty} \frac{\epsilon^{-\mu_s p}}{p^n} dp$$
$$= \frac{\epsilon^{-\mu_s\sqrt{(l^2+z_a^2)}}}{\mu_s\sqrt{(l^2+z_a^2)^n}}\left[1 - \frac{n}{\mu_s\sqrt{(l^2+z_a^2)}} + \frac{n(n+1)}{\mu_s^2(l^2+z_a^2)}\ldots\right]$$
$$= \frac{\epsilon^{-\mu_s\sqrt{(l^2+z_a^2)}}}{\mu_s\sqrt{(l^2+z_a^2)^n}}$$

with a relative error given by the first term neglected, viz.

$$\frac{n}{\mu_s\sqrt{(l^2+z_a^2)}}$$

Hence (except for $\frac{l}{z_a} > 1$)

$$J_z = \frac{1}{\mu_s} \cdot \frac{\epsilon^{-\mu_s\sqrt{(l^2+z_a^2)}}}{l^2+z_a^2}\left[1 + \frac{1}{2}\frac{l^2}{(l^2+z_a^2)} + \frac{3}{8}\frac{l^4}{(l^2+z_a^2)^2} + \ldots\right]$$
$$= \frac{1}{\mu_s} \cdot \frac{\epsilon^{-\mu_s\sqrt{(l^2+z_a^2)}}}{z_a\sqrt{(l^2+z_a^2)}}$$

with an overall relative error of order $\frac{1}{\mu_s\sqrt{(l^2+z_a^2)}}$, i.e. not greater than $\frac{1}{\mu_s z_a}$. Taking $\mu_s = 0{\cdot}12\ \text{cm}^{-1}$ and $z_a = 100$ ft, $\frac{1}{\mu_s z_a} \sim \frac{1}{360}$, i.e. the error is negligible for typical values. The total integral (2) becomes

$$J = \frac{1}{\mu_s z_a}\int \frac{\epsilon^{-\mu_a\sqrt{(l^2+z_a^2)}}}{\sqrt{(l^2+z_a^2)}} dS \qquad (3)$$

Except for special cases (3) may not be integrated directly but provided the linear dimensions of S are small compared with z_a, the quantity $\sqrt{(l^2+z_a^2)}$ and hence J will vary little over the surface area S. It is possible to select a mean range $\bar{l}$ for the whole surface such that

$$J^1 = \frac{S\,\epsilon^{-\mu_a\sqrt{(\bar{l}^2+z_a^2)}}}{\mu_s z_a\sqrt{(\bar{l}^2+z_a^2)}} \qquad (4)$$

The closeness of this approximation may be indicated by the direct integration of (3) for the particular case of the detector situated vertically above the centre of a circular deposit area of diameter cz_a: this may be compared with the approximate form (4) to give the ratio of the approximate and exact maximum intensities. Writing (3) in polar co-ordinates

$$\mu_s z_a \cdot J = 2\int_0^{\pi}\int_0^{\frac{cz_a}{2}} \frac{\epsilon^{-\mu_a\sqrt{(l^2+z_a^2)}}}{\sqrt{(l^2+z_a^2)}}\, l\, dl\, d\phi$$

$$= 2\pi\int_0^{\frac{cz_a}{2}} \frac{\epsilon^{-\mu_a\sqrt{(l^2+z_a^2)}}}{\sqrt{(l^2+z_a^2)}}\, l\, dl$$

$$= -\frac{2\pi}{\mu_a}\left[\epsilon^{-\mu_a\sqrt{(l^2+z_a^2)}}\right]_0^{\frac{cz_a}{2}}$$

$$= \frac{2\pi}{\mu_a}\epsilon^{-\mu_a z_a}\left[1-\epsilon^{-\mu_a z_a\left\{\sqrt{\left(1+\frac{c^2}{4}\right)}-1\right\}}\right]$$

Also from (4), putting $S = \dfrac{\pi c^2 z_a^2}{4}$ and $l = 0$

$$\mu_s z_a \cdot J^1 = \frac{\pi c^2 z_a}{4}\cdot \epsilon^{-\mu_a z_a}$$

$$\therefore \frac{J^1}{J} = \frac{\mu_a z_a}{8}\cdot\frac{c^2}{1-\epsilon^{-\mu_a z_a\left\{\sqrt{\left(1+\frac{c^2}{4}\right)}-1\right\}}}$$

After expansion and to a first approximation,

$$\frac{J^1}{J} \sim 1 + \frac{c^2\mu_a z_a}{16}$$

APPENDIX 2

CALCULATION OF DETECTOR RESPONSE

The detector circuit may be simulated electrically by a parallel combination of resistance and capacity.

Thus it can be shown that, if I is the instantaneous γ flux through the detector, the indicated or recorded current i is given by

$$\frac{di}{dt} + \frac{i}{\tau} = \frac{k}{\tau}\cdot I \qquad (1)$$

where τ is the time constant of the detector circuit.

If $t = 0$ when the aircraft is at the minimum range $\bar{l} = l_0$, integration of (1) gives,

$$i = \frac{k}{\tau}\epsilon^{-\frac{t}{\tau}}\left(\int_{-\infty}^{t}\epsilon^{\frac{t}{\tau}}\cdot I\, dt\right) \qquad (2)$$

From Fig. 2 the mean range $\bar{l}$ may be written in terms of t

$\bar{l} = \sqrt{(l_0^2 + v^2t^2)}$ where l_0 = minimum range

v = aircraft speed

Substituting for $\bar{l}$ in (2) of Section 2.2, and then for I in (2) (Appendix 2).

$$i = \frac{k}{\tau} I_0 \frac{\epsilon^{-\frac{t}{\tau}}}{z_a}\int_{-\infty}^{t}\epsilon^{t/\tau}\cdot\frac{\epsilon^{-\mu_a\sqrt{(z_a^2+\bar{l}_0^2+v^2t^2)}}}{\sqrt{(z_a^2+\bar{l}_0^2+v^2\,t^2)}}\, dt \qquad (3)$$

where $I_0 = \dfrac{NA\rho_s S}{4\pi\mu_s}$.

This integral may be expanded (Maclaurin) in powers of t^2 thus

$$J_t = \int_{-\infty}^{t}\epsilon^{\frac{t}{\tau}}\cdot\frac{\epsilon^{-\mu_a\sqrt{(z_a^2+\bar{l}_0^2+v^2t^2)}}}{\sqrt{(z_a^2+\bar{l}_0^2+v^2\,t^2)}}\, dt$$

$$= \frac{\epsilon^{-\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}}}{\sqrt{(z_a^2+\bar{l}_0^2)}}\int_{-\infty}^{t}\epsilon^{t/\tau}(1-at^2+b\,t^4\ldots)dt$$

where $a = \dfrac{\left[1+\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}\right]v^2}{2(z_a^2+\bar{l}_0^2)}$,

$$b = \frac{\left[3+3\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}+\mu_a^2(z_a^2+\bar{l}_0^2)\right]v^4}{8(z_a^2+\bar{l}_0^2)^2}$$

The expansion may be integrated term by term. To preserve analytical form and avoid numerical computation, terms above t^2 may be ignored, producing in the vicinity of $t = 0$ a fair parabolic approximation to I.

i.e. $J_t \sim \dfrac{\epsilon^{-\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}}}{\sqrt{(z_a^2+\bar{l}_0^2)}}\cdot\tau\,\epsilon^{\frac{t}{\tau}}\left[1-a(t^2-2t\tau+2\tau^2)\right]$

From (3)

$$i = kI_0\cdot\frac{\epsilon^{-\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}}}{z_a\sqrt{(z_a^2+\bar{l}_0^2)}}\cdot\left[1-a(t^2-2t\tau+2\tau^2)\right]$$

By differentiating *w.r.t.* t and equating to zero it can be shown that the maximum value of i occurs at $t = \tau$

i.e.

$$i_{max} = kI_0\frac{\epsilon^{-\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}}}{z_a\sqrt{(z_a^2+\bar{l}_0^2)}}(1-a\tau^2)$$

$$= kI_0\frac{\epsilon^{-\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}}}{z_a\sqrt{(z_a^2+\bar{l}_0^2)}}\left\{1-\frac{v^2\tau^2}{2}\frac{\left[1+\mu_a\sqrt{(z_a^2+\bar{l}_0^2)}\right]}{z_a^2+\bar{l}_0^2}\right\}$$

APPENDIX 3

CALCULATION OF γ-RAY BACKGROUND FLUX

Following Section 2.2 the γ-ray background flux due to earth activity I_e may be written

$$I_e = \frac{N_e A\rho_s}{4\pi}\int_0\frac{1}{r^2}\epsilon^{-\mu_s(r-r_a)}\epsilon^{-\mu_a r_a}\, dV$$

where N_e = γ activity of earth/unit mass/unit time.

ρ_s = density of earth.

The integration now extends over a "deposit" of infinite dimensions. Transforming to spherical co-ordinates ($dV = r^2\sin\theta dr\, d\phi\, d\theta$),

$$I_e = \frac{N_e A \rho_s}{4\pi} \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \int_{z_a \sec\theta}^{\infty} \sin\theta \, \epsilon^{-\mu_s(r - z_a \sec\theta)} \, \epsilon^{-\mu_a z_a \sec\theta} \, dr d\phi d\theta$$

$$= \frac{N_e A \rho_s}{4\pi\mu_s} \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \sin\theta \, \epsilon^{-\mu_a z_a \sec\theta} \, d\phi d\theta$$

$$= \frac{N_e A \rho_s}{2\mu_s} \int_0^{\frac{\pi}{2}} \sin\theta \, \epsilon^{-\mu_a z_a \sec\theta} \, d\theta$$

Putting $q = \sec\theta$

$$I_e = \frac{N_e A \rho_s}{2\mu_s} \int_1^{\infty} \frac{1}{q^2} \epsilon^{-\mu_a z_a q} \, dq$$

This final integral may be evaluated from mathematical tables and is plotted as a function of z_a in Fig. 10.

Putting $z_a = 0$, the ground level value for I_e,

$$I_{eo} = \frac{N_e A \rho_s}{2\mu_s}$$

In like manner the γ background flux due to atmospheric activity is given by

$$I_a = \frac{N_a A}{\mu_a} \left(1 - \tfrac{1}{2} \int_1^{\infty} \frac{1}{q^2} \epsilon^{-\mu_a z_a q} \, dq\right)$$

and the ground level value by

$$I_{ao} = \frac{N_a A}{2\mu_a}$$

where N_a is the γ activity of the atmosphere/unit volume/unit time and is assumed to be independent of z_a.

A three dimensional electrical potential analyser

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An electrical potential analyser has been designed and constructed for the purpose of solving the three dimensional form of Laplace's equation, which has such wide uses in connexion with potential theory. The basic idea of the analyser is that, by means of an electrical analogy, a pure resistance network is used to solve the finite difference form of the potential equation, and a novel form of construction is employed which enables readings of a high order of accuracy to be obtained by the use of the instrument. The analyser has been used to solve several cases of potential flow around aeroplane wings of unconventional shape, but other problems involving the solution of Laplace's equation could equally well be solved by the use of the apparatus.

INTRODUCTION

There are numerous problems in mathematical physics which involve the solution of Laplace's equation in two or three dimensions. In only a few cases is it possible to obtain a mathematical solution to the problem and recourse is usually made to some approximate means, such as the Relaxation Method,[1] in order to obtain a practical solution. As an alternative to a computational method electrical analogies have been devised which have been successfully used for the solution of two dimensional problems in potential theory.[2,3,4,5] Following the use of a two dimensional electrical potential analyser[6] the possibility of constructing a three dimensional instrument was manifest and the present paper describes the design, construction and use of an analyser which was built for the principal purpose of investigating the potential fluid flow around aeroplane wings of unconventional shape. It will be appreciated that the use of the apparatus is not confined to problems of fluid flow but can equally well be used for any problem involving the solution of Laplace's equation.

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THE THEORY OF THE INSTRUMENT

Mathematical Theory.—Laplace's equation for three dimensional potential flow in Cartesian co-ordinates is

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0. \qquad (1)$$

The mathematical solution of this equation involves the obtaining of an analytical expression for the function ϕ over the domain in question, while at the same time satisfying the boundary conditions; only in a few simple cases is a mathematical solution possible. An approximate solution may be obtained by the use of the relaxation technique, the accuracy of the solution depending on the size of the relaxation net used. The labour involved in the use of the relation technique can be very considerable and, in the case of many three dimensional problems, almost prohibitive. The electrical potential analyser possesses the important advantage over the relaxation process in that it provides the final answer to the problem when the electrical potentials have been measured at the nodes of the net, whereas in the relaxation method the solution is only obtained after a series of net relaxations. In addition, only that portion of the net which is of interest need be probed. In both methods

the first step is to set up the equation to be solved in finite difference form, thus equation (1) becomes

$$6\phi_0 - \sum_{n=1}^{n=6} \phi_n = 0 \qquad (2)$$

where ϕ_0 is the value of the required function at a node of a cubical mesh and ϕ_1, $\phi_2 \ldots \phi_6$ are the values of the function at the six surrounding nodal points. A cubical mesh has been used, in the present instance, for general convenience but other mesh forms are possible.

Electrical Theory.—Consider a cubical mesh of resistors where r is the value of each resistor, and $i_2 \ldots i_6$ the currents flowing in the six arms of the mesh from the points 1, 2 . . . 6 to the node 0. If $\phi_0\, \phi_1 \ldots \phi_6$ are the values of the electrical potentials at the nodes considered then, by Ohm's law,

$$6\phi_0 = \sum_{n=1}^{n=6} \phi_n = r(i_1 + i_2 + \ldots + i_6) \qquad (3)$$

but by Kirchoff's first law the sum of the currents at any junction must equal zero and therefore equation (3) reduces to equation (2), the finite difference form of equation (1).

THE DESIGN AND CONSTRUCTION OF THE ANALYSER

From the foregoing argument it will be seen that to provide an electrical analogue to the finite difference form of Laplace's equation it is necessary to provide a mesh of electrical resistors. In the previous two dimensional analyser the mesh arms were composed of wire resistance strain gauges, but this would be too costly except for a small instrument, apart from the fact that it would be impossible to obtain strain gauges sufficiently closely matched in resistance. Precision carbon resistances were considered but were discarded because although they might be closely matched, their resistance values might not be sufficiently stable, and the cost of obtaining them to close resistance limits would have been too high.

In order to provide a suitable resistor a considerable amount of development work was undertaken by Boulton Paul Aircraft Ltd. and British Celanese Ltd. and, as a result, a form of woven resistor was satisfactorily developed. The resistor was formed by weaving a 1-in wide ribbon from 47 s.w.g. Minalpha wire and silk, the warp was composed of silk and the weft of wire, woven in such a manner that a loop was formed every half inch on one side of the ribbon. Sheaths were moulded from Xylonite and tags were riveted to these sheaths at half-inch intervals, the wire loops from the ribbon being soldered to the tags and a cover cemented to the sheath for protection. Series resistance elements consisting of twenty-five units were constructed in this manner. Minalpha wire possesses a negligible temperature coefficient of resistance and is extremely suitable for precision resistors while the accuracy with which the ribbon could be woven ensured a tolerance of $\pm 1\%$ between individual resistance units which had a nominal resistance of 200 Ω.

The analyser was designed to provide nine tiers, each tier being composed of a 25 × 25 square array of points. Fig. 1 shows a general view of the instrument. An insulated panel was drilled to receive an array of plug

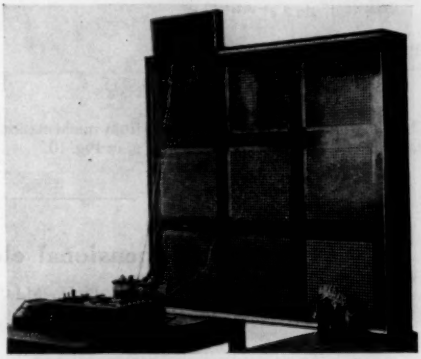

Fig. 1. Potential analyser set up for an experiment

sockets at a half-inch pitch, each socket had bolted to it, on the back of the panel, a metal stirrup and to these stirrups were attached the twenty-five unit resistance elements. Every socket on each tier was connected to

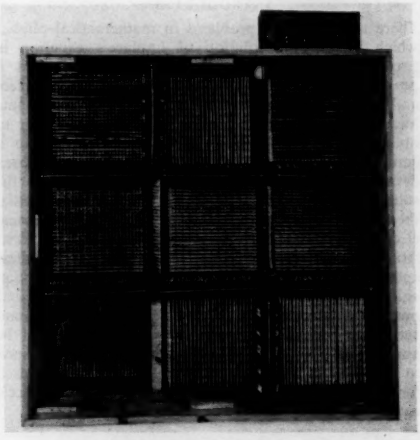

Fig. 2. Rear view of instrument

the corresponding sockets on the tier on either side through a resistor of value identical to that of the unit resistors. The inter-tier resistors were made from woven resistance ribbon, but in this instance separate units were used, the units being mounted in a moulded plastic casing.

The various tiers were connected to the inter-tier resistors by means of harnesses of colour coded leads. Sockets and resistance elements were not assembled on the master tier, but post office type terminals were used instead, these terminals being harnessed to the inter-tier resistors in the normal manner. Resistors, depending on the nature of the particular experiment in hand, were attached to the terminals of the master tier as required. The method of mounting and selecting these resistors will be described later. The resistors on the right-hand side and bottom edges of each tier were doubled in value, while the inter-tier resistors connecting the bottom right-hand corners of each tier were quadrupled in value, thus providing "selvedges" for problems involving a field of single or double symmetry. The reason for using the doubled resistors is that the net can be considered to be slit at the plane of symmetry. When resistance elements are added to the master tier, which is a plane of symmetry, they are also doubled in value. Fig. 2 shows a view taken from the rear of the instrument.

CIRCUIT TESTING

During the construction of the analyser the resistance of each element was checked to ensure that all the units were within a $\pm$ 1% resistance tolerance and during the assembly of the instrument each unit was checked for continuity immediately it was installed. When the final assembly had been completed an overall check was made by applying a potential of 1 V across opposite faces of the analyser and then measuring the voltages at each node. This procedure was carried out for each of the three principal axes. In every case the measured potential at each point was within one millivolt of the correct theoretical value.

EXPERIMENTAL PROCEDURE

The first series of experiments were concerned with potential flow, without circulation, around a circular lamina and various plan forms. As with vortex sheet theory the wings were assumed to be infinitely thin. Wing incidence presents no difficulty; as the field of flow at zero incidence is known the flow conditions for any incidence can be immediately calculated providing the flow normal to the surface is determined.

Similar procedures were used for each experiment. It will be remembered that the master tier on which the model is to be set up has no tier resistors; during the setting up procedure these were provided over the area of the model and as this tier is a plane of symmetry the elements used had double the resistance of the corresponding standard mesh resistors. The plan form was set up on the master tier by connecting resistors to the appropriate terminals. The remainder of the tier was short circuited with a heavy copper wire. In cases where the wing possessed two axes of symmetry in plan form, it was possible to set it up on the master tier so that the axes of symmetry coincided with the mutually perpendicular selvedges of the tier. In cases where only one axis of symmetry existed, one selvedge only could be used. The outer ends of the inter tier resistors placed above the first tier were inter-connected and a potential of one volt was maintained between this point and the short circuited portion of the master tier. The selection of a model size required some judgment, the larger the model the more points on its surface would be available for measurement but, on the other hand, the proximity of the boundary would cause some channel restraint.

A voltage of 1 V was applied to the analyser and the board was scanned by plugging in a probe from socket to socket, the potentials being measured by means of a D.72 A type potentiometer by Muirhead Ltd. This potentiometer is capable of reading accurately to 1 mV and therefore all readings were taken to the nearest millivolt. The electrical circuit which was used for the experiments is shown in Fig. 3. The time taken to set up the apparatus and perform various experiments are given in the following table. One semi-skilled person was employed throughout with the assistance of an unskilled person during the reading and recording stage.

Experiment	*Setting up time*	*Reading and recording*	
		semi-skilled	*un-skilled*
	(h)	(h)	(h)
Rectangular wing	4	24	24
Delta wing	8	32	32
Swept back wing	8	28	28

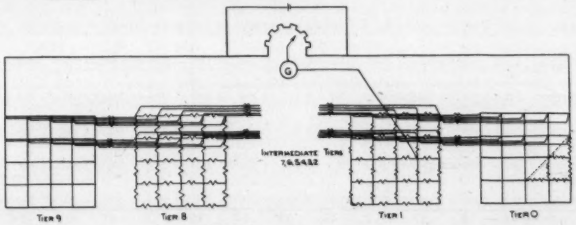

Fig. 3. Electrical circuit for analyser experiments

Three dimensional potential analyser

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
M															M
N		0													N
O		0	0												O
P		0	066	0	0										P
Q		0	106	−4	102	−2	0								Q
R		0	134	−1	164	+1	125	−1	0						R
S		0	155	+1	206	+2	199	−7	142	+4	0				S
T		0	171	+3	236	−2	250	0	225	−2	153	+1	0		T
U		0	182	−4	259	−1	286	−3	281	+4	242	0	160	−1	0	U
V		0	192	0	277	0	313	−3	319	−3	300	+3	251	−4	163	−3	0	V
W		0	199	0	290	0	333	0	347	0	339	+2	309	+1	254	0	163	+5	0	W
X		0	204	−1	299	+2	346	−1	365	−3	364	−2	345	+3	307	+1	247	−1	154	−3	0	X
Y		0	207	0	304	0	354	−1	376	−2	379	+1	365	+1	335	−4	291	+6	226	−3	126	−2	0	Y
Z		0	208	0	306	+1	357	+2	380	+1	384	+2	372	+3	345	+2	304	−1	248	−2	176	+2	086	−2	0	Z
	6	7	8	9	10	11	12	13	14	15	16	17	18	19	

Fig. 4. Three dimensional potential analyser

Problem. Flow normal to flat plate.
Tier 1.

Model size. Delta 12 mesh units semi-span. 12 mesh units root chord uniform voltage of 1·000 at tier 9.

Three dimensional potential analyser

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
M	113	114	114	114	114	114	113	113							M
N	115	116	+2 117	+3 116	−4 116	0	115	115	114	114	113	113	112	112	112	N
O	116	119	0	123	−1 122	0	120	−1 118	−2 117	0	116	115	114	113	113	112	112	O
P	119	−2 126	−2 145	+3 136	−1 129	0	124	−1 121	+1 118	−2 116	−4 115	114	113	113	113	P
Q	123	−1 136	+6 169	+3 172	+1 149	+2 136	+2 128	0	123	0	120	+3 117	−3 116	114	113	114	Q
R	127	0	144	+3 190	−2 208	−1 194	−2 159	−3 141	−2 131	0	125	+1 121	+1 118	116	115	114	R
S	131	+3 151	−3 209	0	240	+2 239	−1 212	−1 168	+1 146	+3 133	−3 126	−1 122	+4 118	116	115	S
T	134	−1 159	+6 225	+2 266	+1 277	−3 263	+1 225	−1 174	0	149	+3 135	+2 126	−4 121	−2 118	117	T
U	137	+2 164	+2 237	−1 287	+1 308	−2 305	0	279	−2 234	+1 178	+2 150	+1 135	+2 126	−1 121	0	118	U
V	139	+2 168	0	247	0	304	+3 333	+3 338	−1 323	−1 289	0	238	+2 178	0	149	+1 134	+3 125	0	120	0	V
W	140	−2 171	0	254	−3 316	+1 351	+2 353	0	356	−1 332	+1 291	−2 236	+1 174	−6 146	+2 131	+1 123	+2	W
X	141	−1 173	−3 260	+2 324	−3 363	−1 380	0	379	+2 361	−5 330	+1 284	−2 227	0	168	+3 140	+2 126	−3	X
Y	142	−2 175	0	263	0	329	−4 371	+4 390	0	392	−1 379	−1 353	0	314	−4 265	0	208	+2 153	−1 131	+2	Y
Z	143	+3 176	+2 264	−1 331	−1 373	1−	393	−2 396	−3 385	−1 361	0	325	+2 279	+4 224	−6 171	+1 135	+4	Z
	6	7	8	9	10	11	12	13	14	15	16	17	18	19	

Fig. 5. Three dimensional potential analyser

Problem. Flow normal to flat plate.
Tier O.

Model size. Delta 12 mesh units semi-span 12 mesh units root chord uniform voltage at 1·000 at tier 9.

RESULTS OF EXPERIMENTS

The presentation and discussion of the results of the various experiments is outside the scope of the present paper but, as examples of the order of accuracy which has been achieved, the values of some of the potentials which were measured on two tiers during one of the experiments are shown in Figs. 4 and 5. The residuals, which have been calculated by the relaxation process from the experimental readings, are also entered on the figures and it will be seen that they are very small; no appreciable improvement could be obtained by the application of the relaxation process to these results.

ACKNOWLEDGMENTS

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The dielectric behaviour of some types of human tissues at microwave frequencies

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The dielectric properties of four types of human tissues have been investigated by a coaxial line method at frequencies corresponding to free space wavelengths in the range 6 to 17 cm. The results have been examined in relation to the Debye dispersion equations and it has been found that the dielectric behaviour of some types of tissues can be described approximately by them if an additional term for ionic conductivity is included. The results have been further discussed with reference to ion and protein hydration and to the ionic conductivities of the intracellular and extracellular fluids.

The object of the measurements to be described was to obtain average propagation constants at microwave frequencies for different types of tissues to aid in the study of the transmission and absorption of centimetre waves in the human body. Several workers have described experiments designed to determine the dielectric properties of animal tissues over a wide wavelength range. The use of condenser field therapy at frequencies corresponding to free-space wavelengths of 3, 6 and 12 m stimulated workers to study particularly tissue dielectric behaviour at these wavelengths. Rajewsky[1] and his collaborators made notable contributions, as also did Osswald.[2] Of interest also are the very high dielectric constants (of the order of 10 000) obtained at much lower frequencies by Iwase[3] using rabbits' tissues. This author reports results from 10^6 cm wavelength (3×10^4 c/s) down to 10^2 cm and discusses briefly the dispersion shown.

At microwave frequencies, the only results on human tissues known to the author are those of England and Sharples,[4] and England.[5] The present work was designed to overlap that of England and to extend the wavelength range used in his investigations to wavelengths longer than 10 cm. Only four types of tissue have so far been investigated, but further work on others is in progress.

EXPERIMENTAL

Method and apparatus.—The method adopted was that described by Roberts and von Hippel.[6] In this method the specimen fills a short length of a waveguide or coaxial line and is terminated by a short circuit. Measurements on a standing wave set up in the guide or line section preceding the specimen then enable the complex dielectric constant, $\epsilon' - j\epsilon''$, to be evaluated.

Signals of wavelengths from 6·5 cm to 17 cm were available, and, to enable measurements to be made on a single specimen over this wavelength range, coaxial line apparatus was employed. This also had the advantage over waveguide apparatus that the specimens required were of much smaller cross-section and bulk.

Normally, standing wave measurements are carried out in an air-filled slotted section of line. For the measurements on bone samples such a procedure was adopted, the apparatus being that used in a previous investigation.[7] In the case of soft tissue specimens a Distrene-filled, slotted, coaxial line was used (Fig. 1). The design was such as to ensure plane specimen inter-

faces both at the Distrene boundary and at the short-circuiting plunger, with a micrometer method of measuring the rather small (a few millimetres) specimen thicknesses which have to be employed with materials of such high dielectric constant and loss. The correct amount of compression to employ to obtain specimens completely filling the space between the Distrene and the plunger, without damage to the tissues, was found to be easily determined from the "feel" of the lock-nut.

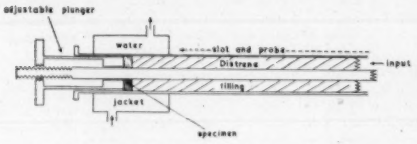

Fig. 1. Diagram (not to scale) of coaxial line apparatus used for measurements on soft tissues

The temperature of the specimens was maintained at $37 \pm 0{\cdot}2°$ C throughout the experiments by means of a thermostat flow system.

Measurements on the standing wave in the Distrene were made by means of a tuned probe incorporating a rectifying crystal. The wire of the probe moved in an air-filled slot in the Distrene 0·5 mm wide and 1 mm deep. The rectified current was measured by use of a sensitive reflecting galvanometer. Calibration of the detection system was made by measurements on a full standing wave in an air-filled coaxial line. On all occasions the galvanometer deflexions were found to be proportional to the square of the signal strength except at deflexions approaching the maximum. Care was taken in the measurements on the Distrene-filled line that deflexions did not exceed that at which the square law broke down, this square law then being assumed in all the work.

The probe distance from the Distrene interface was measured by a vernier sliding against a steel rule. The fixed distance between the interface and the probe when at the zero of the scale was measured as accurately as possible.

Measurements at different wavelengths on the standing wave in the Distrene with no specimen, and the plunger in contact with the Distrene interface, were made during each series of measurements on tissue specimens. The minima were always equally spaced, showing that the dielectric filling was uniform. The position of the minima obtained at the different wavelengths, in conjunction with the wavelengths measured in Distrene, gave estimates, agreeing within 0·1%, of the prob-interface distance when the scale reading was zero. The mean estimate was in excellent agreement with the value of this distance measured by sliding calipers.

By comparison of the wavelengths measured in the Distrene line and those in an air-filled line wavemeter the dielectric constant of the Distrene was calculated to be $2{\cdot}50 \pm 0{\cdot}01$.

The standing wave ratio in the Distrene line with no specimen was 0·015. The minimum readings at different positions were nearly equal, showing that the dielectric loss in the Distrene had a negligible effect. The loss of power causing this non-zero standing wave ratio was, therefore, considered to occur at the plunger. However, since high loss materials were to be measured this loss was neglected.

Computation of the dielectric constant and loss of specimens from the standing wave measurements was made by use of the relations and method given by Roberts and von Hippel.[6] In the case of the Distrene-filled line, the constants so calculated were with reference to those of Distrene. Hence a multiplying factor of 2·50 was used to refer the constants to that of air.

Experimental results.—The dielectric properties of skin, fat, muscle and bone are the main interest when the propagation of microwaves in the human body is considered. Accordingly, fresh specimens of these types of tissue were obtained from the operating theatre and measurements made as quickly as possible after excision. The measurements were normally completed within three hours from the time of excision.

The results of the measurements are presented in Tables 1 and 2.

It is possible that errors as large as $\pm 10\%$ may occur in determinations of ϵ'' (the loss of power at the plunger has been neglected). Owing to the many factors contributing to errors in the measurement using the Roberts and von Hippel method it is very difficult to calculate percentage errors for each tissue specimen. It is unlikely that the values of ϵ' given in the tables are in error to greater than $\pm 3\%$.

DISCUSSION

(a) *Wavelength dependence of the dielectric constants*

As expected there is a spread in the measured values of the dielectric constants of each type of tissue, depending on the anatomical site and probably also on the condition of the patient from whom the tissues were removed. It is not proposed to enter into detailed discussion on the reasons for these differences. Of more immediate interest is the dependence on wavelength of the average dielectric constants of the different types of tissues.

It has already been shown[8] that the dielectric constant at microwave frequencies of whole human blood depends on frequency according to the equations

$$\epsilon' = \frac{\epsilon^* - \epsilon_\infty}{1 + (\omega\tau)^2} + \epsilon_\infty = \frac{\epsilon^* - \epsilon_\infty}{1 + (\lambda_s/\lambda)^2} + \epsilon_\infty$$

$$\epsilon'' = \frac{\epsilon^* - \epsilon_\infty}{1 + (\omega\tau)^2} \cdot \omega\tau + (3{\cdot}6 \times 10^{12})\frac{\pi\sigma}{\omega}$$

$$= \frac{\epsilon^* - \epsilon_\infty}{1 + (\lambda_s/\lambda)^2} \cdot \frac{\lambda_s}{\lambda} + 60\,\lambda\sigma$$

Table 1. *The dielectric constants of skin and muscle at 37° C*

Type of tissue	Specimen from	Wavelength (cm) 16·85 ε′	16·85 ε″	10·07 ε′	10·07 ε″	8·36 ε′	8·36 ε″	6·48 ε′	6·48 ε″
Skin	(*a*) Near faecal fistula	51·5	20·9	51·1	15·2	48·5	13·6	46·1	16·1
	(*b*) Breast	39·7	18·0	40·0	12·3	—	—	38·8	13·4
	(*c*) Instep (sole)	—	—	42·4	13·1	40·0	12·0	39·7	12·5
Muscle	(*a*) Soleus	51·2	24·0	51·0	18·0	48·5	17·5	47·0	20·2
	(*b*) Pectoralis major	50·1	23·3	50·0	17·1	48·0	14·4	46·5	16·3
	(*c*) Pectoralis major	51·5	20·7	52·0	18·9	—	—	48·5	19·5

Table 2. *The dielectric constants of fatty tissue and bone at 37° C*

Type of tissue	Specimen from	Wavelength (cm) 16·85 ε′	16·85 ε″	10·07 ε′	10·07 ε″	8·36 ε′	8·36 ε″	6·48 ε′	6·48 ε″
Fatty tissues	(*a*) Breast	4·2	1·14	3·94	0·87	4·10	0·77	4·06	0·76
	(*b*) Anterior abdominal wall	—	—	4·92	1·46	4·20	1·0	—	—
	(*c*) Near faecal fistula (includes aerolar tissue)	7·24	2·25	7·00	1·75	6·92	1·57	5·75	1·87
	(*d*) Sole of foot	11·1	2·97	11·6	2·25	10·8	1·02	7·74	0·89
	(*e*) Breast (treated by X-rays. Includes fibrous tissue)	16·1	4·53	14·7	3·95	13·5	3·38	8·70	2·40
Bone	(*a*) Anterior medio surface of mid-shaft of tibia	8·4	1·5	8·35	1·32	8·30	1·29	7·83	1·30

where ϵ^* and ϵ_∞ are the values ϵ' approaches on the low and high frequency sides of a dispersion region due to water relaxation,

$\omega = 2\pi \times$ frequency (c/s); $\lambda =$ wavelength (cm)

τ is a single time of relaxation for "free-water" (sec)

λ_r is a "relaxation wavelength" $= (1{\cdot}885 \times 10^{11})\tau$ (cm)

and σ is an ionic conductivity assumed independent of frequency (ohm^{-1} cm^{-1}).

These are the well-known Debye equations containing an additional conductivity term. It was thought of interest to examine whether the present results on skin, muscle, etc., would satisfy these equations approximately.

Skin.—The results for specimens (*b*) and (*c*) show that skin from these sites is similar in dielectric behaviour, whilst that from the site near the faecal fistula gives higher dielectric constants. England obtained the dielectric constants of breast and leg skin at three wavelengths. He reported average values of $43{\cdot}5 - j\,16{\cdot}5$, $35{\cdot}5 - j\,16$ and $23 - j\,13$ at wavelengths of 10, 3·2 and 1·27 cm respectively. His result at 10 cm wavelength is in reasonable agreement with those for specimens (*b*) and (*c*).

Analysis of both England's and the present results then shows that the dielectric behaviour of this type of skin in the microwave region is mainly governed by the relaxation of "free" water and by ionic conductivity. (Effects of inhomogeneity on dispersion of ϵ', as in the case of blood, can be assumed to be negligible at microwave frequencies.) Reasonable agreement (see Fig. 2) between the experimental results and those calculated from the above equations is obtained if, in the latter, the following values are used:

$\epsilon^* = 42$ $\qquad$ $\tau = 0{\cdot}69 \times 10^{-11}$ sec

$\epsilon_\infty = 4$ $\qquad$ $\sigma = 0{\cdot}014$ ohm^{-1} cm^{-1}

The skin from near the faecal fistula can be regarded as abnormal. The results are approximately consistent with:

$\epsilon^* = 51$ $\qquad$ $\tau = 0{\cdot}85 \times 10^{-11}$ sec

$\epsilon_\infty = 4$ $\qquad$ $\sigma = 0{\cdot}014$ ohm^{-1} cm^{-1}

The absence of comparable results at shorter wavelengths makes the derivation of these figures much more approximate than in the previous case, though ϵ^* is more certain.

The increase in ϵ^* and τ over the values obtained for the other skin specimens may be attributed to pathological change. The higher ϵ^* may be due either to a higher total water content or to a change in the proportion of the water which is bound to ions or proteins.

It should be noted that the relaxation time of the water in tissues need not necessarily be equal to that of pure water at the same temperature ($0{\cdot}63 \times 10^{-11}$ sec at 37° C). While inorganic ions in aqueous solution decrease the water relaxation time,[9] it has recently been found[10, 11] that hydrogen bonding between water molecules and the active groups of other types of molecule causes an increase in the relaxation time. This latter effect appears to predominate in the case of tissue water, and its magnitude to depend on the type of tissue.

Muscle.—The three specimens measured give results showing little difference in their dielectric behaviour. Following the same procedure as in the case of skin the results agree approximately (Fig. 2) with values calculated using:

$\epsilon^* = 51$ $\qquad$ $\tau = 0{\cdot}8 \times 10^{-11}$ sec

$\epsilon_\infty = 4$ $\qquad$ $\sigma = 0{\cdot}018$ ohm^{-1} cm^{-1}

England has no comparable results at shorter wavelengths and the absence of these makes the values of τ and σ very approximate.

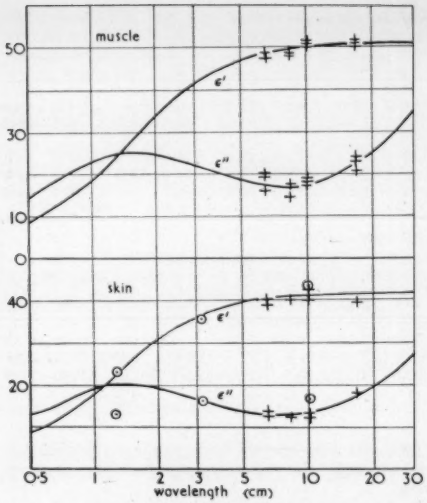

Fig. 2. Variation with wavelength of the dielectric constant (37° C) of skin and muscle tissues

Full line — calculated from Debye equations.
○ — experimental results of England.
× — present experimental results.

Fatty tissue.—The results on different specimens of fatty tissue show that their dielectric behaviour is very dependent on the amounts of other types of tissue present. Specimen (*a*) is the nearest approach to pure human fat and the dielectric properties of this show little change in the wavelength range covered.

In the course of other investigations the author measured the dielectric constant of mixed stearins (body fat constituents) at 37° C and at a wavelength of 8·20 cm, giving a result of $2{\cdot}46 - j\,0{\cdot}03$. It is reasonable to assume that palmitin and olein (also present in human fat) have similar constants. Comparison, then, with the results for specimen (*a*) indicate that its higher dielectric constant and loss are due to the presence of water and ions. The concentrations of these in specimen (*a*) must be much lower than those in skin and muscle. The other specimens give results consistent with increasing water contents due to increasing proportions of other types of tissue. It is noticeable that the fibrosis caused by X-ray treatment produces a marked effect on the dielectric constant of breast fat.

Specimens of fatty tissue investigated by England have given average dielectric constants of $6{\cdot}9 - j\,1{\cdot}6$, $4{\cdot}5 - j\,0{\cdot}95$ and $3{\cdot}4 - j\,1{\cdot}1$ at wavelengths of 10, 3·2 and 1·27 cm respectively. They would, therefore, appear to behave electrically in a manner approximately corresponding to specimen (*c*) of the present series of fatty specimens. The dispersion cannot be explained as simply as in the case of skin and muscle. The latter types of tissues display dielectric dispersion approximately in the manner of homogeneous materials at microwave frequencies. This can be attributed to the possibility that the dielectric constants (operative at these frequencies) of the extra-, inter- and intracellular fluids are not very different.

Fatty tissue is more heterogeneous at microwave frequencies since the cells containing most of the fat have a low dielectric constant and are dispersed in a medium of much higher dielectric constant. So dispersion in such a material would not be expected to follow, even approximately, that calculated from simple water relaxation alone.

Bone.—The results for the single specimen used in the present work are in fair agreement with the results of England. He obtained, for a specimen taken from the femur, dielectric constants of $7{\cdot}6 - j\,1{\cdot}45$ and $6{\cdot}3 - j\,1{\cdot}1$ at wavelengths of 3·2 and 1·27 cm respectively. Thus dispersion of ϵ' from 8·4 to 6·3 occurs between wavelengths of 16·85 cm and 1·27 cm. This corresponds approximately to a Debye dispersion with a single relaxation time of $0{\cdot}7 \times 10^{-11}$ sec. The values of ϵ'', however, do not fit in with this type of dispersion spectrum, with an additional contribution from ionic conductivity. So, as in the case of fatty tissue, the dispersion is probably influenced by inhomogeneity and analysis becomes fraught with difficulties.

(*b*) *Variation of absorption and reflexion coefficients with wavelength*

Calculations of microwave propagation in the human body (to be described elsewhere) have been facilitated by the derivation of the absorption coefficient of various types of tissues at different wavelengths and of the reflexion coefficients of the tissue interfaces (e.g. skin-fat, fat-muscle).

Fig. 3 shows the way in which the signal strength attenuation constant (absorption coefficient) varies with wavelength. The curves for skin and muscle have been calculated from the constants given in the previous section. It has been assumed that the only absorption processes present in the wavelength range 0·5 cm to 30 cm are those of ionic conduction and water relaxation. The curve for fat relates to specimen (*c*) of Table 2, which may be taken as representative of average fatty tissues. This curve, and that for bone, have been drawn through points calculated from the experimental results in Table 2.

Points of interest when the therapeutic use of microwaves is contemplated are that the absorption changes slowly with wavelength from 30 cm down to the region of 10 cm, and that the ratio of absorption coefficients of any two types of tissues remains relatively constant over a greater wavelength range. It is noteworthy that the skin absorption coefficient for microwaves of 1·25 cm

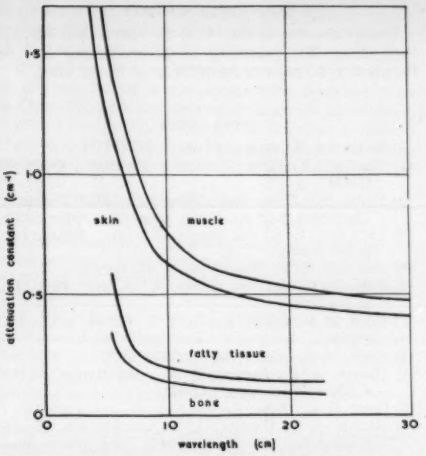

Fig. 3. Variation with wavelength of the attenuation constant of different tissues. (Energy absorption coefficient = 2 × attenuation constant)

wavelength is approximately equal to that for the infra-red radiation from a conventional therapeutic heat lamp.

The reflexion coefficient, $r'e^{jr''}$, of a fatty tissue-muscle interface is given in Table 3.

Table 3. *Reflexion coefficient of fat-muscle interface*

λ	r'	r''
6·5 cm	0·49	0·997 π
8·4 cm	0·47	0·971 π
10·0 cm	0·46	0·987 π
17·0 cm	0·46	0·984 π

The small dependence on wavelength also extends to other tissue interfaces (such as skin-fat, muscle-bone). The coefficients of these interfaces would not be very different from those in Table 3 (if due allowance is made for a change of sign).

(*c*) *The dependence of* ϵ^* *on free-water content and hydration*

The work on human blood[8] enabled an estimate of haemoglobin hydration to be made from a comparison of ϵ^* of blood with the corresponding constant for pure water. The theory of Fricke[12] for inhomogeneous dielectrics was applied. In the case of skin and muscle insufficient information regarding extracellular volume, cell shape, etc., is available for this theory to be used to obtain a worthwhile estimate of average protein hydration.

However, it can be shown that the different values of ϵ^* for skin and muscle are in accord with their different water, ion and protein concentrations if it is assumed that the dielectric constant is given by a simple volume proportion relation such as that used by Iwase.[3] It is thus assumed that the dielectric behaviour of tissues corresponds to a homogeneous dispersion of their constituents.

The chemical composition of human muscle and skin has been published by many workers. The results of Mitchell and others[13] have been used here. Taking average protein hydration in these tissues as 0·40 gm per gm,* the static dielectric constant of water at 37° C as 74·1, the depression of this due to ions as given by Hasted and others,[9] and assuming that the dielectric constant of bound water, protein, fatty compounds, etc., is 2·0, it can then be calculated that the values of ϵ^* for average skin and muscle at 37° C are 43·7 and 53·8. These are in fair agreement with the experimental values of 42 and 51.

Owing to the lower water contents of fatty tissue and bone, calculations of their dielectric constants from simple volume considerations are of doubtful value and have not been carried out.

(*d*) *Ionic conductivity of skin and muscle*

The electrical conductivity of the tissues of the forearm was measured by Burger and van Milaan[14] using direct current. They obtained a conductivity of about 0·004 ohm^{-1} cm^{-1}. It can be assumed that the currents used in their experiments were set up mainly in the extracellular fluids (blood plasma and interstitial fluid) of the skin and muscle tissues of the forearm, the membrane reactances of the cells being assumed infinite at zero frequency.

In the present case of microwave frequencies the operative ionic conductivity in skin and muscle is 0·014 and 0·018 ohm^{-1} cm^{-1}, much higher than the zero frequency figure. This increase is due to membrane reactances becoming negligible at microwave frequencies and allowing the intracellular fluid to contribute to the ionic conductivity.

Taking the extracellular volume of skin and muscle tissues as 27%, and applying Fricke's theory[12] (using a cell form factor of 2) to the d.c. conductivity of these tissues, it can then be calculated that the conductivity of the extracellular fluid is 0·02 ohm^{-1} cm^{-1}. The microwave conductivities are lower than this, indicating that the ionic conductivity of the intracellular fluids is lower than that of the extracellular liquid. A similar result was found for whole blood.[8] The conductivity differences between interstitial fluid, blood plasma, and intracellular fluid are related to the different protein concentrations in these liquids, as well as to differences in their ionic concentrations.

* This figure is approximately double the hydration which might be estimated from dielectric measurements on skin and muscle proteins if allowance is made for inhomogeneity. When using a dielectric constant relation based on simple volume proportions, the doubling of the hydration factor compensates roughly for the neglect of inhomogeneity.

CONCLUSIONS

The investigation shows that the dielectric behaviour at microwave frequencies of skin and muscle tissues can be approximately described in terms of the Debye dispersion equations with a single relaxation time, if additional dielectric loss due to ionic conductivity is allowed for. It is likely that other tissues in which the electrical behaviour of the intra- and extracellular phases is not too dissimilar would behave in the same way. However, in tissues such as fat and bone, where the phases have large differences in dielectric constant at microwave frequencies, the dispersion is not so simply described.

Reasonable explanations of the dielectric constants and ionic conductivities operative in the microwave region in terms of the hydration of ions and proteins and of differences in conductivity of the extra- and intracellular fluids have been given. It seems possible that microwave measurements on tissues, giving results of higher accuracy than those reported here, could, in conjunction with zero or low frequency conductivity measurements, lead to reasonable estimates of protein hydration and fluid volumes and conductivities.

The propagation constants, which can be derived from the dielectric constants, together with the reflexion coefficients shown in Table 3, have enabled calculations to be made of the field strength and energy distributions in parts of the human body exposed to plane wave radiation. These will be the subject of another paper.

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The preparation of high melting point metal single crystals and bicrystals with pre-determined crystallographic orientation

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A method is described for the preparation, in metals of high melting points, of single crystals and bicrystals with predetermined crystallographic orientations. The crystals are grown horizontally in an atmosphere of argon; heating is by graphite rod resistors. The method has been applied to copper and nickel.

The importance of the investigation of metal single crystals has been demonstrated in recent years. However, many properties of a metal in its usual condition are related to the fact that it is an aggregate of many small crystals. The effect which each crystal exerts on the neighbouring crystals has a powerful influence on the properties of the metal. To investigate these effects, it is necessary to study the regions between the crystals—that is, the crystal boundaries. It is more fruitful to study the properties and effects of a single boundary between two crystals than to infer them from specimens containing many crystals and boundaries. A specimen consisting of two crystals of different, but controlled, crystallographic orientation, joined by a single boundary, is called a bicrystal. Fig. 1 illustrates a bicrystal and the single crystal seeds from which it is grown.

Many techniques have been developed for the preparation of metal single crystals. Holden[1] gives an extensive list of references with a discussion of the basic methods employed. The Bridgeman method[2] of producing large single crystals from the liquid metal in a vertical furnace has been refined by Goetz[3] so that the orientation can be controlled by the use of seed

Fig. 1. Diagram of a bicrystal

crystals. Horizontal growth from a seed crystal has been adapted by Chalmers[4,5] to the preparation of bicrystals.

These are grown from two seed crystals placed in such a position that upon slow solidification of the melt the two crystals will grow in the same direction and be joined by a crystal boundary. The apparatus used by Chalmers for growing bicrystals and single crystal seeds is limited to the metals with low melting points such as tin, lead, zinc and aluminium. This article describes a method which is used for growing single crystals and bicrystals of copper and nickel. Some advantages of this method are:

(*a*) A smooth, thermally-etched and relatively strain-free surface is obtained on the top of the crystal. This eliminates the need for preparing the surface by the usual metallographic techniques prior to investigating such properties as micro-hardness, ferro-magnetic domain structures, etc.

(*b*) The top surface of the crystal may be observed while growth is taking place. The position of the interface between solid and liquid is visible and this simplifies "seeding on" and control of the rate of growth of the crystal.

(*c*) The graphite rod heating elements are inexpensive and easily replaced. They can be in close proximity and directly exposed to the metal.

PRINCIPLE OF THE METHOD

In order to achieve the highest chance of success in growing a crystal, it is necessary that the greatest possible proportion of the heat which leaves the liquid should do so by conduction through the interface and into the solid. This is brought about by maintaining a steep temperature gradient and by allowing the interface to move slowly so that the rate of release of latent heat is slow. The problems encountered in applying these principles to metals with high melting points are: (1) close control of the temperature distribution in the specimen, and (2) the necessity for protecting the specimen from chemical reaction with its surroundings. In the present apparatus, the specimen is heated by means of graphite resistors in close proximity to it. The temperature gradient is maintained by water cooling near the end of the specimen. The interface is caused to move by changing the current through, and hence the temperature of, the resistors; this method avoids the use of moving parts. An atmosphere of argon is maintained round the specimen which is supported in an alumina "boat."

DESCRIPTION OF APPARATUS

The furnace is constructed as shown in Fig. 2. It consists of two parallel, horizontal graphite rod heating elements *A*, the ends of which are clamped to water-cooled copper terminals *B*. Power leads *C* are of $\frac{1}{4}$ in copper tubing which also carries the cooling water to the terminals. The terminals are encased in lead castings *D*, and rest on a water-cooled rack *E* from which they are separated by rubber insulators. The rack is supported in a cantilever position by the end plate *F*. The furnace casing *G* is a 10 in diameter steel cylinder of $\frac{1}{4}$ in wall thickness, water cooled by $\frac{1}{8}$ in copper tubing *H* soldered to it. The casing is mounted on a truck on tracks and, when rolled into operating position (as in Fig. 2), is held against the end plate by two tie-rods *J* and sealed gas tight with a Neoprene gasket *K*. Two glass windows *M* in the top of the casing permit observation of the melt which is necessary particularly during the "seeding on" operation. A Wilson seal *N* in the casing wall, through which a $\frac{1}{8}$ in silica rod *P* is inserted, allows the introduction of mechanical motion such as stirring or skimming the melt. The Wilson seal is designed to allow the rod a lateral motion, the Neoprene gasket in the seal acting as a fulcrum. To form the heating chamber inside the furnace casing, a 1 in thick layer of highly refractory insulating firebrick (not illustrated) is built around the graphite rods. This forms a chamber about $1\frac{1}{2}$ in high, $2\frac{1}{2}$ in wide and 14 in long. The bricks are fitted loosely together and are supported by the water-cooled rack. A slit about $\frac{1}{4}$ in wide and 4 in long is left in the top of the brickwork so that the melt and seed crystals can be observed from the windows. The silica rod is also passed through the slit. Sheet iron radiation shields (not illustrated) at either end of the chamber protect the terminals from direct radiation. Temperatures obtainable in the furnace are limited by the power input available, by the quality, quantity and arrangement about the

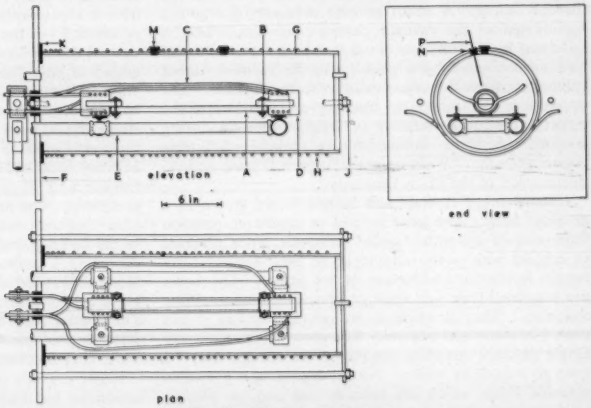

Fig. 2. Diagram of apparatus

elements of the refractory materials used, and by the sublimation temperature of the graphite elements. The maximum temperature used is about 1 600° C which is attained half-way between the terminals. Thus, there are two zones, from the centre of the furnace to either end, in which conditions are satisfactory for the growth of metal single crystals from the melt. Either zone can be used or both simultaneously. The important conditions provided in this way are: (1) a steep temperature gradient of about 1 600° C in 8 in. in the case of nickel, and (2) control of the rate of growth to any desired speed by varying the rate of lowering the power input. The furnace is intended for use with an inert gas atmosphere, argon in this case. It is connected through the end plate to a high vacuum rotary pump for evacuation prior to introducing the argon. The vacuum is measured with a McLeod type gauge connected to the furnace through the casing on one leg of a two-way glass stop-cock, the other leg of which serves as a vacuum release or atmosphere outlet. The atmosphere inlet is through the furnace casing. A small positive pressure of argon is maintained in the furnace during operation. This is indicated by the inflation of a 6 in rubber meteorological balloon connected by a glass tee in the vacuum rubber tubing line from the argon cylinder to the furnace. The pressure is controlled by bleeding argon through the outlet valve during heating or letting argon in during cooling. A static atmosphere is maintained during crystal growth. "Welding grade" argon is used and no purification of the gas is necessary.

Copper single crystals and bicrystals are grown in a graphite boat. The boat is held in cantilever position from one of the water-cooled terminals, since graphite in contact with oxide refractories at these temperatures results in chemical reduction of the oxides. The boats are machined from $\frac{1}{4}$ in thick graphite plates on a milling machine. They provide a trough 8 in × 1 in × $\frac{3}{8}$ in with $\frac{1}{8}$ in walls and bottom. For growing the narrower single crystals, graphite inserts in the trough may be used to reduce its width. Nickel crystals are grown in alumina boats which are open at one end for placing seed crystals. Single crystals are grown in boats 5 in × $\frac{1}{2}$ in wide × $\frac{3}{8}$ in deep and bicrystals in boats 4 in × 1 in wide × $\frac{1}{4}$ in deep. The walls and bottom of the boats are $\frac{1}{16}$ in thick. The alumina boats are supported for their full length on slabs of insulating firebrick. They have a life of three or four melts and fail by warping and cracking. Graphite boats last for many melts.

The power supply is obtained from a 200 kVA, 25 c/s, single phase, step-down transformer with output voltage tapped at 122 V. This feeds two 7 kVA, 230 V, 60 c/s, voltage control Variac transformers connected in parallel to carry a maximum current of 62 A at 122 V. The Variacs feed a 30 kVA, 25 c/s, single phase, 9·5 : 1, step-down transformer. The furnace is connected across the secondary of this transformer. This arrangement permits continuous control of current input to the furnace from 0 to 600 A at 0 to 13 V approximately. The central section of the $\frac{1}{2}$ in diameter, 16 in long, graphite rod heating elements are machined flat along one side for 14 in in a milling machine, so that connected in parallel, their total resistance is about 0·02 Ω when cold. This allows the maximum power input to the furnace of 7 kVA. The current is measured with an ammeter in the Variac secondary circuit.

METHOD OF OPERATION

To grow the initial single crystal, which can thereafter be used as a seed crystal, it is necessary to cause a single nucleus to form and to grow. To do this a $\frac{1}{4}$ in diameter copper rod is soldered to one of the water-cooled terminals. The rod projects into the heating chamber, and embedded in the end of the rod is a $\frac{1}{4}$ in length of $\frac{1}{4}$ in silica rod ground to a point. This serves as a "heat wick." The copper rod is bent so that the pointed silica tip touches the cool end of the pool of liquid metal. This is most easily done by having the boat at an angle of about 1° to the horizontal so that the metal will flow toward the cool end and rest against the silica tip. The action of this "heat wick" is to super-cool as small a volume of the metal as possible so that only one nucleus will form. Formation of this nucleus and growth of a single crystal from it is brought about by slowly lowering the power input to the furnace. In this way the interface between solid and liquid moves toward the centre of the furnace and in so doing the metal is crystallized progressively. The progress of growth may be followed by observing the position of the interface through a window in the furnace casing. Crystals are grown at a rate not exceeding 2 mm/min. By having the heating elements along either side of the boat and a water-cooled terminal at the cold end, ideal heat flow conditions are approached. The copper rod "heat wick" and the seed crystals also serve to promote withdrawal of heat from the cold end while heat flows into the liquid phase.

Single crystals may be grown in any crystallographic direction, but there is a preferred direction of growth which is usually one of the principal axes of the crystal. Thus, if the crystal is grown in a direction in which growth is more difficult, there is more likelihood of "stray" crystals forming. A bicrystal may also be grown with any crystallographic directions, but for nickel, and probably for other high melting point metals, it is necessary that the orientations of the crystals should be symmetrical to within a few degrees about the axis of the boat. This is to cause the boundary to grow parallel to the axis of the boat.[6] If the condition of symmetry is not satisfied, the boundary will grow at an angle to the axis of the specimen, and it may then move back until it is between one of the seed crystals and the rest of the specimen. The much smaller cross-section of the seed crystal then corresponds to a smaller area and, therefore, lower energy of the boundary. The migration of the boundary is rapid at the melting point of nickel.

Instead of a bicrystal, a large single crystal of the same orientation as one of the seed crystals is obtained. As with single crystals, "strays" are more likely to occur when difficult growth directions are used for bicrystals. Slow rates of growth, about 0·5 mm/min, may help to stop stray crystals forming and minimize the formation of striations. Chalmers[6] has analysed the conditions governing the direction of growth of the crystal boundary in bicrystals of tin and lead.

The procedure for growing a crystal is to fill the boat with the metal to be melted. The seed crystals or copper rod heat wick are placed in position. The cool ends of the seed crystals are supported on blocks or layers of sheet of the same metal as the crystal itself. This helps to keep the seeds cool and prevents their melting back too far. The two seed crystals of a bicrystal are separated by a very thin alumina shield for the distance that the seed crystals are to be melted back. The shield is ground to about 0·005 in thick on a fine abrasive wheel. The volume of the shield is reduced in this way so that it will reach the same temperature as the metal and not cause nucleation of unwanted crystals. The shield is prevented from being floated out of position by placing a block of alumina across its thick end. The refractory lining is placed in position and the cooling water is turned on. The furnace is sealed and the chamber evacuated to about 50 μ of mercury. The chamber is then filled with argon so that a slight positive pressure is indicated by the balloon. The power is turned on and the temperature raised rapidly. A nickel charge of 50 to 200 g is melted in about 20 min. As the liquid metal is observed to join on to the seed crystal, the power input is lowered by an amount sufficient to allow about $\frac{1}{4}$ in of the seed to melt. The power is then slowly decreased and the solid-liquid interface moves toward the centre of the furnace. When growth is completed, the furnace may be cooled rapidly so that in about one hour the crystal can be removed. The procedure for growing a bicrystal 3 in long takes about four hours, including the setting up, heating, growing and cooling times.

The seed crystals used for growing a bicrystal should be nearly parallel to each other and in a horizontal plane. This facilitates the "seeding on" operation. Thus, it is necessary to grow seed crystals with the required crystallographic orientation prior to using them for a bicrystal. The orientation of the initial seed crystal is determined from a Laue back-reflexion photograph. The seed crystal is then mounted in the furnace at such an angle that the crystal grown in a horizontal plane by seeding on to it will be of the desired orientation. It may be necessary to go through several stages before the required single crystal is obtained. For example, a nickel seed crystal with the [100] direction as its axis of growth and the (100) plane parallel to the top surface can be grown in a [110] direction with the (100) plane still parallel to the top surface. The geometry of the boat and furnace makes it difficult to do this in one operation with the seed in a horizontal plane at 45° to the required axis of growth. Therefore, the seed is rotated 90° about its long axis and tilted 45° in a vertical plane. The crystal grown from this has a [110] direction of growth and a (110) plane parallel to the surface. The original seed is cut off and the new crystal is rotated 90° about its long axis and another crystal is grown straight from it. This last crystal has a [110] direction of growth and a (100) plane parallel to the top surface.

It is often necessary to cut a crystal and, to avoid straining the crystal, the cutting is done electrolytically. The part of the crystal to be immersed is wrapped in an acid resisting adhesive tape (electroplating tape) except for a $\frac{1}{8}$ in strip around the crystal where it is desired to cut through. The crystal is then suspended vertically in a 10% sulphuric acid-water bath. It is made the anode, and a water-cooled copper tubing coil is used as the cathode. A current of about 10 A cuts through the crystal in a few hours.

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The effect of oxide impurities upon the initiation of arcs

During recent experiments with the electric arc in hydrogen at low gas pressures it was found that the purity of the metal constituting the cathode affected the value of the potential difference at which the arc could be initiated between fixed electrodes. The arrangement was similar to that described by Edels.[1] The impulse necessary to start the arc came from a 2 kV generator and in one set of experiments, with the fixed distance between the tungsten electrodes 3·9 mm and the gas pressure 10^{-3} mm of mercury, the arc struck at 210 V arc supply voltage, the polarity of the impulse discharge voltage being the same as that of the arc voltage. The presence of small traces of oxide, such as calcium oxide, barium oxide, etc., on the cathode, decreased the value of the arc supply voltage at which the arc was initiated to 145 V.

This effect occurred with other metallic electrodes including nickel and aluminium. As pointed out by Edels, the conditions for arc initiation by impulse require that the impulse discharge voltage must be less than that of the arc supply voltage, the inequality being maintained for a time sufficient for the current from the arc source to attain an arc value. The oxide impurities increase the current passing after the initial spark, i.e. the conductivity in the gap between the electrodes is increased and the arc current is built up at a lower arc supply voltage. Llewellyn Jones and Davies[2] have shown that a thin oxide film on the cathode can produce enhanced pre-breakdown electron emission. In addition, Salvage[3] found that the electric strength of various organic liquids depends upon the electrode metal. Traces of oxides produce marked effects on arc or spark discharges and affect the working of Geiger counters.

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Magnetostriction storage systems for a high speed digital computer

A continuously variable delay line using the magnetostriction effect in nickel, has recently been described[1] for use with pulses of 1 or 2 μsec duration in a navigational aid system. Delay lines of similar type have been developed in these Laboratories as storage elements for an electronic binary digital computer.

For use in a storage system similar to those incorporating the mercury delay line,[2] a 60 μsec delay line consists of a length of about 30 cm of thin tape (nickel or 50% nickel-iron are suitable materials) suitably terminated at each end to suppress reflexions and threaded through a small transmitting coil near one end and a receiving coil near the other end. That portion of the tape within the transmitting coil is magnetized by an adjacent permanent magnet so that a pulse of appropriate shape is launched in the tape when a current pulse is passed through the transmitting coil. A pentode type E.F. 55 coupled to a pulse transformer, is used to drive the transmitting coil and three stages of video amplification are required to produce an output signal of about 30 V. Clock pulses from the computer at 333 000 c/s are gated by this output and the system made regenerative by mixing the resulting output with the signal input to the driving circuit.

The chief advantages of the magnetostrictive delay line store over the mercury delay type may be summarized as follows: (i) greater inherent simplicity and ruggedness; (ii) smaller insertion loss, which is almost independent of the total delay; (iii) the output is easily available at any point along the delay line and may be obtained without interference with the acoustic pulse in the line.

Methods are being developed whereby a long delay line (e.g. 1 msec) may be enclosed in a small space. These methods, apart from purely space-saving considerations, will also facilitate the use of temperature control which may be necessary for longer delay times.

We have also shown that a static storage system can be constructed using the magnetostrictive line. The nature of the output pulse obtained from the receiving coil depends upon the state of magnetization of the tape within it and information can be stored in the tape as remanent magnetization. Thus it is possible to represent the digit "1" by, say, positive remanent magnetization and the digit "0" by zero or negative remanent magnetization. One receiving coil is required for each storage position, the coils being spaced so that the pulse travels from one coil to the next in a time equal to the period between digits. If, for example, the coils are connected in series then when a sampling pulse is transmitted down the line the stored information is reproduced as a series of pulses at the output. The advantages of this system are that the storage is permanent and is not destroyed by the action of reading out the information.

A fuller description of these systems, parts of which form the subject of a patent application, will be published later.

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Summarized proceedings of conference on metals Leamington Spa, April, 1951

The annual spring conference of the X-ray analysis group of The Institute of Physics was held at Ashorne Hill, Leamington Spa, on 12–13 April, 1951. The subject was "The texture and structure of metals." Recent work in three important branches of this subject was presented in eighteen papers and a number of short contributions. An evening discourse was delivered by Professor N. F. Mott, H. H. Wills Professor of Physics, University of Bristol, and an exhibition of photographs and apparatus was held. The discourse, papers and the discussions are summarized in this report.

TEXTURAL STUDIES

The first paper in the opening session was given by DR. G. B. GREENOUGH (Royal Aircraft Establishment, Farnborough), who reviewed work on "Lattice strains in deformed metal aggregates."[1]

The occurrence of residual strains at zero stress has been discussed by many authors.[2,3,4,5] There has been some controversy as to whether or not a necessary condition for the occurrence of residual lattice strains is that the applied stresses must exceed the elastic limit of the material. Trivial effects, such as non-axial loading of the test specimen, are not the main causes of the observed residual strains. It is possible that a stress system develops owing to the different degrees of hardness within the material. Some evidence has been given to show that surface grains are softer than those within the bulk of the material,[6] and Smith and Wood[7] have suggested that the boundaries of the grains become amorphous. Another possible cause of the residual strains in polycrystalline materials is the difference in yield strengths of grains in various orientations within the metal; experimental evidence for this view was provided by the speaker.[8]

"A simple method of X-ray microscopy" was described by DR. R. W. K. HONEYCOMBE (Cavendish Laboratory, Cambridge). The experimental technique is similar to that of Berg[9] and Barrett.[10] An image of a crystal surface can be formed by the reflexion of characteristic radiation from a line source of X-rays; with suitable experimental conditions, the image can be enlarged 50 to 100 times to reveal significant microscopic features. A full account of the experimental technique has been given elsewhere.[11]

It is found that within a totally reflecting area, the intensity of the X-ray image is not always uniform. When trivial causes, e.g. non-uniformity of source, are eliminated, narrow regions of high intensity remain. These usually occur in images from deformed crystals, and, by comparison with optical micrography, it is found that heavily distorted regions of the crystal, e.g. "kink bands," are responsible. These differences in intensity in X-ray images provide a very sensitive way of studying the effects of plastic deformation.

The author stressed the fact that the way to accurate interpretation of X-ray micrographs lies in careful comparison with the results of optical micrography. The two techniques reveal different features of plastic deformation, and when taken together provide a more complete picture.

In a contribution "Microbeam experiments on copper and other metals," MR. P. GAY (Cavendish Laboratory, Cambridge) described the results of the application of the fine beam technique[12] to the deformation of copper and nickel. Microbeam photographs taken during the early stages of deformation show the resolution of the continuous diffraction rings, obtained using ordinary back-reflexion methods, into broadened spots clustered into arcs; however, for heavier deformations, the resolution of the spots within the arcs decreases until, for a specimen of copper reduced 50% in thickness by rolling, no substructure is visible.

The interpretation of the photographs for small deformations is similar to that described for aluminium.[12,13,14] Measurements of the shapes of the spots show that there is considerable physical broadening, which can be interpreted in terms of either small particle size or distortion effects. As the mean particle volume is known, by consideration of various shapes of particle, e.g. equiaxial, rods or lamellae, it can be shown that particle-size broadening is not effective. Further, the value of the density of dislocations in boundaries for a 7% rolled copper specimen is found to be 10^9 dislocations/cm^2.

For the more heavily deformed specimens, an upper limit is found for the particle-size by evaluating the condition for continuity of the arcs. It is shown that, for the 50% rolled copper specimen, this limit is 1 μ. By attributing the whole of the measured line breadth to small particle-size effects, a lower limit of particle-size of 3×10^{-6} cm is obtained. For this material, the values of the dislocation density which can be derived depend on the assumption made as to the manner in which the breakdown of the original grain occurs. A value of 4×10^9 dislocations/cm^2 is found, if it is assumed that the grains have been bent and subsequently polygonized. If, however, the mosaics are randomly orientated in the parent grain, it is concluded that the limits within which the dislocation density must lie are 3×10^{10} and 9×10^{11} per cm^2.

The examination of "The structure of an α/β brass" by microscopic and X-ray methods was discussed by MR. R. J. DAVIS (Inorganic Chemistry Laboratory,

Oxford). The alloy (42% Zn) was extruded in the homogeneous β region, aircooled and reduced 10% by cold-work. The brass then contains about 30% by volume of α precipitate, much of which is found to originate as intergranular dendrites which subsequently branch, giving rise to a family of needles like the teeth on a series of parallel combs.

The brass was examined by an electropolishing-etching method developed to give fine pitting on the β phase. By this method, a substructure is revealed in the large β grains, the boundaries enclosing areas of about 20 μ diameter. The subgrains appear fibrous when viewed parallel to the α needles.

On annealing at 500° C, the α needles spheroidize to angular polyhedral grains, whose corners appear to coincide with the substructure boundaries.

X-ray examination at Cambridge indicated that a specimen annealed for 11 days at 500° C contains β phase as grains of diameter about 70 μ in agreement with the metallographic grain size. The grains occur in groups of nearly the same orientation, the members of a group being misorientated by about 0·4° over a total range of 3°.

The angles at the α-grain corners in annealed specimens were measured. A preliminary distribution curve gives an average dihedral angle of 160°, in contrast to 110° for annealed polycrystalline α/β brass.

The fibrous form of the substructure in the original alloy suggests that it cannot be ascribed to either hot or cold working. Annealing at 800° C destroys the substructure, which reappears with α precipitate on cooling to 500° C. Mr. Davis suggested that the precipitation caused fragmentation of the β matrix, through the volume changes involved in precipitation of α from β.

The outline of a treatment by which it is possible to predict the deformation textures of f.c.c., b.c.c. and hexagonal metals was given in a paper on "The development of preferred orientation in metals on deformation" by MR. E. A. CALNAN and DR. C. J. B. CLEWS (National Physical Laboratory, Teddington). The behaviour of the polycrystalline aggregate is deduced from a knowledge of the deformation modes of the single crystals, whilst the interaction of crystals on one another and the maintenance of cohesion at grain boundaries are taken into account.

Some account of this work has been published,[15] and only the general principles for the simple case of a f.c.c. metal under compression are given here. Slip takes place on {111} planes in a [110] direction; the resolved shear stress on the most favourable slip system varies with orientation in the manner shown in Fig. 1. Thus for an isolated single crystal with compression axis at P, slip occurs when 0·46 of the applied stress is equal to the critical shear stress. In the polycrystalline aggregate, slip will not occur in general when this condition is satisfied, owing to the constraints imposed by the surrounding grains. This implies that the initial shear stress on this most favourable system has not been reached, and therefore that the effective stress on the grain may be represented by a point on a lower shear stress contour. The most probable path for the movement of the effective stress, T_e, is along the line of

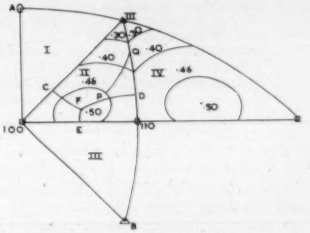

Fig. 1. Stereographic projection showing shear stress contours for a face-centred cubic metal
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quickest descent to Q, and thence along the triangle edge to the minimum at the [111] direction. If deformation occurs when T_e lies between P and Q single crystal rotation occurs; if it is between Q and [111], the resolved shear stresses on two systems are equal and there is duplex rotation: while, if T_e has reached [111], multiple slip on six symmetrically disposed systems occurs and there is no rotation. When slip occurs, the lateral stresses are relieved, the applied and effective stresses again become coincident and the cycle of operations may be repeated. Thus the deformation is visualized as a step-like process consisting of small elements of single, duplex or multiple slip. These compression rotations for various orientations throughout the reference triangle are shown in Fig. 2(*a*), and their combination to give the general trends of rotation in

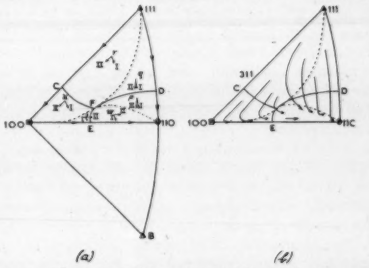

Fig. 2(*a*). Lines of quickest descent from the shear stress peak (F), and directions of rotation due to single crystal slip (→ I) and duplex slip (→ II) in a face-centred cubic metal in compression

Fig. 2(*b*). General trends of the rotations in a face-centred cubic metal in compression
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Fig. 2(*b*). The resulting predicted texture is compared with the experimental work in Fig. 3.

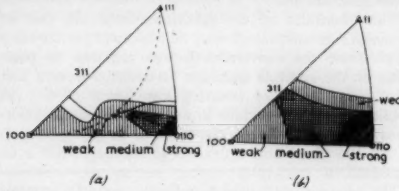

Fig. 3(*a*). Predicted compression texture for a face-centred cubic metal

Fig. 3(*b*). Texture in aluminium after 98% compression

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Mr. Calnan also demonstrated the method for deriving rolling textures, and mentioned that twinning can be taken into account in deriving the rolling textures of hexagonal metals.

In all cases, the satisfactory agreement between the predicted and observed deformation textures for metals of all common structures give some confidence in the general validity of the method.

In the discussion which followed this session, DR. L. G. FINCH (Hadfields Ltd.) gave examples illustrating the absence of residual lattice strain in low-carbon steels after application of strictly axial tensile stress below the upper yield stress, and of pre-yield residual strain in a sample previously subjected to non-axial tensile loading. An abrupt and inconsistent residual body strain recorded after discontinuous yielding can be explained by the heterogeneous and, in certain respects, fortuitous, deformation attending the formation of a Lüder line, which effectively produces a state of non-axial loading in an initially axially loaded specimen.

Dr. Finch described a method[16, 17] for the correction of errors arising from a radial shift in the position of the apparent doublet peak relative to the true $K\alpha_1$ peak. The considerable significance of the correction was illustrated for the case of a stress versus residual lattice strain characteristic for a mild steel loaded incrementally in tension to its breaking stress.

DR. T. LLOYD RICHARDS (Imperial Chemical Industries Ltd., Birmingham), referring to the paper by Calnan and Clews, presented evidence that deformation of polycrystalline metal by normal crystal slip is limited to amounts less than 100% elongation, or 50% reduction in cross-section, further deformation proceeding by a second mechanism, termed shear.[18] The use of slip as a basis for analysis is therefore only strictly valid for the determination of the stress-strain curve of annealed metal, and not for predicting textures resulting from heavy cold working. The direction of shear lay in the surface of the unextended cone, and rolling textures in f.c.c. and b.c.c. metals can be explained merely on the basis that the atomic close-packed directions [110] and [111] respectively lay in shear planes inclined at about 35° to the strip surface.

MR. R. J. DAVIS agreed with the comment by DR. GREENOUGH on his paper that it is surprising, in view of theoretical considerations, that the slightly misorientated subgrain boundaries have an interfacial tension about one-third that of the ordinary β-grain boundaries.

In the final contribution to the discussion, MR. P. GAY gave an account of investigations of the deformation of the softer metals, tin and cadmium, using ordinary back-reflexion methods. The evidence shows a polygonized structure for small deformations, with recrystallization occurring for heavier deformations. A value of 3×10^8 per cm^2 is found for the density of dislocations in boundaries in 35% rolled tin.

TECHNIQUES OF X-RAY ANALYSIS

In this session, now a regular feature of the spring X-ray analysis group conference, short contributions on a wide range of subjects were presented.

A precision-made back reflexion microbeam X-ray camera was described and exhibited by DR. R. W. CAHN (Atomic Energy Research Establishment, Harwell). The primary purpose of the camera is to take Laue photographs of individual grains in coarse-grained metal specimens, the grains being previously selected under a microscope. The specimen is prepared as for metallographic examination and mounted on a stage unit which can be removed from the camera and set up on a "Vickers" metallurgical microscope. When the grain has been selected, the stage unit is transferred to the camera. By using the appropriate adjustments on the stage, it can be ensured that the X-ray beam is incident on the selected grain. This camera has been used successfully to determine the twinning planes in coarse-grain uranium.

DR. P. B. HIRSCH and MR. P. GAY (Cavendish Laboratory, Cambridge) described an X-ray technique for the study of sub-structures in materials. A back-reflexion photograph is taken with an X-ray beam of such diameter that a spotty ring is obtained. The mean particle size of the specimen can be determined from the known area of cross-section and divergence of the beam, by counting the number of spots on the same ring for the different exposures.[13] If some of the particles are mosaics within larger metallurgical grains, the spots lie along arcs around the ring; the angular extension of the arcs shows the total range of misorientations within one grain.

Dr. Hirsch illustrated the method by examples of its application to substructures in α/β brass, an Ag-Al alloy and tempered martensite. He believed that it could be applied to many problems, particularly in the metallurgical field, where standard metallurgical methods failed to give conclusive results.

New methods of measurement of temperature distribution in a high-temperature Debye-Scherrer camera were described by MR. R. L. BERRY (University of Birmingham). The standard methods of thermal

calibration and their limitations were discussed. In particular, the use of an exploring thermocouple was shown to be fundamentally unsound owing to the conduction of heat by the leads to its tip. These limitations have been overcome in recent work by the use of two new methods. They are, firstly, the use of a dummy, thermally insulated specimen, to which fine thermocouple wires are attached so that their leads pass between the furnaces. Secondly, the use of a 1 mm high X-ray beam to measure the parameter of a silver specimen at various positions and so determine the temperature distribution along it.

Measurement of temperature distribution by these methods has shown that it is necessary to standardize conditions rigorously after calibration. The principle conditions include: (i) constancy of surface emissivity of specimens; (ii) standardization of the positions of local thermocouples; (iii) standardization of the specimen relative to the furnaces, and (iv) complete thermal insulation of the specimen.

Dr. H. Lipson (College of Technology, Manchester) described work carried out in his laboratory by Mr. C. A. Taylor and Mr. A. W. Hanson on the development of the ideas of Bragg on Fourier summation by optical interference methods.[19] Bragg has shown that an adequate representation of the structure of diopside can be obtained in this way, but for more general centrosymmetrical structures, phase differences of π must be introduced into separate beams of light. This has been accomplished by using disks of mica punched from a uniform sheet; between crossed nicols, two perpendicular extinction directions exist for the mica, and when the plate is turned through an extinction direction, the phase of the light changes by π. The amount of light transmitted depends on the angle through which the mica is turned, and so the magnitude of the structure amplitude can be represented by the angle of rotation and its sign by the direction.

The method has been applied to durene and shows that it is possible to distinguish between a possible and an impossible combination of signs for the coefficients of the Fourier series.

Some striking electron microscope photographs of screw dislocations in paraffin crystals were shown by Dr. V. Vand (University of Glasgow). Dr. Vand expressed the opinion that no theory of polymorphic transformations or phase changes in any crystal is complete without taking into account the screw dislocations present in all crystals.

Dr. A. Hargreaves (College of Technology, Manchester) showed that the signs of the structure factors, F_A and F_B, of a pair of centro-symmetrical isomorphous compounds, A and B, can be simply determined from plots over limited ranges of the Bragg angle θ of F_A/ϕ against F_B/ϕ, where

$$\phi \simeq \Sigma(f_A - f_B)\cos 2\pi\left(\frac{hx}{a} + \frac{ky}{b} + \frac{\iota z}{c}\right)$$

represents the difference in contribution of the replacement atoms, co-ordinates (xyz) to F_A and F_B, for any given reflexion $hk\iota$.

The intensities of the reflexions from the pair of isomorphous compounds may be measured on arbitrary scales since the conversion factors required to place them on the absolute scale can be deduced from a plot of F_A/ϕ against F_B/ϕ covering low values of θ. An application of the method to an actual structure determination enabled the correct signs to be given to each of the 165 observable reflexions in the particular zone examined.

The accuracy with which a Geiger counter spectrometer can measure very small effects was illustrated by an account of studies of the effects of cold work on the Debye-Scherrer spectrum, given by Dr. G. K. Williamson (Department of Metallurgy, University of Birmingham). Measurement of the background levels of two worked specimens of aluminium show that a small rise of 12–15 counts is produced by the cold work. This corresponds to approximately 7% of the total reflected intensity although the shift of the background is only 0·1% of the peak value of the neighbouring line.

Experimental measurements of the change in line breadths on recovery of super-pure and commercially pure aluminium suggest that recovery corresponds to an ordering of dislocations to reduce the long range strain field, and hence the line breadths, without any appreciable change in the short range strains and the background level.

It appears that the annealed specimens exhibit some line broadening: it is not possible to account for this broadening by small particle-size effects and it is concluded that some lattice strain persists in the annealed material.

Dr. A. J. C. Wilson (Viriamu Jones Laboratory, Cardiff) presenting a paper by Dr. D. Rogers of his laboratory, discussed new methods for extracting a direct structure image from Patterson maps.[20, 21, 22, 23, 24, 25, 26]

For two isomorphs, containing one replaceable atom, the difference Patterson,

$$\Delta P(r) = \sum_{H=-\infty}^{+\infty}(I_1 - I_2)_H \exp 2\pi i(H \,.\, r) \qquad (1)$$

(where I_1, I_2 are corresponding intensities from each isomorph) gives a map representing those few images containing vectors from the replaceable atom, if atomic positions in the two structures correspond reasonably well.[27] This overlapping limits the usefulness of the method.[28] If, however, the replaceable atom positions are known, these images may be rearranged and brought into coincidence. The map of the function

$$\phi(r) = \frac{1}{f_{A1} - f_{A2}} \sum_{H=-\infty}^{\infty}\left[\frac{I_1 - I_2}{F_A^*}\right]_H \exp 2\pi i(H \,.\, r) \qquad (2)$$

(where $[f_{A1}, F_A]_H$, $[f_{A2}, F_A]_H$ are the contributions of the replaceable atoms to the structure factors) gives the desired structure directly if it is centrosymmetrical. For

non-centrosymmetrical cases, with F_A systematically real, the two replaceable atoms appear in their correct positions but are common to two centrosymmetric related and superposed structures.[29] For a non-centrosymmetric case with F_A systematically complex, one structure appears on a weak, undulating background, and here the method is a useful extension of the isomorphous replacement technique.

The main difficulty is that for negligible F_A, the amplitude $\left[\frac{I_1 - I_2}{F_A}\right]_H$ is indeterminate and must be omitted; this causes serious difficulties if it occurs systematically.

The method may be applied to any Patterson synthesis. Thus if we locate p atoms within the cell, then the map of

$$\sum_{H=-\infty}^{+\infty}\left[\frac{I}{F_K^*}\right]_H \exp 2\pi i(H \,.\, r) \qquad (3)$$

where $F_K^* = \sum_{j=1}^{p} f_j \exp 2\pi i(-H \,.\, r)$, has p of the N images in coincidence, whilst the remaining $(N - p)$ are scattered diffusely throughout the cell. The prominent image has weight $\sum_{j=1}^{p} f_j$, so that the likelihood of recognizing the required image from the background increases with the weight and number of known atoms.[30]

The last contribution was given by Dr. I. G. Edmunds (College of Technology, Manchester). He described a method of measuring the intensity distribution in a section of a diffuse reflexion taken normal to the rotation axis in reciprocal space. The screens on a Weissenberg moving film camera are replaced by a screen with a very narrow, but accurately uniform, slot of adjustable width ($\sim 0{\cdot}1$ mm); the intensity distribution is recorded on one photograph, thus obviating the use of any monitoring device. To examine other, parallel sections, the position of the screen is altered by means of a calibrated, screwed collar.

THE BEHAVIOUR OF ELECTRONS IN ALLOYS

In the evening lecture, Professor N. F. Mott reviewed the problems to be explained by the electron theory.

The free electrons may be described either in terms of the London-Heitler theory[31] in which each electron has an atomic wave function, or in terms of the Bloch collective electron theory[32] in which each electron has a wave function extending through the whole lattice. The collective electron theory can be used to establish the distinction between conductors and insulators; however, some difficulties occur, as, e.g., in the case of nickel oxide, which should theoretically be a metallic conductor, whereas it is found that it is actually an insulator. If a substance is an insulator, then either of the viewpoints can be adopted; if the substance is a conductor, some consideration is needed to see which theory is more convenient in application.

The speaker then turned to other physical properties, in particular, cohesive forces and ductility in metals. He pointed out that for the f.c.c. metals, the repulsive forces of the ions in contact had been used as a basis for the calculations of their elastic constants.[33] Professor Mott concluded by discussing the conditions for the ductility of metals. If a metal is not ductile, it means that the dislocations are unable to move; it can be shown that the condition for anchorage of dislocations is a short surface of lattice misfit.

STRUCTURAL STUDIES (PHASES)

Professor G. V. Raynor (University of Birmingham) opened the second session with a brief review of the contributions of X-ray structure analysis to the theory of alloy formation. He included a detailed account of recent work on the development of order in alloys. The work described illustrates the importance of interactions between solute metals in a ternary system.

The addition of tin above a critical content to the ordered Ag-Mg structure causes the original cubic cell to change to one of twice its dimensions. A similar phenomenon is not found in the Al-Mg-Zn system. On heating, changes are observed at two temperatures, the first corresponding to the disappearance of the doubly ordered structure, the second to the disappearance of the initial phase. It is suggested that the tin, on account of its high valency, gives intense local lattice distortion, which is relieved by a regular arrangement of the tin atoms. Zinc would give much smaller distortion. Secondly, the electrochemical interaction of magnesium and tin atoms would lower the free energy in the ordered state; here again the effect would be much smaller for zinc.

Dr. W. Hume-Rothery (Inorganic Chemistry Laboratory, Oxford) presented some new results on gamma-phases obtained in collaboration with Mr. J. O. Betterton during an investigation of the Cu-Ga phase diagram. In the gamma-phase region of this field, three modifications of the gamma structure are present and are denoted γ_1, γ_2 and γ_3. The structural changes can be correlated with changes in the number of atoms in the unit cell, and it is established that, from a full complement at the Cu-rich limit, this number falls slightly with increasing gallium content in the γ_1 region, then much more rapidly in the γ_2 and γ_3 fields, so as to maintain a constant number of electrons per unit cell, at a value slightly less than that corresponding to a full Brillouin zone. The Cu-rich limit of the γ_1-phase occurs at the same electron-atom ratio as that in the corresponding boundary in the Cu-Zn system, in which the structure then reverts to a body-centred cube array with some vacant sites. It is suggested that the existence of gamma-phases between two body-centred cubic phases is favoured by a reasonable difference in the sizes of the atoms concerned. For example, no such phases are found in the Ag-Al system where the atomic diameters are nearly the same. For the Cu-Ga structures the normal valencies of 1, 2, 3 for Cu, Zn, Ga serve better to correlate the phenomena than do Pauling valencies.

The importance of crystal structure analysis in helping to elucidate some of the factors controlling the formation of intermetallic compounds was demonstrated in the next contribution by DR. K. ROBINSON (Cavendish Laboratory, Cambridge).

He described first the recently determined structure of Mn_3SiAl_9. The Brillouin zone of this compound approximates to a Fermi ellipsoid containing 1·68 electrons per atom. This value can be interpreted by assigning a negative "valency" to the transition metal atom.[34] If this negative valency means an absorption of extra electrons into the 3d-bands of the atoms, then electron density syntheses, and a study of the intensities of certain low-angle reflexions, should support this theory.

A feature of Mn_3SiAl_9 is that it is isomorphous with Co_2Al_5[35] and $(Fe,Ni)_4Al_{10}$[36] except that two of the atomic sites occupied in the two latter are empty in Mn_3SiAl_9. This may be due to the controlling influence of the electron-atom ratio.

The sigma phase figured prominently in this session, and in the first paper on this topic, DR. A. M. B. DOUGLAS (Cavendish Laboratory, Cambridge) reported on the recent determination of the structure. This phase is of considerable technical importance because it causes serious embrittlement of some steels. It occurs at varying compositions in the systems Fe-Cr, Mn-Cr, Co-Cr, Mn-V, Fe-V, Co-V, Ni-V and Fe-Mo,[37, 38, 39] and in many ternary systems.

Early attempts to solve the structure by interpreting X-ray powder photographs were not successful, although the tetragonal,[40] orthorhombic[38] and hexagonal unit cells suggested were later shown to be related to the true unit cell.

In the Co-Cr system[41] it was possible to obtain single crystals. From these the symmetry was found to be tetragonal, space group P 4/mnm, P 4mn, or P $\bar{4}$n2, with 30 atoms per unit cell. These results are similar to those reported by Tucker[42] for β-uranium. The similarity is confirmed by Fourier (hk0) projections. More data are now being collected with a view to exact determination of the space-group and of the z-parameters, and work is proceeding on Fe-Mo.

Many other groups of workers are interested in the structure of the sigma phase. In America, Shoemaker and Bergman[43] are working with single crystals of Fe-Cr, and Kasper, Decker and Belanger[44] with single crystals of Co-Cr, and both confirm the similarity of the structure with that of β-uranium. Dr. Douglas also reviewed other work being done in this country, which was presented later in the session. There has been some discussion as to whether the sigma phase is an electron compound, and the suggestion of DR. SULLY (presented later in the session) is that the electron-rich boundary of the range of composition of all the isomorphous phases occurs at an electron-atom ratio of 2·25.[45] This does not agree with the electron-atom ratio (1·72) which would correspond to the approximate filling of the first prominent Brillouin zone, which is indicated by the first group of strong lines on the powder photograph. No satisfactory explanation of this result has so far been advanced.

The structures of β- and γ-uranium were the subject of a paper by DR. J. THEWLIS (Atomic Energy Research Establishment, Harwell) entitled "The X-ray examination of uranium at various temperatures." Successful X-ray photographs of β- and γ-uranium have been obtained, using filings sealed off in an evacuated silica capillary tube, and, prior to Tucker's work[42] on the structure of a quenched uranium-chromium alloy containing 1·4 atomic per cent of chromium, the photographs of β-uranium had been indexed by Ito's[46] method on a monoclinic unit cell containing 30 atoms and having the dimensions $a = 9{\cdot}99$ A, $b = 7{\cdot}85$ A, $c = 8{\cdot}67$ A and $\beta = 101° 42'$. Following the announcement of Tucker's work, however, an X-ray examination of a 1·4 atomic per cent uranium-chromium alloy showed that its structure was indeed similar to that of β-uranium, as had been claimed by Tucker. The intensity differences between the photographs of β-Ur and those of the uranium-chromium alloy indicate that the two structures, although similar, are not identical. The conclusion to be drawn is that the detailed structure of β-uranium will have to be worked out from the element itself. Figures for the unit cell dimensions and density of α-, β- and γ-uranium, and of the uranium-chromium alloy, are given below. The values for α-uranium are based on Jacob and Warren's results,[47] bearing in mind that their unit cell dimensions were almost certainly in kX units and not in Angstrom units as stated in their paper, and that they used a wrong value for Avogadro's constant.

An interesting property of the uranium-chromium alloy is that it assumes the α-uranium structure when subjected to filing, but takes up its former structure again when quenched from 720° C.

In the final paper of this session, DR. H. J. GOLDSCHMIDT (B.S.A. Group Research Centre, Sheffield) described work on "The constitutions of some alloy and carbide systems related to heat-resisting materials." Although alloy steels often contain up to twelve elements,

Table 1. *Unit cell dimensions and density measurements for U*

	$\alpha - U$ (room temp.)	$\beta - U$ (720° C)	$U - Cr$ (1·4% Cr) (room temp.)	$\gamma - U$ (805° C)
a (A)	2·858	10·759 ± 0·001	10·594 ± 0·005	3·524 ± 0·002
b (A)	5·877	—	—	—
c (A)	4·955	5·656 ± 0·001	5·631 ± 0·004	—
ρ	19·00	18·11	18·56	18·06

the number of phases found is relatively small—so far the author has found twenty-six isomorphous families.

The first main group, the metallic carbides, falls into three structural classes,[48] the cubic (TiC) type, the hexagonal (WC) type and the cementite (orthorhombic) type. These classes form clearly defined fields in the periodic system. Chromium is in a transitional position, having three more complex carbides. In steels, chromium can be made to enter either austenite, ferrite, cementite, $Cr_{23}C_6$, Cr_7C_3, or the sigma phase. This property gives it a key position in heat resisting steels.

The structure of the κ-carbide, $Cr_{23}C_6$, was then described.[49] The structural features are reflected in the quaternary Fe-Cr-C-W system. The carbide bears a simple relation to both the body- and face-centred cubic structures, which explains how it is formed in austenite 18/8 type steels from austenite and as a stable alternative to martensite. The η-carbide, Fe_4W_2C,[50] is linked structurally to W_2C, the carbon environment being the same in both phases. Two types of 6-fold carbon co-ordination are found in carbides, while the 8-fold carbon co-ordination in the κ-carbide is a transition between η and Fe_3C.

The ternary systems Fe-Cr-W and Fe-Cr-Mo have recently been investigated.[51, 52] The emphasis here is on the competitive character of the formation of σ and ξ compounds. The ξ structure suffers several minor modifications from that determined by Westgren and a more detailed study is required.

The speaker then considered several features of the heat-resisting steels. The most important feature described was the existence of the ψ phase[53] in extracts from austenitic steels. This is a true carbide which bridges the gap between the face-centred cube of austenite and defect lattice formation in VC. There is a systematic tendency in carbides of the first long period to lose carbon (e.g. TiC, V_4C_3, $Cr_{23}C_6$); iron austenite, with one carbon to twelve metal atoms for saturation, fits in with this trend. Manganese stabilizes austenite and increases its carbon solubility. If size factors permit, it should be possible for cubic carbides to form solid solutions with austenite; the κ-carbide apparently dissolves in this way at high temperatures. However, chromium carbide does not form a close-packed cubic lattice, as do vanadium carbide and austenite. The "missing-link" was found when a carbide phase with an NaCl type structure was extracted from a quenched pure-chromium steel. It looks at first like austenite, and may be confused with it in X-ray examinations. It is a true carbide containing 30 atomic per cent carbon and can be regarded as an "austenite-like" hybrid carbide, while austenite itself is a special type of carbide.

In a discussion on Dr. Hume-Rothery's paper, Professor Raynor suggested that in the silver-aluminium system, the silver atom would tend to expand the lattice on account of its greater size and to contract it because of its smaller valency. These two effects might cancel out, although local distortions would still be present.

Dr. A. J. Bradley (B.S.A. Group Research Centre, Sheffield) emphasized the difficulty he had found in similar work on the copper-aluminium system. For order-disorder transformations the agreement of observed and calculated intensities must be very good before the interpretation is considered certain.

Mr. T. J. Heal (Fulmer Research Institute, Stoke Poges) described the work of Dr. Sully on the *e/a* ratio σ phase, to which reference was made in Dr. Douglas's paper.

Dr. Sully assumes that in binary alloys of the transition elements, an exchange of electrons can occur between Pauling's bond and atomic orbitals so that the latter are filled to the fullest permitted extent of 4·88 electrons per atom. The column on the right in Table 2 gives the ratio of atoms to electrons in excess of those required to fill the 3d atomic orbitals. It is assumed, after Pauling, that vanadium has 5 electrons available for bonding and the elements chromium to nickel all have 5·78; it is also assumed that vanadium, like the elements Cr to Ni, has 2·44 atomic orbitals accommodating 4·88 electrons.

Table 2. *Analysis of bond and atomic orbitals in transition elements*

element	*electrons in 3d and 4s bonding orbitals*	*electrons in 3d atomic orbitals*	*vacancies in atomic orbitals*	*excess of bonding electrons over vacancies in 3d atomic orbitals*
V	5	0·0	4·88	0·12
Cr	5·78	0·22	4·66	1·12
Mn	5·78	1·22	3·66	2·12
Fe	5·78	2·22	2·66	3·12
Co	5·78	3·22	1·66	4·12
Ni	5·78	4·22	0·66	5·12

Fig. 4 shows the variation with composition of the ratio of atoms to electrons, in excess of those required to fill 3d orbitals, for alloys of nickel, cobalt and iron with manganese, chromium and vanadium. A line is

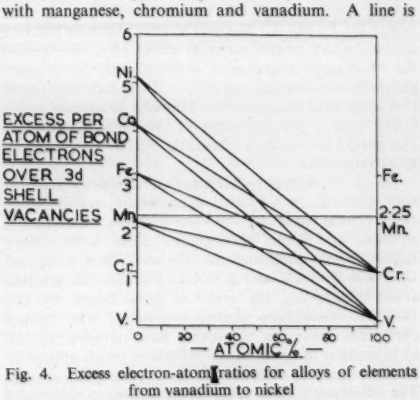

Fig. 4. Excess electron-atom ratios for alloys of elements from vanadium to nickel

drawn across the diagram at the proposed critical value of the ratio of 2·25 which is assumed to be the maximum, thus predicting the electron rich phase boundary. The diagram predicts the occurrence of σ phase in all these systems where it is known to occur and shows equally clearly that it will not occur in FeCo, FeNi where it is known not to exist. Also in the systems where σ does occur, Sully's views predict boundaries in fair agreement with those boundaries which have been established with any degree of precision: in particular the theory explains why the phase does not always occur at equiatomic composition.

In addition Sully explains the non-occurrence of the structure in elemental manganese and in the very high vanadium, vanadium-manganese alloys, by assuming that a critical amount of distortion resulting from the solution of elements of different atomic size is necessary to cause the change from the α-manganese to the sigma structure.

Dr. K. W. Andrews (Research and Development Department, United Steel Companies, Ltd.) described the occurrence of "chi phase," a constituent which had been found in certain types of chromium-nickel-molybdenum steels, and which appeared to be closely related to sigma phase. The experimental evidence suggested that it might be a metastable modification of the sigma phase but that when molybdenum was present the free energy difference became very small, so that either constituent might appear, or both constituents together, under similar conditions, thus suggesting an analogy with the behaviour of cobalt.

Furthermore, "χ phase" has a powder pattern closely similar to that of α-manganese. This may be significant in view of suggestions that there may be some relationship between the sigma phase and manganese. The binary alloy systems in which sigma phases are formed have one component on one side of manganese in the same row of the periodic table and one on the other side, except for Fe-Mo, Cr-Mn and V-Mn. In these last two systems sigma phases occur at about 75% Mn, but in the other cases the range of stability seems to be based generally on an equiatomic ratio. These facts may imply that some kind of superlattice tendency is a factor which contributes to the formation of the sigma phases, but this would not exclude electronic effects which may be more important.

Dr. J. W. Christian (Inorganic Chemistry Laboratory, Oxford), in a written contribution, reports some single crystal results for the V-Ni σ phases. Oscillation, rotation and Laue photographs taken with copper radiation have confirmed that the unit cell is tetragonal with $a = 8{\cdot}95$ kX and $c = 4{\cdot}63$ kX and the possible space groups are the same as those found by Dr. Douglas. Oscillation photographs taken with filtered chromium radiation, however, have resulted in the identification of some of the reflexions which appear to be characteristic absences in the copper photographs. The difference in atomic scattering powers of nickel and vanadium is 7·0 for chromium and only 0·55 for copper so that these results presumably indicate that the different atoms are in ordered positions in the unit cell. These positions may correspond to the layers in the structure described by Dr. Douglas, in which case the fourfold inversion axis of symmetry is lost and the true symmetry is orthorhombic, but the possibility of another kind of ordering leading to a lower tetragonal symmetry cannot be ruled out. V-Ni is especially suitable for the investigation of possible ordering since, with the exception of Fe-Mo, it is the only known σ phase which can yield a considerable difference in atomic scattering factors by suitable choice of radiation.

Finally, Dr. J. F. Brown (Imperial Chemical Industries Ltd., Billingham) described some work on the σ phase in 25/20 Cr-Ni steels using the electron microscope. Electron micrographs show quite clearly the increase in the amount of σ present on prolonged heat treatment within a temperature range of 850°, and the growth of the needle-like crystals with time can be followed.

σ phase and carbide $Cr_{23}C_6$ were isolated from the phase by electrolysis and the extract examined under the electron microscope. The σ crystals retained their characteristic shape. No crystals large enough for X-ray work were found, so the micro-crystals were selected in a three-stage microscope (5 000 × magnification) and electron diffraction patterns obtained. 18/8 steels were used here as they gave better shaped crystals and less carbide. Excellent single crystal spot patterns were obtained. These gave a tetragonal unit cell, with $a = 8{\cdot}8$ A., $c = 4{\cdot}5$ A., in good agreement with the X-ray data (which were not available at that time). Interesting data on the carbide $Cr_{23}C_6$ were also obtained. This appears as thin flakes which on heat treatment grow into a mass of inter-connected triangles. The diffraction patterns show that the (111) face is perpendicular to the beam. Electron diffraction results are limited by the uncertainties in operating voltage and specimen position, but it would seem that the identity, crystal habit, and even the crystal symmetry and unit cell can be found from a crystal of about 0·5 μ square and 500 A thick.

STRUCTURAL STUDIES (TRANSFORMATIONS)

The third session of the conference was concerned with the study of transformations. The first contribution was by Dr. H. Lipson on "The kinetics of the order-disorder transformation."[54]

At the X-ray analysis group conference in Birmingham in 1947 the author described preliminary work on single crystals of $AuCu_3$. This substance can be obtained in various states of order by quenching after annealing for different lengths of time, and partially ordered states are revealed by the appearance of diffuse superlattice spots. These spots, however, are in general elongated, their relationships being most conveniently expressed in reciprocal space (Fig. 5).

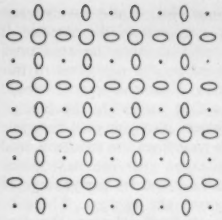

Fig. 5. Section of reciprocal lattice of $AuCu_3$

[Reproduced by courtesy of *Acta Crystallographica*

The suggestion made by Wilson[55] is that the alloy has the structure shown in Fig. 6. In this diagram are seen anti-phase domains at the boundaries of which the gold atoms tend to avoid each other. This structure could result if order started at particular points where short range or "nearest neighbour" order happened to be high; but then it is difficult to see how the sizes of the anti-phase domains would increase.

An alternative suggestion is that the increase in order is due entirely to the mutual avoidance of gold atoms. If a random arrangement of gold and copper atoms is considered, as in Fig. 7, there are some regions in which gold atoms predominate; these can be reduced by interchanging gold and copper atoms, as, for example, by changes indicated by the arrows. This is equivalent to increasing the short-range order coefficient as defined by Peierls,[56] but the long-range order coefficient as defined by Bragg and Williams[57, 58] remains zero. This process leads to a partially ordered structure, and if continued leads to anti-phase domains which increase in size (Fig. 8). A test of this theory has been made by means of gratings which show with visible light all the phenomena exhibited by the alloys with X-rays.[59, 60] This work is being continued by the examination of Cd-Mg alloys: it has been found that hexagonal $CdMg_3$ shows similar effects.

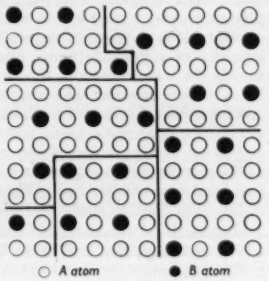

Fig. 6. Anti-phase domains for Wilson's theory

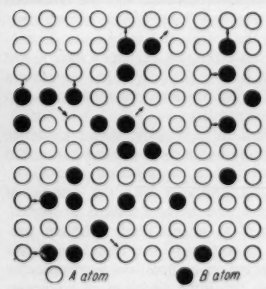

Fig. 7. Random arrangement in $AuCu_3$, with gold atoms tending to move as shown by arrows

[Reproduced by courtesy of *Butterworths Scientific Publications Ltd.*

Dr. J. B. Newkirk (Cavendish Laboratory, Cambridge) presented the results of an investigation of the order - disorder transformation in Co-Pt alloys.* The order and disorder phase fields seem to be separated by a definite two-phase region (Fig. 9) which would mean that the ordered and disordered states can be considered as separate phases obeying the Phase Rule.[61] For alloys near 50 atomic per cent composition, the tetragonal ordered phase appears first as nuclei which grow into platelets coherent with the (110) planes of the cubic disordered matrix[62] (Fig. 10). This condition gives rise to strains which account for the maxima found in hardness and cohesive force at intermediate stages of ageing.[63] These strains can be relieved on continued ageing, by thermal recovery and self-deformation at high reaction temperatures or by total recrystallization at low temperatures.

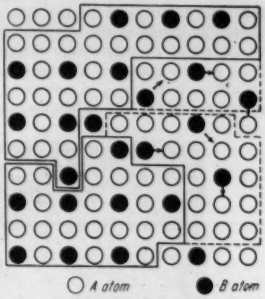

Fig. 8. Partially ordered structure resulting from movements in Fig. 7

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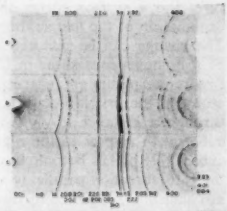

Fig. 9. Debye-Scherrer patterns illustrating three equilibrium conditions of Pt-Co containing 53·8 at. per cent Co

(*a*) Disordered f.c.c. phase—quenched from 1 000° C.

(*b*) Disordered and ordered phases coexisting in equilibrium—held 256 h at 800° C.W.Q.

(*c*) Ordered f.c.c. phase—held 91 h at 725° C.

"A theory of the transformation of pure cobalt" was presented by Dr. J. W. Christian (Inorganic Chemistry Laboratory, Oxford). The allotropic change of pure cobalt on cooling from high temperatures from a face-centred-cubic to a close-packed-hexagonal structure has been experimentally investigated by X-ray methods. Edwards and Lipson[64] and Troiano and Tokich[65] showed that the transformation takes place over a range of temperatures depending on the grain size, and that

* Co-operative project between Metals Research Laboratory, Carnegie Institute of Technology, Pittsburgh, and Research Laboratories, General Electric Company, Schenectady.

the relative amounts of the two phases can only be altered by changing the temperature. These effects are characteristic of martensitic reactions, but in addition there are the results of Edwards, Lipson and Wilson[66] on diffuse broadening of reflexions which can be interpreted as due to faults in stacking of close packed layers.

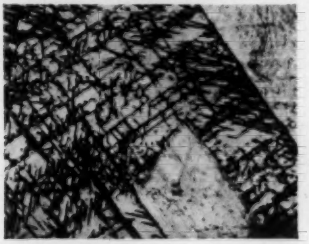

Fig. 10. Microstructure of Co-Pt alloy near 50 atomic per cent composition, showing pattern of ordered phase in disordered matrix

Quantitative measurements show that the hexagonal faults cannot be in thermal equilibrium with the structure, and a surface energy theory is inadequate as an explanation of the temperature range of transformation. The change can be pictured as taking place by means of a homogeneous shear in the $(11\bar{2})$ direction if every two atomic planes are regarded as locked together. The atoms in the first, third, fifth, etc., planes are displaced through a distance $a/\sqrt{6}$ (where a is the side of the cube), with respect to the plane beneath them, and the cubic (111) planes become hexagonal (0001) planes with the $[1\bar{1}0]$ direction parallel to the $[11\bar{2}0]$ direction. This mechanism is supported by the observed crystallographic relations and lattice parameters.

A shear of this kind must take place by a dislocation mechanism. Heidenreich and Shockley[67] have pointed out that lattice dislocations in f.c.c. materials probably split into half-dislocations so that the atomic displacements in two half-dislocations add vectorially to give a lattice vector. The region between the half-dislocations is hexagonal for about two atomic planes and the higher energy of this region provides an attractive force to balance the repulsive force. There is thus an equilibrium separation of the half-dislocations, and this increases as the temperature is lowered and the free-energy difference becomes smaller. Below the ideal transformation temperature, the attractive force disappears and is replaced by a repulsive force. The half-dislocations then separate rapidly and may accelerate to high velocities.

Frank[68] has shown that a fast moving dislocation in a simple cubic lattice will be reflected at a free surface or a grain boundary as a dislocation of opposite sign on the same atomic plane. Geometrical considerations show that this process is impossible in the case of an isolated half-dislocation since this would involve impossibly close distances of approach of the atoms. It is, however, possible for reflexion into the next or a succeeding atomic plane to occur, and repetition of this process down a whole series of atomic planes would lead to the production of a f.c.c. twin. In the case of cobalt repetition of a reflexion into the next but one plane would be necessary to transform the structure, and this is preferred, since reflexion into the next plane destroys the hexagonal layer, formed by the passage of the dislocation, and thus raises the energy. There is, however, a finite probability that reflexion into the next plane will occur and it is suggested that these "wrong" reflexions are responsible for the occurrence of hexagonal faults.

In a paper on "An X-ray method of preparing isothermal transformation diagrams" Mr. H. T. Heal[69] (British Iron and Steel Research Association) described a technique which can be used for following phase changes in steel where other methods cannot be applied. The principle of the method is to measure the intensity of diffracted beams from one set of planes during the transformation, the changes being followed by a Geiger counter and counting-rate meter. The specimen is in the form of a ribbon—wide, so that there are many crystals in the beam, and thin, to allow cooling. A heating current is passed through the specimen and as the current is changed the counter records the diffraction from a pre-selected line. The statistical fluctuations of the meter limit the method if rapid changes are studied.

Dr. K. H. Jack (King's College, Newcastle-upon-Tyne) gave a summary of work on "Structural transformations during the tempering of carbon and nitrogen martensites." Quenched high-carbon steel consisting of tetragonal martensite with retained austenite undergoes three successive structural changes during low-temperature tempering, each stage being accompanied by characteristic changes in physical properties. The second stage is undoubtedly the decomposition of the retained austenite. Previous workers[70] have also shown that during the first stage the tetragonal martensite decomposes into ferrite and an unknown carbide which is not cementite, but which is too finely divided to give a diffraction pattern. During the third stage this unknown carbide transforms to cementite.

New X-ray observations[71, 72] give direct evidence that the transition carbide has a close-packed hexagonal iron-atom arrangement with a composition near Fe_3C. Because of its structural similarity with the ϵ-phase of the iron-nitrogen system, the name ϵ-iron is proposed. ϵ-iron carbide is formed as an oriented, coherent precipitate with marked relaxation of the Laue conditions for X-ray reflexion except in a direction perpendicular to the (101) planes, this being the direction of maximum growth or of minimum lattice strain.

During the third stage of steel tempering, ϵ-iron carbide transforms to give a fine dispersion of very thin platelets of cementite with the planes of the platelets

parallel with the (001) lattice planes of the cementite structure. The platelets are only one or two unit cells in thickness but their mean diameter is of the order of fifty unit cells. Thus, the observed X-ray diffraction pattern is essentially a two-dimensional one and is markedly different from that of high-temperature crystalline cementite. Included among the strongest reflexions are those corresponding with previously unidentified reflexions observed by earlier investigators[73] and ascribed by them to a new unknown iron carbide. With increasing tempering temperatures the gradual growth and recrystallization of the cementite platelets are accompanied by a reduction in the lattice strain of the matrix. Each of the two stages of the transition of martensite to cementite requires small movements of iron atoms. The proposed structural changes not only account for the changes in hardness but are also in excellent agreement with the observed dimensional changes[74] in steel tempering. The behaviour of nitrogen martensites[71, 75] on tempering is different from that of the corresponding carbon alloys. Below 250° C a new phase, designated as α'', is formed as an intermediate in the decomposition.

$$\text{nitrogen martensite } (\alpha') \rightarrow Fe_4N\ (\gamma')$$

α'' has a structural unit made up of eight ($2 \times 2 \times 2$) distorted body-centred tetragonal units of the original martensite structure. The contents of the α'' unit cell (space group: I 4/mmm) are ideally $Fe_{16}N_2$ but up to one half of the number of nitrogen-atom sites may be vacant. The structure is essentially a martensite in which the interstitial atoms are completely ordered. It can also be interpreted as the Fe_4N structure in which removal of alternate nitrogen atoms has caused considerable distortion. This α''-nitride is also identified as the previously unknown iron nitride which has been observed[76, 77] to be precipitated by heat-treatment of nitrogen-ferrite.

Although the tempering transformations for carbon and for nitrogen martensites are dissimilar in detail, they are basically related and are analogous with those occurring in other age-hardening systems. In each case a coherent transition phase is produced. The transition structure is intermediate between the structures of the initial and final phases, and its formation and decomposition occur by relatively small atomic movements. Thus, nucleation and therefore precipitation from solid solution can occur more readily than by a single-stage process.

In a paper entitled "An analysis of X-ray diffraction line shapes from martensite" DR. R. A. SMITH (Mond Nickel Co. Ltd.) presented some results on an analysis of martensite line-shapes measured with a Geiger-counter spectrometer at the University of Birmingham. Averbach and Warren[78] have shown how Fourier analysis of diffraction line-profiles can give values for particle-size and lattice strain. The method has been applied to martensite lines, using iron-nickel alloy specimens. The results confirm earlier work[79] in that the line-broadening is mainly due to lattice distortion. A particle-size effect is also detected. The distortion decreases and the particle-size increases on tempering, as found by Krainer.[80] The distortion is greater and the particle-size smaller, the greater the carbon content. The general conclusion drawn from the various results obtained is that the internal structures of martensitic and cold-worked metals and alloys are very similar, differing only in degree.

MR. D. V. WILSON (University of Birmingham), in a contribution "Some effects of cold work on cementite in steel," described an investigation by X-ray and magnetic methods on the condition of carbide particles in cold-worked high-carbon steels. When a carbon steel is plastically deformed, the cementite Curie temperature changes;[81] the possibility that this is due to a change in carbide structure has been examined. Photographs of unetched[82] specimens taken in a glancing angle X-ray camera show no evidence of a new carbide phase. Two types of effect due to cold work are observed. The first corresponds to the development of preferred orientations in the cementite in which a direction near [001] tends to be alined with the compression axis. The second is a systematic movement of the carbide lines, which indicates high compression strains in the cementite particles. Directionally, these are opposite to those observed in the ferrite of cold-worked steels.[83] The average value of the directional component is high; values in excess of 1% mean strain value are observed. This directional component may be eliminated by applying stress in reverse, which also results in considerable recovery towards the normal Curie temperature. If a quantitative relationship between Curie point displacement and strain in the cementite can be established, magnetic analysis, which is not confined to surface layers, may provide a method complementary to X-ray analysis for the investigation of fine-scale stress systems in steel.

In the discussion which followed the last paper, MR. J. CRANGLE (University of Sheffield) gave a short account of some recent magnetic work on the tempering of carbon martensite. The primary precipitate from high-carbon martensite is an ϵ-phase iron carbide with a composition between Fe_3C and Fe_2C. This phase breaks down at above 250° C to form iron percarbide ($Fe_{20}C_9$), the Curie point of which is rather higher than that of cementite. At temperatures of up to 550° C the iron percarbide reacts with free iron to produce cementite. It was tentatively suggested that a small amount of cementite which is formed at somewhat above 200° C might be the same as the cementite platelets reported by Jack; it is probably produced by direct precipitation from the low-carbon martensite remaining after the first tempering stage.

In a short contribution on the mechanism of the austenite-martensite transformation, DR. T. LL. RICHARDS (Imperial Chemical Industries Ltd.) pointed out that

Jaswon and Wheeler,[84] using one of the twenty-four possible variants of the Kurdjumow-Sachs orientation relationship, namely,

$$\{111\}\ \text{FCC} \parallel \{110\}\ \text{BCC};\ \langle 110\rangle\ \text{FCC} \parallel \langle 111\rangle\ \text{BCC}$$

had determined the atomic co-ordinates before and after transformation. These the speaker noted were precisely those required by the mechanism initially proposed by E. C. Bain,[85] that is, a compression along a cube axis of the face-centred cubic structure. Jaswon and Wheeler also established that three face-centred cubic planes remained parallel in the transformation, namely the (111), a plane within 1° of (111) and the (225), but no reason was advanced to explain why the latter is preferred as the habit plane for steels with less than 1·4% carbon.

In conjunction with MR. D. E. YEOMANS the speaker had calculated on the same basis that the face-centred cubic (225) habit plane was exactly parallel to the body-centred cubic ($\bar{2}11$), a twin plane of body-centred cubic lattices, while the face-centred cubic (259) habit plane, observed for steels with more than 1·4% carbon was almost parallel to body-centred cubic ($1\bar{1}1$), a secondary twin plane. Furthermore, the transformation could be regarded as the simple Bain mechanism coupled with a rotation of about 12° on a plane close to a face-centred cubic (225). Relief of mutual strains between martensite plates and matrix would result from twinning of the plate and the midrib frequently observed is therefore in all probability a (112) or (111) twin plane.

MR. P. R. POMEROY (British Non-Ferrous Metals Research Association) described the copper corner of the Cu-Ni-Fe phase diagram. This Research Association has, over the past few years, developed two Cu-Ni-Fe alloys which have remarkably high resistance to corrosion by impingement of sea-water containing entrapped air bubbles.[95] One of them, containing 5% nickel and 1·3% iron, is a soft material which can be coppersmithed as readily as copper. The other, containing 10% nickel and 2% iron, is harder and is best used in straight lengths, as in ships' condensers.

The alloys are normally produced in a condition which, according to the X-ray work of Bradley, Cox and Goldschmidt,[86] should be single phase. However, the colour of the protective films formed during exposure to aerated sea water is sometimes light and sometimes dark; when it is dark, the corrosion resistance is not quite so good. It was found that impurities could not be responsible either for this difference or for the fact that the alloys can be produced in a condition in which they are brittle on hot-working, and eventually it was concluded that when material quenched from temperatures round 900° C is reheated for a short period at about 600° C, it begins to undergo a transformation of the form $\alpha \rightarrow \alpha + \alpha'$. This conclusion is in agreement with the phase diagram of Köster and Dannöhl[87] which was determined by magnetic measurements. A magnetic change accompanies the transformation. Microscopically nothing is visible until a fairly late stage of ageing, and using X-rays, nothing is obvious even then. Using the electron microscope, however, structures are found which suggest that the transformation follows a similar course to that studied by Daniel and Lipson[88, 89] and later by Hargreaves[90] in copper-nickel-iron alloys of higher nickel and iron content. Consideration of tie-lines on Bradley's diagram indicates that the parameters of the two stable phases in these compositions may be very nearly equal, and hard to distinguish by X-ray techniques. If this proves to be the case, we have an example where the electron microscope has given information which neither the light microscope nor X-ray methods are able to do.

DR. B. A. BILBY (University of Birmingham) mentioned that in reference to Dr. Christian's hypothesis, there is now a controversy[91, 92] about the speed at which dislocations will move, and it is not yet established that they will achieve kinetic energies sufficient for the proposed reflexion mechanism to occur. This is of particular importance here, since the reflected dislocation must be formed not even on the adjacent plane, but on the next plane but one. It has been shown, however, that slip bands[93] and the homogeneous deformations involved in twinning and martensitic transformations[94] can occur by the motion of dislocations without reflexion, the latter processes by a so-called "pole mechanism."

A reflexion process (or "pole mechanism") will cause, in the region of the crystal which is twinned or transformed by the dislocations from any one stacking fault nucleus, not only a shear transformation of the unit cell to give the new crystal structure, but also a macroscopic homogeneous shear. The presence or absence of a macroscopic shear in cobalt apparently has not been established beyond doubt, and experiments to this end would be useful. Confirmation of the existence of such a shear would be a valuable pointer to the transformation mechanism; its absence would not, however, preclude a reflexion or pole mechanism, since if the regions originating from any one nucleus are sufficiently small, a random averaging over the three types of nuclei possible in any (111) plane would produce a mean macroscopic shear of zero.

More quantitative results are desirable on the variation of faulting with grain size in powders and fine-grained material. A possible cause of faulting not hitherto considered arises because any (111) plane in which partial dislocations are moving will be intersected by other dislocations whose Burgers vectors have components perpendicular to (111). In fact all $a/2$ $\langle 110\rangle$ vectors not lying in (111) have a component perpendicular to (111) equal to the (111) spacing. By "wrapping round" or "cutting through" these intersecting dislocations, regions of the nuclei can spread to neighbouring planes and cause cubic faults.

DR. A. TAYLOR (The Mond Nickel Company Ltd.) stated that the results of his own work on carbonyl iron powders were in close accord with those of Dr. Smith

on martensite, and emphasized the necessity of carrying out line-broadening measurements over a wide range of Bragg angles. Such a series of measurements enabled both crystal size and lattice stress to be computed. It was pointed out that solid specimens gave too few convenient spectra to enable a complete analysis of the line broadening to be made, and that filings, though ideal on account of the wide range of diffraction spectra which they produced, were, however, not completely satisfactory, since the degree of cold work to which they were subject and the rise in temperature during preparation were both uncertain quantities. Dr. Taylor then described a controlled method of cold working compacts of annealed powders, using powdered silicon as reference material for the line-broadening determinations.

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ORIGINAL CONTRIBUTIONS

A method of measuring the interface resistance and capacitance of oxide cathodes

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A method of measuring the interface resistance and capacitance of oxide cathodes is described. The interface impedance causes a feed-back that is frequency-dependent. Another frequency-dependent network is added in order to make the gain independent of frequency. When this is achieved the interface components are equal to the measuring components. The apparatus is briefly described.

It has become apparent that a most important effect in some valves with oxide-coated cathodes is the gradual growth, during life, of an interface impedance between the cathode core and the coating material. This interface has been commented on in several published papers,[1–5] and its formation is now ascribed to chemical action between the coating material and impurities—notably silicon—in the core.

The following facts are known about this impedance. It consists of a resistance and capacitance in parallel whose values are independent of the cathode current. In a new valve the resistance is zero and it grows to a value of the order of 100 Ω. In an old valve the capacitance is of the order of 0·005 μF, and in a fairly new valve it is larger but difficult to measure owing to the much lower resistance across it. The interface resistance and capacitance in a well-aged valve depend on the cathode area: the resistance is smaller, and capacitance larger, for a larger cathode than they are for a smaller one. The figures quoted above are intended only to convey the order of magnitude of the interface components in small receiving valves with cathodes requiring a heater power of, say, about 2 W.

Having established these facts it is clear that the

measuring apparatus might have taken several forms; the form described in this paper has the merit of simplicity and is direct-reading.

The interface impedance will cause a feed-back that is frequency-dependent. The method used is to add another frequency-dependent network, and to make the gain in the circuit independent of frequency. Since the impedance is associated only with the cathode of the valve it is unnecessary to use more than the control grid and anode. Therefore all valves are strapped as triodes, however many electrodes they may contain. Also, the apparatus may be adapted for the measurement of diodes. The diode is biased positively by connecting a suitable high resistance between its anode and h.t. positive. Not all valves are really suitable for this measurement, since the interface impedance does not affect the operation of low mutual conductance valves as much as valves with high slopes. With low-slope valves the impedance exists but is difficult to measure. Valves which are most suitable for this measurement have a mutual conductance in excess of 5 mA/V.

In the next section the theory of the measurement is given, and in the following section a more detailed description of the apparatus. The routine measurement of interface impedance is very simple with this apparatus.

THEORY OF THE METHOD

The effect of the components of interface impedance is to make the gain of the test valve frequency-dependent. The method of measurement is, however, to arrange the test circuit with its measuring components in such a way that the gain is made independent of frequency. Having achieved this, it happens that the interface components are equal to the measuring components. In practice it is sufficient to adjust the circuit at three frequencies.

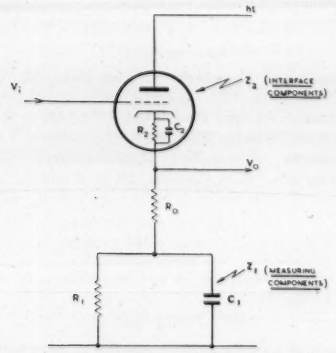

Fig. 1. Schematic form of measuring circuit

The circuit is shown schematically in Fig. 1. The interface components are R_2 and C_2 (Z_2) and the measuring components are R_1 and C_1 (Z_1). R_0 is an additional adjustable circuit element.

The gain is

$$\frac{V_0}{V_1} = \frac{g(R_0 + Z_1)}{1 + g(R_0 + Z_1 + Z_2)}$$

where g is the mutual conductance of the valve. This is an approximation derived on the assumption that the amplification factor μ is so large that $(\mu + 1)$ can be replaced by μ; and also that the effect of the valve capacitances can be neglected (in particular the grid-cathode capacitance in comparison with the mutual condutance). In practice, for valves with an amplification factor of over 50 and a mutual conductance of about 5 mA/V, the errors due to the approximation are negligible. If R_0 is adjusted so that—

$$gR_0 = 1$$

then the gain is—

$$\frac{1 + gZ_1}{2 + g(Z_1 + Z_2)}$$

If the gain is now adjusted to be equal to $\frac{1}{2}$, then—

$$Z_1 = Z_2$$

The most convenient way of making the adjustments is to choose three frequencies, a high frequency, a low and an intermediate, in that order, and to make the gain equal to $\frac{1}{2}$ at each frequency in turn.

At the high frequency Z_1 and Z_2 can be regarded as being zero, so that the gain is—

$$\frac{gR_0}{1 + gR_0}$$

and the adjustment of R_0 to make this equal $\frac{1}{2}$ makes $gR_0 = 1$.

At the low frequency, Z_1 can be regarded as being composed of R_1 alone and Z_2 of R_2 alone, the gain then is—

$$\frac{g(R_0 + R_1)}{1 + g(R_1 + R_2 + R_0)}$$

$$= \frac{1 + gR_1}{2 + g(R_1 + R_2)}, \qquad \text{since } gR_0 = 1.$$

Thus, if the gain is $\frac{1}{2}$, then—

$$R_1 = R_2$$

Finally at the intermediate frequency the adjustment of C_1 to make the gain equal to $\frac{1}{2}$ makes $Z_1 = Z_2$, from whence it follows that—

$$C_1 = C_2$$

Both C_1 and R_1 can be direct reading components, and

so a simple method of measuring the interface components is established.

It should also be remarked that the input and output voltage have the same phase, and this is of great convenience as it enables an ordinary peak valve-voltmeter to be used, even if the input waveform is not exactly sinusoidal.

DETAILS OF THE APPARATUS

A unit type of construction has been adopted; there are 5 units, viz.:—

1. A 3-frequency oscillator.
2. The main test panel.
3. An amplifying peak-reading valve-voltmeter.
4. A stabilized h.t. power-pack.
5. Heater supply for the test valve.

The oscillator can be switched to deliver one of three frequencies, 50 kc/s, 500 kc/s, or 5 Mc/s. The output voltage, across a low impedance cable, can be independently adjusted for each frequency.

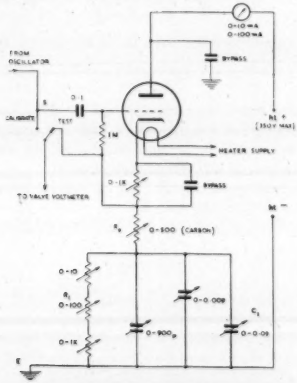

Fig. 2. The measuring circuit

The voltmeter has a uniform sensitivity over the above frequency range, and a full-scale deflexion of 0·1 V. The scale of the instrument has been calibrated 0–10. It measures the positive peak of the input wave-form.

Both the h.t. power-pack and the heater supply are of conventional design.

It is of interest to describe the main test panel in greater detail: the circuit diagram is shown in Fig. 2. The switch S connects the oscillator either to the valve-voltmeter or to the test valve; in the first position the voltmeter measures the input voltage, and in the second position the voltmeter measures the output from the test valve. The cathode current of the test valve is adjusted to a convenient level by varying the h.t. voltage and/or the bias resistor, which is by-passed by a very large capacitor. R_1 is a 3-decade resistor (1 110 Ω max.) and C_1 is a 3-decade capacitor (0·0999 μF max.). These components correspond to those in Fig. 1.

Briefly, the procedure of measurement is as follows:—

1. Set up the test valve and adjust the cathode current to a convenient value. Ensure that R_1 and C_1 are both zero.
2. Place switch S in the "calibrate" position and adjust the oscillator controls so that the voltmeter reads 0·1 V (scale reading 10) for each frequency in turn.
3. Place S in the "Test" position, and with the 5 Mc/s input frequency, adjust R_0 so that the voltmeter reads 0·05 V (scale reading 5).
4. Change the frequency to 50 kc/s and adjust R_1 so that the voltmeter again reads 0·05 V.
5. Change the frequency to 500 kc/s and adjust C_1 so that the voltmeter reads 0·05 V.

The readings thus obtained of R_1 and C_1 are equivalent to the interface resistance and capacitance respectively.

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Fluid flow in ducts with a uniformly distributed leakage

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An analysis of the problem is given for any power law relationship between flow rate and pressure gradient and between leakage rate and excess pressure in a duct. The results relate the pressure drop, inlet flow rate and outlet flow rate for a leaking duct to the pressure drop and flow rate in a similar leakless duct. It is shown that these quantities can be conveniently obtained using a nomogram involving a resistance constant and a leakage constant for the duct. Assumptions involved are discussed and a description is given of a model experiment which confirms the results of the theoretical treatment.

This paper considers the problem of the flow of fluid through a duct with a uniformly distributed leakage, for any power law relationship between flow rate or leakage rate and pressure. The problem arose specifically in connexion with auxiliary ventilation systems in coal mines but it may have other applications. Ventilation ducts in coal mines are of 1½ to 3 ft diameter and they cover distances up to 10 000 ft. Complete elimination of leakage at the seams and joints is impossible in practice and the estimation of fan requirements has in the past involved rough allowances for leakage.

An analysis of the problem is given below for the case of a duct with a leakage which is uniformly distributed along its length. The flow and leakage rates at any point are related to the pressure by two simple differential equations. Their solution gives the fan pressure and delivery rate for any required output from a duct in terms of a resistance constant and a leakage constant for the duct. A general solution is given to cover different power law variations between flow rate and pressure gradient. Presentation of the results is simplified by introducing dimensionless quantities involving the ratios of fan pressure and delivery volume in a leaking duct to those in a leakless duct with the same output. For the particular case of square law relation between pressure and flow rate the solution is given in nomogram form. The assumptions involved are discussed and a description is given of a model experiment which confirms the results of the theoretical treatment.

EQUATIONS OF FLOW AND LEAKAGE

The relation between pressure gradient and flow rate in a leakless duct of uniform cross-section may be expressed in the form

$$\frac{dP}{dx} = rQ^{\lambda} \qquad (1)$$

where P = static pressure in duct.
x = distance along duct.
r = resistance constant.
Q = volume rate of fluid flow.
λ = constant depending on Reynolds number and roughness of duct.

Expressed in the above differential form equation (1) will also be true for a leaking duct, although Q will in this case vary along the duct in a manner dependent on the nature of the leakage. For a uniformly distributed leakage the variation of Q is given by

$$\frac{dQ}{dx} = kP^{\mu} \qquad (2)$$

where k = leakage constant.
μ = constant depending on the nature of the leakage holes.

In equations (1) and (2) P is taken to be the pressure difference between inside and outside the duct, the outside pressure being assumed constant. Before solving these equations, a number of points relating to their form and validity will be considered.

(1) The equations cover the cases of forcing or exhausting systems provided that P and Q are regarded as positive in each case and x is measured in the direction of increasing P.

(2) A constant fluid density is assumed. This implies isothermal conditions and also that the variations in absolute pressure are small. In ventilation systems the fan pressure rarely exceeds 24 in w.g. so that variation of absolute pressure along a duct is less than ± 3%.

(3) The index, λ, in equation (1) for turbulent flow in smooth ducts varies from 1·70 to 2·0 according to the following function* of Reynolds number, R_e.

$$\lambda = 2 - \frac{29{\cdot}7}{85 + R_e^{0{\cdot}35}}$$

Also λ increases with the roughness of the duct. In most formulae used by ventilation engineers λ is assumed to have the value 2·0.

(4) The leakage index, μ, in equation (2) may have any value between 1·0 and 0·5 depending on whether the leakage involves viscous flow through capillary channels or non-viscous flow.

(5) It is assumed that the leakage is not influenced by velocity pressure. In the ventilation systems considered here the velocity pressure will be only about 5% of the static pressure.

(6) Pressure losses at bends and end losses are neglected.

* OWER, E. *The Measurement of Air Flow*, 3rd Ed., p. 29 (London: Chapman and Hall, Ltd., 1949).

GENERAL FLOW THEORY FOR LEAKING DUCTS

Eliminating Q from equations (1) and (2) gives the following differential equation relating pressure and distance along a leaking duct:—

$$\left(\frac{dP}{dx}\right)^{1/\lambda}\frac{d^2P}{dx^2} = r^{1/\lambda}k\lambda\frac{dP}{dx}P^{\mu} \qquad (3)$$

Integrating and taking boundary conditions $P = 0$, $Q = Q_0$, $x = 0$ gives

$$\frac{\lambda}{\lambda+1}\left(\frac{dP}{dx}\right)^{(\lambda+1)/\lambda} = \frac{r^{1/\lambda}k\lambda P^{\mu+1}}{\mu+1} + \frac{\lambda}{\lambda+1}r^{(\lambda+1)/\lambda}Q_0^{\lambda+1} \qquad (4)$$

It is convenient to generalize this equation by introducing dimensionless parameters y and t given by

$$P = \left[\frac{\mu+1}{k(\lambda+1)}\right]^{1/(\mu+1)} r^{1/(\mu+1)}Q_0^{(\lambda+1)/(\mu+1)}y \qquad (5)$$

and

$$x = \left[\frac{\mu+1}{k(\lambda+1)}\right]^{1/(\mu+1)} r^{-\mu/(\mu+1)}Q_0^{(1-\lambda\mu)/(\mu+1)}t \qquad (6)$$

Equation (4) may now be reduced to

$$\left(\frac{dy}{dt}\right)^{(\lambda+1)/\lambda} = y^{(\mu+1)} + 1 \qquad (7)$$

so that

$$t = \int_0^y \frac{dy}{(1+y^{(\mu+1)})^{\lambda/(\lambda+1)}} \qquad (8)$$

This integral is in general only soluble by numerical methods, although it is of interest to consider the values of λ and μ for which it is integrable.

Thus if we write

$$z = \frac{y^{(\mu+1)}}{1+y^{(\mu+1)}}$$

$$(1+\mu)t = \int_0^z z^{[1/(\mu+1)]-1}(1-z)^{[(\lambda\mu-1)/(\lambda+1)(\mu+1)]-1}dz \qquad (9)$$

The new variable z has a range from 0 to 1 as y varies from 0 to ∞.

The integral (9) is the incomplete beta function

$$B_z\left[\frac{1}{\mu+1}, \frac{\lambda\mu-1}{(\mu+1)(\lambda+1)}\right]$$

and as such is a non-tractable integral except in the case where the integrand reduces to a rational function of the form

$$R[\phi, \sqrt[n]{(a\phi+b)}]$$

where n is an integer.

It may be shown that the expression (9) reduces to this form, if, and only if $\lambda\mu = 1$ and $\mu = r/s$ where r and s are integers.

By writing $\theta = z^{1/(r+s)}$ in the particular case of $\lambda\mu = 1$ and $\mu = r/s$ equation (9) reduces to

$$t = \int_0^{\theta} \frac{s\theta^{(s-1)}}{1-\theta^{(r+s)}}d\theta \qquad (10)$$

and is therefore integrable. In practice this is most easily accomplished by numerical methods.

The solution of equation (8) for the relevant values of λ and μ thus enables the pressure, P, at any point x along a duct to be obtained from equations (5) and (6). There is however a more convenient method of expressing the results of the analysis, by introducing the ratios of pressure and volume rate of flow in a leaking duct to those in a leakless duct with the same delivery rate.

Thus from equations (5) and (6) $P/x = rQ_0^{\lambda}y/t$

For a leakless duct $P_0/x = rQ_0^{\lambda}$,

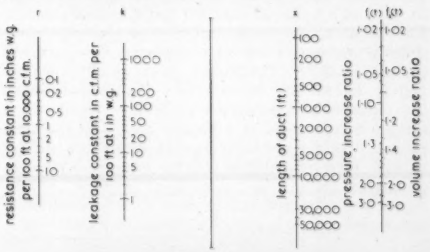
So that the pressure increase ratio $P/P_0 = y/t$ (11)

Similarly it can be shown that volume increase ratio

$$Q/Q_0 = [1+y^{(\mu+1)}]^{1/(\lambda+1)} \qquad (12)$$

The power increase ratio is of course the product of the above two ratios.

Evaluation of the ratios can be quickly carried out by using a nomogram. Thus for values of λ and μ relevant to the types of flow and leakage in the problem considered, a nomogram is constructed according to equation (6). This gives t in terms of x, k, r and Q_0 and the t scale on the nomogram can be scaled in the functions of t represented by the pressure and volume increase ratios. A nomogram for one pair of values λ, μ is described below.



Nomogram for air flow in leaking ducts.
Basic relationship $t = (2kr^{\lambda})^{\frac{1}{2}}x$

It will be noted in equation (6) that t is independent of Q_0 if $\lambda\mu = 1$. In these cases the pressure and volume increase ratios are independent of the volume rate of delivery from the duct and simple relations exist for changing the flow in the duct. Thus if the output is to be doubled the input must be doubled and the fan pressure must be increased by 2^{λ}.

A particular case of $\lambda\mu = 1$ is when $\lambda = 2{\cdot}0$ and $\mu = 0{\cdot}5$. This is the case of square law relationship between flow rate and pressure gradient and between leakage rate and pressure. For applications in ventilation engineering these laws will be approximately true

and it is of interest to consider the effect of small changes in the values of λ and μ on the results of the analysis. This is shown in Table 1.

Table 1. *Effect of small changes in λ and μ*

P.I.R. = Pressure increase ratio. V.I.R. = Volume increase ratio.

	$\mu = 0{\cdot}5$ $\lambda = 1{\cdot}5$		$\mu = 0{\cdot}5$ $\lambda = 2{\cdot}0$		$\mu = 1{\cdot}0$ $\lambda = 1{\cdot}5$		$\mu = 1{\cdot}0$ $\lambda = 2{\cdot}0$	
y	P.I.R.	V.I.R.	P.I.R.	V.I.R.	P.I.R.	V.I.R.	P.I.R.	V.I.R.
0·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00
0·25	1·03	1·05	1·03	1·04	1·01	1·03	1·01	1·02
0·50	1·07	1·13	1·09	1·10	1·04	1·09	1·05	1·08
0·75	1·13	1·22	1·14	1·18	1·10	1·20	1·11	1·16
1·00	1·19	1·32	1·22	1·26	1·16	1·32	1·19	1·26
1·50	1·32	1·52	1·36	1·41	1·30	1·60	1·35	1·48
2·00	1·45	1·71	1·51	1·56	1·46	1·90	1·53	1·72
2·50	1·57	1·90	1·65	1·69	1·62	2·21	1·71	1·94
3·00	1·70	2·08	1·79	1·84	1·79	2·51	1·90	2·16
3·50	1·82	2·24	1·93	1·98	1·95	2·81	2·08	2·38
4·00	1·94	2·41	2·07	2·08	2·11	3·11	2·27	2·58

For turbulent flow λ will be between 1·70 and 2·0 and for non-viscous leakage μ will be close to 0·5 so that in these cases the errors involved in using $\lambda = 2{\cdot}0$ and $\mu = 0{\cdot}5$ will be small.

For the flow conditions in ducts in coal mines λ is about 1·9 and μ varies from 0·5 to 0·7. Square law relations have been assumed for this application so that resistance and leakage "constants" are obtained which vary slightly with flow rate. In practice mean values of the constants are used which are appropriate to the range of flow rates involved.

When $\lambda = 2{\cdot}0$ and $\mu = 0{\cdot}5$ the equations given earlier reduce to

$$y = \left(\frac{2k}{r}\right)^{2/3} \frac{P}{Q_0^2} \tag{13}$$

$$t = (2kr^{1/2})^{2/3}x \tag{14}$$

$$t = \int_0^y \frac{dy}{(1 + y^{3/2})^{2/3}} \tag{15}$$

$$\text{Pressure increase ratio} = y/t \tag{16}$$

$$\text{Volume increase ratio} = (1 + y^{3/2})^{1/3} \tag{17}$$

The nomogram given in outline in the figure has been used for the ducts in coal mines. It determines the pressure and volume increase ratios in terms of r, k and x. A line joining the appropriate r and k values gives a point on the unscaled line which is a measure of $2kr^{\frac{1}{2}}$ and is the characteristic of the duct. A second line from this point through the appropriate x value indicates the pressure and volume increase ratios on the end scale. The end scale is derived by first scaling it in terms of t and then inserting the functions of t derived from equations (15), (16) and (17). Having obtained these ratios the actual fan delivery flow rate and pressure in a leaking duct is obtained by applying the ratios to the required output flow rate and to the pressure which would be needed for this flow rate in a similar leakless duct. For many practical purposes the pressure and volume increase ratios can be taken to be equal in magnitude.

Values of r for air flow in different diameter ducts are fairly well established. Values of k have been determined for ducts in coal mines in terms of the type of jointing between sections and the type of duct seam, i.e. welded or riveted.

DESIGN AND RESULTS OF A MODEL EXPERIMENT

A small-scale experiment was carried out to verify the results of the theoretical treatment. The details can be best described in relation to the design of the model. It was desirable that the maximum static and velocity pressures in the model duct should cover the same range as those in a full-scale duct so that a constant air density could be assumed and so that the leakage in the model and full-scale ducts would occur under similar flow conditions. This limited the static pressure to about 24 in w.g. and the air velocity to about 50 ft/sec. In order that a useful range of air velocities could be used it was desirable that the lower limit should extend to 5 ft/sec. The diameter of the duct was thus chosen so that the flow was fully turbulent down to 5 ft/sec. This requires a Reynolds number of at least 2 000 at 5 ft/sec and therefore a diameter of at least $\frac{3}{4}$ in. Finally the length of the duct was chosen so that the pressure drop would be about 24 in w.g. at 50 ft/sec. A convenient duct was made from 150 ft of $\frac{3}{4}$ in diameter bore electric conduit. A uniformly distributed leakage was obtained by drilling $\frac{1}{16}$ in holes at 3 ft intervals. This gave a leakage rate of the same order as the delivery flow rate.

The tubing was laid out in a double circuit and rotameters at each end of the tubing measured the flow rate

Table 2. *Comparison of theoretical and observed results for a model duct. Duct length 125 ft, diameter $\frac{3}{4}$ in. Forcing system*

Leakage constant (c.f.m. per 100 ft at 1 in w.g.)	*Resistance constant (in w.g. per 100 ft at 1 c.f.m.)*	*Outlet flow rate (c.f.m.)*	*Inlet flow rate (c.f.m.)*		*Inlet pressure (in w.g.)*	
			observed	*theoretical*	*observed*	*theoretical*
0·85	0·230	2·30	3·0	3·20	1·90	2·00
0·85	0·185	5·00	7·0	6·80	7·70	7·65
0·85	0·165	7·45	10·0	10·00	15·50	14·90
1·25	0·235	1·95	3·0	3·15	1·70	1·75
1·25	0·185	4·55	7·0	7·05	7·25	7·25
1·25	0·165	7·15	11·0	10·80	16·10	15·30
1·70	0·245	1·15	2·0	2·10	0·60	0·75
1·70	0·190	3·85	7·0	6·90	6·20	6·20
1·70	0·160	6·85	12·0	11·60	16·25	15·70
2·10	0·240	1·45	3·0	3·10	1·30	1·30
2·10	0·190	3·45	7·0	7·00	5·50	5·75
2·10	0·160	6·55	13·0	12·60	17·00	16·50
2·50	0·240	1·30	3·0	3·10	1·15	1·20
2·50	0·185	3·50	8·0	7·70	6·40	6·35
2·50	0·160	6·20	14·0	13·20	17·25	16·60
2·95	0·215	2·00	5·0	5·20	2·60	2·90
2·95	0·185	3·60	9·0	8·70	7·30	7·40
2·95	0·170	4·90	12·0	11·60	12·50	12·25

into and out of the pipes. A pump at one end of the pipe passed air through the system by either "blowing" or "sucking" and an auxiliary pump at the other end of the pipe was adjusted to give zero (atmospheric) pressure at the pipe outlet (thus overcoming the resistance of the rotameter). The static pressures at the inlet and outlet of the pipe were measured with U-tube water manometers. Before drilling the leakage holes, the relation between pressure, P, flow rate, Q, and resistance constant, r, was found to be

$$P = rQ^{1.73} \tag{18}$$

The low value of the index of Q was due to the unavoidably low Reynolds number (2 000 to 20 000). The resistance "constant" was also evaluated on the basis of a square law relation between P and Q and its value was tabulated in terms of flow rate. The leakage rate through the holes drilled in the pipe was determined by measurements with short lengths of pipe with sealed ends. A square law relation was obtained between leakage rate and pressure and a leakage constant was obtained in cubic feet per minute (c.f.m.) per hole at 1 in w.g. Tests were then carried out with a leaking pipe system. Inlet and outlet flow rates were measured at different pump pressures. The results were compared with values predicted by the theoretical treatment using appropriate resistance and leakage constants. Table 2 shows the agreement obtained in one series of tests, in which flow rates were varied from 2 to 14 c.f.m. and pressures from 1 to 17 in w.g. with leakage constants from 0·85 to 2·95 c.f.m. per 100 ft at 1 in w.g. These results are for a forcing system. Equally good agreement was obtained with an exhausting system and with different lengths of piping.

The design and use of an admittance bridge for piezoelectric crystals

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A radio frequency bridge for the rapid measurement of resistance and Q-factor of piezoelectric crystals is described. The limitation of the accuracy of measurement due to frequency fluctuations of the generator used and to variations in the stray capacity of the variable resistance arm of the bridge is discussed. Examples of the use of the bridge at 250 kc/s are given.

Very few references are to be found to the use of bridges for the measurement of piezoelectric crystal constants. The two described in greatest detail[1,2] are used semi-quantitatively for the detection and estimation of the piezoelectric effect in small crystals.

The bridge developed by the author has been used at frequencies of 250 kc/s and 500 kc/s for the measurement of the equivalent circuit of the crystal and also the Q-factor. By using three different ratio arms the resistance can be measured to $\pm 1\%$ or $\pm 8\ \Omega$ whichever is the greater in the range between 16 Ω and 40 000 Ω. Q-factors up to 10^5 can be measured with reasonable accuracy.

THEORY

The equivalent electrical circuit of a piezoelectric crystal near one of its resonant frequencies is shown in

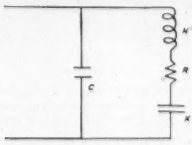

Fig. 1. Crystal equivalent circuit

Fig. 1. On the admittance plane this is represented by a circle of diameter $1/R$ and centre at $1/2R$; $2\pi f_0 c$ (Fig. 2). As the frequency is increased from a low value the circle is traversed clockwise.

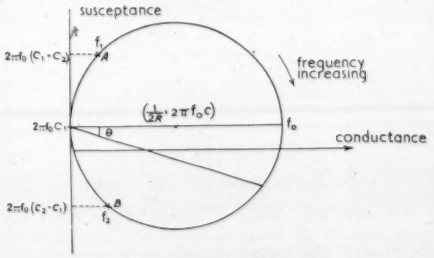

Fig. 2. Crystal admittance circle

A point on the circle defined by the angle θ is related to the frequency by the expression

$$\tan\theta = 2Q(f - f_0)/f_0 \tag{1}$$

where Q (or $1/2\pi f_0 KR$) is the Q-factor and f_0 the resonant frequency of the NKR arm of the equivalent circuit.

As a Q of 10 000 is quite an average value for a crystal the admittance varies very rapidly with frequency. High frequency stability is therefore required of the oscillator

used with the bridge. It is this factor which limits the accuracy of low resistance (high Q) measurements. The uncertainty δR caused by a frequency instability $\delta f/f_0$ can be estimated by equating the out-of-balance voltages produced by a change of δR in R to that produced by a deviation in frequency of δf from f_0. This gives

$$\delta R \simeq (1/2\pi f_0 K)\delta f/f_0 \tag{2}$$

or

$$\delta R/R \simeq Q\delta f/f_0 \tag{3}$$

The frequency must keep within δf of f_0 for the second or so which is required to observe the out-of-balance voltage for two settings of the bridge. For an average crystal at 250 kc/s, δR is 60 Ω when $\delta f/f_0$ is 10^{-6}.

In a system which is not phase sensitive, such as the crevasse method,[3] less frequency stability is required of the generator used. The equation corresponding to (3) is

$$\delta R/R = (Q\delta f/f_0)^2 \tag{4}$$

OPERATION OF THE BRIDGE

The bridge works on about 1 V fed from a variable frequency Franklin oscillator. The out-of-balance signal from the bridge is amplified and displayed on a cathode-ray tube. A schematic diagram of the bridge is shown in Fig. 3. C_2 is normally out of circuit, its function being to check the setting of C_1 and Q-measurement.

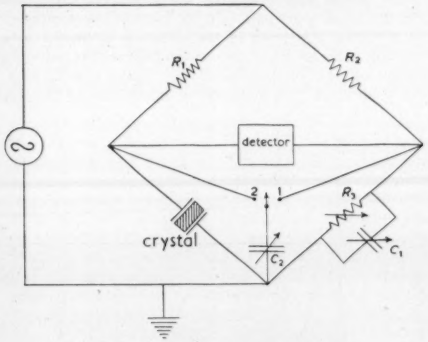

Fig. 3. Schematic diagram of bridge
$R_1 = R_2 = 10$ kΩ. R_3 = variable resistor 0–40 kΩ.
$C_1 = C_2$ = variable capacitors 0–100 pF.

Assuming C_1 to be set correctly, R is found by adjusting the frequency and the variable resistance for zero out-of-balance voltage. The frequency at balance is f_0.

To measure Q, the resistance arm is set to a value R' three or four times R. C_2 is switched to position 1. The balance of the bridge can be obtained by adjustment of C_2 and frequency. The crystal admittance will then correspond to point A (Fig. 2). On switching C_2 to position 2, balance can be restored by adjusting the frequency (point B, Fig. 2). If θ_1 and f_1 correspond to the point A, and θ_2 and f_2 to the point B, then $\theta_2 = -\theta_1$ and $f_0 - f_1 = f_2 - f_0$. Hence

$$\left(\frac{R' - R}{R}\right)^{\frac{1}{2}} = Q\frac{f_2 - f_1}{f_0} \tag{5}$$

A typical graph of $[(R' - R)/R]^{\frac{1}{2}}$ against $(f_2 - f_1)$ for a 1 in diameter crystal enabling an accurate determination of Q to be made is shown in Fig. 4.

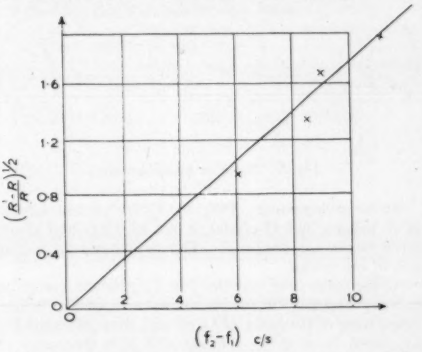

Fig. 4. Determination of Q for a 250 kc/s X-cut 1 in diameter crystal
$R = 1{\cdot}8$ kΩ; f_0/Q = 1/slope = 5·7 c/s; therefore Q = 44 000.

The adjustment of C_1, the factor which sets the upper limit to resistance measurement, must now be considered. The approximate setting can be found by open circuiting the variable resistance arm of the bridge, and, at a frequency remote from any crystal response, adjusting C_1 for reactive balance. Any out-of-balance signal observed when the frequency is changed indicates a piezoelectric response of the crystal.

The precise setting of C_1 is found by successive adjustments until the two balance points, corresponding to C_2, being first in one arm of the bridge, then being in the other, can be achieved by adjustment of frequency only. C_1 then equals the electrostatic capacity of the crystal plus that of the associated leads.

THE VARIABLE RESISTANCE ARM OF THE BRIDGE

The setting of C_1 obtained as described above will be correct for any setting of the variable resistance arm so long as the stray capacity of the arm does not change. If there is a change in capacity with setting an error will be introduced. The fractional error $\delta R/R$ in R caused by a capacity change δC is given by

$$\delta R/R \simeq (R2\pi f_0 \delta C)^2 \tag{6}$$

It is this factor which limits the accuracy of high resistance (low Q) measurements.

The design of the variable resistance arm shown in Fig. 5 was aimed at minimizing capacity changes by retaining, so far as possible, the same geometrical arrangement of resistance no matter what the setting. Tenth watt resistances and miniature switches, because of their low capacities, were used throughout. D.c. calibration of the variable resistance is necessary.

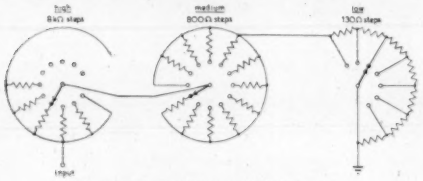

Fig. 5. Variable resistance arm

By observing values of C_1 and C_2 for various settings of R' using a high Q crystal, a plot of the crystal admittance circle was obtained. This constitutes an overall test of the bridge.

At the same time variations in C_1 gave the variations in stray capacity of the resistance arm of the bridge. These were of the order of $1\frac{1}{2}$ pF and thus correspond to 1% error in R at $R = 40\ \text{k}\Omega$ and at a frequency of 250 kc/s.

THE BALANCE-TO-UNBALANCE TRANSFORMER OF THE DETECTOR

Some trouble was experienced in the development of the balance-to-unbalance transformer, which is used to couple the bridge to the detector amplifier. It consists of a tuned primary closely coupled to a few turns of a secondary winding. In the original design it was found that, on reversal of the primary—a general test of the balance of a bridge—the balance changed by as much as 10%. This was due to capacitative coupling between primary and secondary. The effect was reduced to $\frac{1}{4}$% by having a very high step-down ratio and putting a Faraday screen between the two windings.

ASSOCIATED EQUIPMENT

An oscillator of the Franklin type, with undecoupled cathode loads in each valve to increase the frequency stability, is used. The accuracy of $\pm 8\ \Omega$ obtained in crystal measurements indicates a short term of stability of better than 1 in 10^6. A cathode follower stage provides the necessary buffering to prevent any reaction of the crystal on the oscillator frequency.

A three-stage tuned amplifier with oscilloscope display of the out-of-balance signal makes up the detector system. For a long series of resistance measurements it was found convenient to modulate the oscillator in frequency to a depth of just over 1 in 10^6 at 50 c/s. Less frequent adjustments of oscillator frequency were necessary and, in addition, the sense of an out-of-balance signal could be determined.

THE APPLICATION OF THE BRIDGE TO VARIOUS CRYSTAL MEASUREMENTS

The table shows values of the crystal equivalent circuit components for various 250 kc/s X-cut crystals. The acoustic loading is indicated by the drop in resistance

Measurements on 250 kc/s X-cut crystals

crystal	*R (atmospheric)*	*R (vacuum)*	*inductance*	*Q (vacuum)*
1 in² with different mounting losses	9·1 kΩ	4·3 kΩ	67 H	24 000
	6·7 kΩ	2·9 kΩ	62 H	33 000
	3·2 kΩ	0·3 kΩ	54 H	280 000
Straubel contour crystal 4 cm²	7·5 kΩ	3·6 kΩ	110 H	50 000
Circular 1 in diameter	6·3 kΩ	2·7 kΩ	57 H	50 000

when the crystal chamber is evacuated. The change of resistance with pressure is strictly linear to quite high pressures. It is possible that circumstances may be encountered which make the use of this effect for gas pressure measurement quite convenient.

The bridge was originally developed for ultrasonic interferometer work only. In one form of interferometer the electrical resistance of the crystal is measured as a function of the length of gas column between the radiating surface of the crystal and a reflector placed parallel to it. Fig. 6 shows a typical system of resonances of the gas

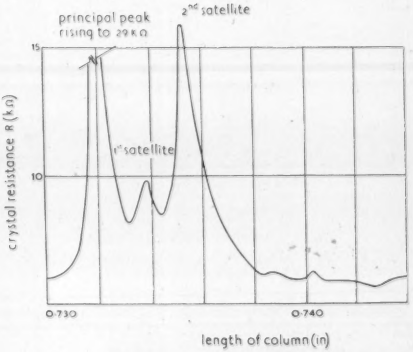

Fig. 6. A group of interferometer resonances

column. The principal peak is due to the resonance of plane waves excited by the crystal. The smaller peaks are due to the excitation of complex modes of vibration.

The electromagnetic counterpart of these modes occurs in cavity resonators and waveguides.

FREQUENCY LIMITS OF THE BRIDGE

Equation (5) shows that the upper limit of resistance measurement falls off as the square of the frequency. The resistance of crystals due to acoustic loading falls off as the square of the frequency.[3] The two factors thus to a certain extent compensate.

The bridge has been used without modification at 500 kc/s. By reducing all resistance values by a factor of about ten, the extension of measurements to frequencies of the order of megacycles per second would be possible.

ACKNOWLEDGMENTS

The development of this instrument was begun at University College, Hull, and completed at King's College, Newcastle.

The author wishes to thank Professor L. S. Palmer of Hull and Professor W. E. Curtis of Newcastle for the facilities put at his disposal.

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A note on the measurement of damping in vibrating rods

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The theory of the determination of dynamic elastic modulus and damping from observations of the resonance curves of thin rods vibrating longitudinally or in torsion is discussed. Formulae are derived for these quantities in terms of resonance frequency and bandwidth for cases where the higher-order harmonics are used and where the damping is neither very small nor independent of frequency. A formula given by Lethersich and Pelzer in this Journal[3] is amended.

Considerable attention has been devoted in recent years to the measurement of the elastic and inelastic properties of solids, particularly plastics, subjected to alternating stresses.[1,2] The results have been of value both practically, as for example in the design of tyres and vibration absorbers, and also in advancing theoretical knowledge of the relation between molecular structure and mechanical properties. Experimental work in this field consists in the measurement of the elastic and dissipative components of an effective stress-strain modulus as a function of frequency. A method commonly employed for this in the acoustic and ultrasonic range of frequencies is the observation of the harmonic resonances of a rod of the test material excited electrically in its normal modes of longitudinal or torsional vibration. When the dissipative component, and hence the damping of the resonances, is very small, the position and sharpness of the resonance curves in the frequency scale are simply related to the elastic modulus and the damping coefficient respectively at the resonant frequency. If the damping is moderately high, however, as is often the case in plastics, the relations between the observed quantities and the parameters of the material become more complicated. The present note describes the derivation of these relations in a fairly general case.

THEORY OF THE VIBRATING ROD

For mathematical convenience, it is common in problems on vibrations to consider the relation between the stress s and the strain e in a solid to be of the form:

$$s = Ee + \eta \frac{de}{dt} \qquad (1)$$

where E and η are appropriate elastic and viscous moduli. The equation of longitudinal motion in a thin rod or tube is then[3]

$$\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial^3 u}{\partial t \partial x^2} \qquad (2)$$

where x = longitudinal co-ordinate
u = longitudinal displacement
ρ = density of rod
E, η = Young's modulus and the associated viscosity coefficient of the rod respectively.

The same equation is valid for torsional vibration if E, η and u are replaced by the rigidity modulus, its associated viscosity and the angular displacement respectively. Assuming a steady-state solution of the form:

$$u(x, t) = U(x) \exp j\omega t$$

(2) becomes:

$$\rho\omega^2 U + (E + j\omega\eta)\frac{\partial^2 U}{\partial x^2} = 0 \qquad (3)$$

In practice, however, it is found that equation (1) does not adequately represent the behaviour of most solids. It is now widely accepted that, at least in the high polymers, a complete model of the solid must comprise an indefinitely large number of elements, all obeying equation (1) but with differing individual values of E and η. The observed strain is then the linear superposition of the strains in the individual elements. If this linear model is valid, then it is legitimate to specify the response of the system to a sinusoidal stress in terms of an effective elastic modulus and viscosity coefficient for the frequency concerned.[4] This is exactly analogous

to the specification of a complicated electrical network by its effective transfer impedance at any frequency. On this basis equation (2) is no longer valid in general, but it can readily be shown that an expression identical with equation (3) is derivable from first principles for the steady-state case, with the distinction that E and η are to be regarded as functions of frequency. The solution of equation (3) can then proceed along the conventional lines. If a rod of length l is excited by a sinusoidal stress of peak amplitude S at one end, the amplitude of vibration observed at the other is given by the expression:

$$A = |U(l)| = \frac{S}{\omega(1 + 1/Q_\omega^2)^{\frac{1}{4}}}\left[\frac{2}{E\rho(\cosh \alpha l - \cos \beta l)}\right]^{\frac{1}{2}} \quad (4)$$

where $Q_\omega = E/\eta\omega$

$$\alpha^2 = \frac{2\rho\omega^2}{E}\left[\frac{(1 + 1/Q_\omega^2)^{\frac{1}{2}} - 1}{1 + 1/Q_\omega^2}\right]$$

$$\beta^2 = \frac{2\rho\omega^2}{E}\left[\frac{(1 + 1/Q_\omega^2)^{\frac{1}{2}} + 1}{1 + 1/Q_\omega^2}\right]$$

The resonances occur at or near multiples of the frequency $\omega_0/2\pi = (E/\rho)^{\frac{1}{2}}/2l$. Equation (4) is in agreement with the result given by Lethersich and Pelzer.[5]

RESONANCE CURVES

It is now necessary to find from equation (4) the exact position and width of the resonance peaks, and to facilitate this certain simplifying assumptions will be made. It will first be assumed, as an arbitrary though convenient limit, that:

$$Q_\omega \geqslant 10 \quad (5a)$$

The functions of Q_ω in equation (4) can now be greatly simplified by expansion in terms of $1/Q_\omega^2$. If it is also assumed that:

$$\frac{\omega}{\omega_0}\,\frac{1}{Q_\omega} \leqslant 0{\cdot}44 \quad (5b)$$

and

$$\left|\frac{\omega - m\omega_0}{\omega_0}\right| \leqslant 0{\cdot}22 \quad (5c)$$

where m is an integer, the order of the harmonic, then the following approximate relations

$$\begin{aligned}\cosh\theta &= 1 + \theta^2/2 + \theta^4/24\\ \cos\theta &= 1 - \theta^2/2 + \theta^4/24\end{aligned} \quad (6)$$

can be used in deriving a simplified expression for the amplitude near the resonance peaks, viz.:

$$A = \frac{2S}{\pi(E\rho)^{\frac{1}{2}}}\left[\frac{\omega^4}{\omega_0^3 Q_\omega^2}(1 - 3/4Q_\omega^2) + \frac{\pi^2\omega^6}{12\omega_0^4 Q_\omega^4}(1 - 2/Q_\omega^2) + \frac{4(\omega - m\omega_0)^2\omega^2}{\omega_0^2}(1 - 1/4Q_\omega^2) - \frac{4\pi^2(\omega - m\omega_0)^4\omega^2}{3\omega_0^4}(1 - 1/Q_\omega^2)\right]^{-\frac{1}{2}}. \quad (7)$$

Equation (7) gives A to an accuracy of 1% or better provided that conditions (5) are satisfied. These conditions cover most of the cases in which the experimental system under consideration is likely to be used. Conditions (5b) and (5c) may be regarded as setting an upper limit to the order of the harmonic for which equation (7) is valid.

In determining the shape of the resonance curves from equation (7) it is necessary to know how Q_ω, which is a useful measure of the damping, and E vary with frequency. It is convenient to assume that Q_ω obeys a power law of the form:

$$Q_\omega\omega^k = \text{constant}$$

where k is a constant whose magnitude is of the order of unity or less. This representation is only required to hold over the width of any one peak and in practice is likely to be sufficiently accurate for the purpose. E will be treated as independent of frequency over the width of any one peak. It can be shown that errors due to this assumption are negligible unless E varies more rapidly than, say, $\omega^{\frac{1}{2}}$, which would be very unlikely in practice.

With the above assumptions, the maxima of the amplitude are found to occur when $\omega = \omega_m$, where:

$$\omega_m = m\omega_0\left[1 + \frac{2 + k}{4Q^2} + \frac{(3 + 2k)\pi^2 m^2}{48Q^4}\right]^{-1} \quad (8)$$

Q signifying, in this case and henceforth, $Q(\omega_m)$. From equation (8) ω_0 can be found, and hence $E(\omega_m)$. For $k = 1$ the value of $(m\omega_0 - \omega_m)$ in the most extreme case allowed by conditions (5) is only about $0{\cdot}01\ \omega_m$. (It must be remembered, however, that in longitudinal vibration in rods of finite thickness the resonance frequencies are somewhat affected by radial motion, here assumed negligible.)

The resonant amplitude A_r can be found by substitution of $\omega = \omega_m$ in equation (7). The frequencies at which the amplitude has dropped by a factor of $\sqrt{2}$ (i.e. 3 db) on either side of the peak are then given by equation (7) when A is put equal to $A_r/\sqrt{2}$. Putting $(\omega - \omega_m)/\omega_m = z$, the fractional detuning at the 3 db points, in the resulting equation and expanding powers of ω as power series in z, neglecting only those terms which are appreciably less than 1% of the main terms in the equation, we get finally the fifth-degree equation for z:

$$(1 - \epsilon_2)z^2 + 2\phi z^3 - 2\lambda(1 - \epsilon_4)z^4 - 4\lambda z^5 = (1 + \lambda/2Q^2)(1 + \epsilon_0)/4Q^2 \quad (9)$$

where

$$\left.\begin{aligned}\lambda &= \pi^2\omega_m^2/6\omega_0^2\\ \epsilon_0 &= [(1 + 4k + k^2) + (3 + 10k + 3k^2)\lambda/2Q^2 + (6 + 2k + k^2)\lambda^2/4Q^4]/4Q^2\\ \epsilon_2 &= [(3 - 3k - 2k^2) + (21 + 10k - 2k^2)\lambda/2Q^2 + 3(2 + k)(3 + 2k)\lambda^2/Q^4 + 3(3 + 2k)^2\lambda^3/4Q^6]/4Q^2\\ \epsilon_4 &= [1 + (10 + 4k)\lambda/Q^2 + (6 + 4k)\lambda^2/Q^4]/2\lambda\\ \phi &= 1 + (2 + k)\lambda/Q^2 + (3 + 2k)\lambda^2/2Q^4\end{aligned}\right\} \quad (10)$$

The two smallest roots of this equation (9) give the fractional detuning at the 3 db points and hence the bandwidth.

SOLUTION OF THE BANDWIDTH EQUATION

For the simplest case, where the damping is very small (specifically, $1/Q$ negligible), so that the bandwidth and therefore z are small, the only significant terms of (9) are:

$$z^2 = 1/4Q^2$$

with the positive and negative solutions:

$$\left.\begin{aligned} z_p &= (\omega_p - \omega_m)/\omega_m = 1/2Q \\ z_n &= (\omega_n - \omega_m)/\omega_m = -1/2Q \end{aligned}\right\} \qquad (11)$$

where ω_p and ω_n are the angular frequencies of the upper and lower 3 db points. The overall fractional bandwidth δ is then:

$$\delta = z_p - z_n = (\omega_p - \omega_n)/\omega_m = 1/Q \qquad (12)$$

which is a well-known result for small damping.

A more accurate solution of equation (9), say to 1% in all cases allowed by equation (5), may be obtained as follows. First the terms in z^3 and z^5 are neglected and the resulting quadratic solved to give the first approximations z_{1p} and z_{1n}. ($z_{1p} = -z_{1n}$). The solution is

$$z_{1p}^2 = \frac{1-\epsilon_2}{4\lambda(1-\epsilon_4)} \left\{1 - \left[1 - \frac{(2\lambda/Q^2)(1+\lambda/2Q^2)(1+\epsilon_0)(1-\epsilon_4)}{(1-\epsilon_2)^2}\right]^{\frac{1}{2}}\right\} \qquad (13)$$

z_1 is then inserted in the z^3 and z^5 terms and the new quadratic solved to give z_2. A final repetition of the process gives z_3. Now the z^3 and z^5 terms are not necessarily small compared with z^2. However, they are odd functions of z, and while the individual positive and negative solutions z_{3p} and z_{3n} may differ appreciably from z_{1p} and z_{1n} respectively, the quantity $(z_{3p} - z_{3n})$, in which we are interested, differs from $(z_{1p} - z_{1n})$ by only a few percent in all cases for which conditions (5) hold, and further approximations are unnecessary. This being so, it is practicable to express the final solution in terms of z_1 and a small correcting term μ, thus:

$$(z_{3p} - z_{3n}) = (z_{1p} - z_{1n})(1+\mu) = 2z_{1p}(1+\mu) \qquad (14)$$

where

$$\mu = \frac{z_1^2\phi}{2(1-4\lambda z_1^2)^{\frac{1}{2}}}\left[5\phi - (12\phi + 28)\lambda z_1^2 + 80\lambda^2 z_1^4\right] \qquad (15)$$

Combining (13), (14) and (15) and inverting the solution to give Q in terms of $(z_{3p} - z_{3n}) = \delta$, the overall fractional bandwidth, we obtain as the final result:

$$1/Q^2 = (1/\lambda)\left\{\left[1 + 2\lambda\delta^2(1 - \tfrac{1}{2}\lambda\delta^2 - \gamma)\right]^{\frac{1}{2}} - 1\right\} \qquad (16)$$

where

$$\gamma = \tfrac{1}{4}\delta^2\left[(8 + k - k^2) + \lambda\delta^2(17{\cdot}5 + 13k + k^2) + \lambda^2\delta^4(32{\cdot}3 + 29k + 9k^2) + \lambda^4\delta^8(56{\cdot}5 + 53{\cdot}8k + 18{\cdot}4k^2)\right]$$

The small term γ includes μ and the ϵ's of equation (10). Its derivation involves replacing z_1^2 in μ by $\frac{1}{4}\delta^2$ and $1/Q^2$ in the ϵ and ϕ by the value given by equation (16) when γ is assumed zero. Equation (16) is accurate to 1% or better in all cases covered by equation (5). The greatest difference between the values of Q given by equation (16) and by the simplest formula (12) is about 15%. It may be noted that k, which describes the variation of Q with frequency, appears in equation (16) only in the small term γ, and so has relatively little influence on the result obtained for Q. In practice it is in general necessary to find approximate values of Q from experimental measurements of δ over a range of frequencies (i.e. of harmonics) by using equation (16) with γ assumed zero, thence to derive k and γ, and finally to re-substitute in equation (16) to obtain a more accurate value of Q.

Equation (16) is valid under conditions (5). It can be shown that (5*b*) will always be satisfied when (5*c*) is, and when rewritten in a form more convenient for practical work, using equation (8), (5*c*) becomes approximately:

$$\frac{D}{\omega_m} \leq \frac{0{\cdot}22}{m} - \frac{2+k}{4Q^2} \qquad (5c)$$

where D is the difference in the angular frequencies of the peak and the lower 3 db point of the resonance curve.

If $\lambda\delta^2 < 0{\cdot}1$ the radical in equation (16) can be simply expanded, giving:

$$1/Q = \delta(1 - \lambda\delta^2 - \gamma)^{\frac{1}{2}}$$

If, moreover, $\lambda\delta^2$ and terms of the order of $1/Q^2$ (though not necessarily $1/Q$) can be neglected in comparison with unity, this reduces to:

$$1/Q = \delta \qquad (17)$$

This is identical with equation (12), i.e. the overall bandwidth is still given by the formula found for the more extreme case of $1/Q$ negligible. The result given by Lethersich and Pelzer,[5] who have considered the case of $1/Q$ finite, is thus in error in giving a correction of the order of $1/Q$ to equation (17).

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Interaction between currents as a relativistic second order effect of electrostatic forces

E. G. Cullwick, in his book *The Fundamentals of Electromagnetism* (Cambridge University Press, 1949), purports, as indicated in the title of this note, to derive the force acting between electric currents from Coulomb's law and relativity (or in other words to derive the relation $\mu_0\kappa_0c^2 = 1$) without using magnetic concepts. A rather similar programme has been carried out successfully by Page and Adams in their *Electrodynamics* (Chapman & Hall, Ltd., 1940). Cullwick's approach, although less general, is much shorter and easier. Unfortunately his derivation is based on an approximate treatment leading to certain residuals and he is led to the conclusion that ". . . a stationary charge should be acted upon by a stationary current circuit . . ." (p. 137) so that unnecessary doubts of the basic proposition might be raised. Now a more rigorous treatment which follows that of Professor Cullwick step by step can give an exact result and the residuals disappear.

Cullwick's premises are the transformation laws of electric field strength and of mechanical force which can be stated as follows:

If from a reference system, denoted by a prime, in which there is an electric field E' due to a charge at rest in this system, one goes over to a system, denoted by two primes, which moves with velocity $v = \beta c$ with reference to the first system (c being the velocity of light), then the components parallel and normal to v of the field E'' in this second system are related to those in the first by

$$E''_{\parallel} = E'_{\parallel} \tag{1}$$

$$E''_{\perp} = \frac{E'_{\perp}}{\sqrt{(1-\beta^2)}} \tag{2}$$

Secondly, let a particle be at rest in a system denoted, say, by an asterisk and F^* be the force acting on the particle in this system. In a system moving with velocity $v = \beta c$, denoted by two asterisks, the force will be F^{**} where

$$F^{**}_{\parallel} = F^*_{\parallel} \tag{3}$$

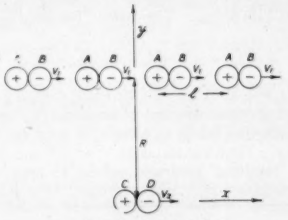
$$F^{**}_{\perp} = F^*_{\perp}\sqrt{(1-\beta^2)} \tag{4}$$

are the transformation relations for the force components parallel and normal to v. This transformation law for F can be deduced from purely mechanical considerations. This is important, since Cullwick in his book derives the transformation law for F from a magnetic argument, which might give the impression of assuming the laws which are to be proved.

The main reason for Cullwick's surprising result arises from his attempt to deduce the force between current elements instead of considering the whole of at least one closed circuit. This immediately leads to a difficulty, because a current element cannot be treated as an indivisible unchanging entity, but has to be composed of charges of opposite sign in relative motion, the usual model being a positive charge at rest and a negative charge in motion. Now if at one instant the charges coincide, they will not do so in the next, nor can they have coincided in the instant before. As the interaction between charges is not instantaneous but propagated with the velocity of light, the history of at least one of the two systems of charges is relevant. On the other hand, in considering the action of the whole of one closed circuit on an element of another circuit at the instant when the charges composing the latter coincide, the history of the former system is determined by the fixed geometry of its closed circuit. The simplest circuit for the purpose is an infinite straight conductor (which is as good as a closed one in having no free end in any finite region of space), represented by a row A of positive charges q at rest, spaced apart at distances l, and a row B of negative charges $-q$ moving in the same straight line with velocity v_1. This represents a current

$$I_1 = qv_1/l \tag{5}$$

Let the current element acted upon by this circuit be at a distance R from the straight line conductor (assuming $l \ll R$), and consist of C, a positive charge q at rest and of D, a negative charge $-q$ moving with velocity v_2 parallel to I_1. This corresponds to a current element $I_2ds = qv_2$ (see figure).



Conduction model

It is now required to calculate (*a*) the repulsion on C exercised by charges A, (*b*) the attraction of B on C, (*c*) the attraction of A on D, and (*d*) the repulsion of B on D.

(*a*) Both A and C are at rest so that the ordinary static law holds, namely the action of a long uniformly charged wire on a point charge which is (using here the unrationalized M.K.S. notation as does Cullwick):—

$$F_a = -2q^2/\kappa_0 Rl = -2qI_1/\kappa_0 v_1 R \tag{6}$$

(*b*) To an observer moving with the charges B these will be farther apart, at a distance l_1, than they appear to an observer at rest to whom this distance is subject to the Lorentz contraction so that $l = l_1(1-\beta_1^2)^{-\frac{1}{2}}$ where $\beta_1 = v_1/c$. (Professor Cullwick has kindly drawn attention to a misunderstanding to which this statement might give rise, namely the misinterpretation that, if the current were switched off, two adjacent negative charges when coming to rest would behave as if fixed to the ends of a measuring rod which is transferred from velocity v_1 to rest. This interpretation should not be read into the sentence heading this paragraph. The actual situation, if switching off is contemplated, is that, for an observer at rest relative to the wire and positive charges the distance between two adjacent negative charges will always be the same as the distance between the positive charges, whether the negative charges be moving (current flowing) or at rest (no current); this must be so as in this system of reference the wire never carries a net charge whether current flows or not. But of course in any system in relative motion against the conductor the spacing between the negative charges must then be different from that between

the positive ones if a current flows, and any section of the wire carries a net charge in such a system which is proportional to the scalar product of the vector of the velocity of the wire and the vector of the current through this section of the wire. An observer moving with the charges B will experience, at a distance R from the conductor AB a field E_1 from the charges B, at rest to him, where E_1 is given by the static formula

$$E_1 = 2q/\kappa_0 R l_1 = 2q(1-\beta)^{\frac{1}{2}}/\kappa_0 R l \tag{7}$$

To an observer at rest with C the field E_1 appears according to equation (2) as a field E which is $E_1(1-\beta_1^2)^{-\frac{1}{2}}$, so the force on C is

$$F_b = qE = 2q^2/\kappa_0 R l = 2qI_1/\kappa_0 v_1 R \tag{8}$$

Accordingly $F_a + F_b = 0$.

Thus a current exercises no force on a charge at rest, if the positive and negative charge densities in the current are equal. Cullwick's approximation, using the model of two current elements, leads to a net residual force on p. 137. This residual has not a mathematically significant order of magnitude in the approximation and disappears altogether in the present more rigorous treatment.

(*c*) The force on the moving charge D due to the fixed charges A as observed by an observer at rest also turns out to be of the same value as F_b. To an observer at rest the field at CD due to A is $2I_1/\kappa_0 v_1 R$; to an observer moving with D this field according to (2) is $2I_1/\kappa_0 v_1 R(1-\beta_2^2)^{\frac{1}{2}}$ where $\beta_2 = v_2/c$; therefore to him the force on D is $2qI_1/\kappa_0 v_1 R(1-\beta_2^2)^{\frac{1}{2}}$. To obtain the force in the system at rest is required the law of the transformation of the force from the system in which the moving body D is at rest, to a system (in which A is at rest) moving with velocity $-v_2$ against it in a direction at right angle to the force. According to this law, eqn. (4), force transforms differently as compared with field: it is now necessary not to divide but to multiply by $(1-\beta_2^2)^{\frac{1}{2}}$ and thus we obtain

$$F_c = 2qI_1/\kappa_0 v_1 R \tag{9}$$

(*d*) The field at a distance R from the moving charges B to an observer moving with them is $2q/\kappa_0 R l_1$ as was already evaluated as E_1 under (*b*). To an observer moving with the charge D this field is $2q/\kappa_0 R l_1(1-\beta_{12}^2)^{\frac{1}{2}}$ where β_{12} is the relative velocity of the one observer against the other, divided by c. This relative velocity has to be determined from Einstein's addition theorem and yields

$$\beta_{12} = (\beta_1 - \beta_2)/(1-\beta_1\beta_2) \tag{10}$$

The force on the charge D to an observer moving with it is $-2q^2/\kappa_0 R l_1(1-\beta_{12}^2)^{\frac{1}{2}}$. To an observer at rest and therefore moving with $-v_2$ with respect to the body on which the force acts this will appear as a force

$$-2q^2(1-\beta_2^2)^{\frac{1}{2}}/\kappa_0 R l_1(1-\beta_{12}^2)^{\frac{1}{2}}$$

Substituting for l_1

$$F_d = -2q^2(1-\beta_1^2)^{\frac{1}{2}}(1-\beta_2^2)^{\frac{1}{2}}/\kappa_0 R l(1-\beta_{12}^2)^{\frac{1}{2}} = -\frac{2qI_1}{\kappa_0 v_1 R}\left[\frac{(1-\beta_1^2)(1-\beta_2^2)}{1-\beta_{12}^2}\right]^{\frac{1}{2}} \tag{11}$$

Since the forces F_a and F_b cancel, the net force F is

$$F = F_c + F_d = \frac{2qI_1}{\kappa_0 v_1 R}\left\{1 - \left[\frac{(1-\beta_1^2)(1-\beta_2^2)}{1-\beta_{12}^2}\right]^{\frac{1}{2}}\right\} \tag{12}$$

Insert now the value of β_{12} into the expression under the square root; the following result is then obtained:

$$\frac{(1-\beta_1^2)(1-\beta_2^2)}{1-\beta_{12}^2} = (1-\beta_1\beta_2)^2\frac{(1-\beta_1^2)(1-\beta_2^2)}{(1-\beta_1\beta_2)^2-(\beta_1-\beta_2)^2} = (1-\beta_1\beta_2)^2 \tag{13}$$

Inserting this into the expression for F yields:

$$F = 2qI_1\beta_1\beta_2/\kappa_0 v_1 R = 2qI_1 v_2/\kappa_0 c^2 R \tag{14}$$

But $qv_2 = I_2 ds$, whence

$$F = \frac{1}{\kappa_0 c^2}\left(\frac{2I_1 I_2 ds}{R}\right) \tag{15}$$

or, as Cullwick also gives,

$$F = \frac{1}{\kappa_0 c^2}\left(\frac{2I_1 I_2}{R}\right)$$

per unit length of a conductor carrying I_2.

This has now been shown to be exactly true and not just an approximation. The ordinary electromagnetic law of force is

$$F = \frac{\mu_0 2I_1 I_2}{R} \tag{16}$$

whence

$$\kappa_0\mu_0 c^2 = 1 \tag{17}$$

It should be noted that this is obtained as an exact relation whilst in Cullwick's book it appears only as an approximation.

Accordingly the equation $\kappa_0\mu_0 c^2 = 1$ may be regarded as inherently essential in the theory of the electromagnetic field unified according to modern theoretical physics.

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E. G. CULLWICK'S REPLY

My deduction of the magnetic force between two parallel current-elements was not intended to be more than an approximate application of relativistic transformations, and I am indebted to Drs. Whitehead and Pelzer for their rigorous treatment, in which the residual force on a stationary charge disappears.

The method suggests that all the phenomena of constant-current electromagnetism may be deduced by combining electrostatics with the Restricted Theory of Relativity. As a further example, we may continue the case under discussion as follows:

Considering a straight conductor carrying a steady current, let σ_+ and σ_- be the positive and negative charge per unit length, referred to a stationary observer 0, so that $\sigma = q/L$ and $I = \sigma v$, and let σ'_+ and σ'_- be the same quantities referred to an observer 0′ who moves with the negative charge-cloud.

Since the circuit as a whole must appear uncharged to 0 under all conditions, we must have:

$$\sigma_+ = \sigma_- \tag{1}$$

whether current flows or not. Referred to 0′, however, the conductor suffers the Lorentz contraction and since the positive charge moves with the wire relatively to 0′ we must have:

$$\sigma'_+ = \sigma_+\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{2}$$

The negative charge-cloud is stationary relative to 0′, whereas to 0 it represents a convection current σv. The quantity σ_-

therefore transforms as follows, thus assuming its minimum value for 0′:

$$\sigma'_- = \sigma_-\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} \qquad (3)$$

Then, although for 0 there is no electric field, the observer 0′ will measure an electric field given by:

$$\mathscr{E}' = \frac{2(\sigma'_+ - \sigma'_-)}{\kappa_0 R} = \frac{2\sigma v^2}{\kappa_0 R c^2}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \frac{2\mu_0(\sigma v)v}{R}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

But $$\frac{2\mu_0(\sigma v)}{R}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = B'$$

the magnetic field of the current $I' = v\sigma'_+$ as measured by 0′, so that:

$$\mathscr{E}' = \boldsymbol{B}' \times (-\boldsymbol{v}) \qquad (4)$$

where $-\boldsymbol{v}$ is the velocity of the conductor relative to 0′.

The result given by (4) is exactly that required by the relativistic transformation of the electromagnetic field, for a unit charge moving with velocity $\boldsymbol{v}$ relative to the stationary wire experiences a force equal to $\boldsymbol{v} \times \boldsymbol{B}$, and $\mathscr{E} = 0$.

The existence, for 0′, of a resultant charge on the wire is consistent with the invariance of the total charge on the complete circuit, since other parts of the circuit, such as a parallel return wire, assume charges of opposite sign.

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The flow characteristics of capillary leaks

In the course of an investigation we proposed to make use of an equation to calculate the flow characteristics of small capillaries such as would occur in a vacuum leak. Such an equation is given on page 191 of the excellent book on vacuum practice by Guthrie and Wakerling, *Vacuum Equipment and Techniques*. On further scrutiny, however, we have come to the conclusion that this equation has been wrongly derived, for the following reasons, and we would be glad of the hospitality of your columns to make our views known. Owing to the importance to vacuum technologists of this type of flow, we feel that the derivation as well as the result is of interest.

The problem may be reduced to the case of gas flow through a cylinder of length L (cm), very large compared with its diameter, D (cm), with atmospheric pressure, P, at one end and vacuum at the other. At higher pressures, where the mean free path λ of the gas molecules is very much less than D (i.e. $\lambda \ll D$) and laminar flow is assumed, the flow through the cylinder is given by the well-known Poiseuille equation for viscous flow, whereas at low pressures when $\lambda \gg D$ it is given by Knudsen's molecular flow formula. Knudsen has also given a semi-empirical formula which is valid for all pressures. Assuming (i) that the flow is not turbulent in any part of the tube, and (ii) that the pressure difference between the ends is not so great that the mechanism of flow (viscous or molecular) changes along it, Knudsen showed that the conductance C is given by

$$C = \frac{\pi}{128}\frac{D^4}{\eta}\frac{\bar{P}}{L} + \frac{1}{6}\sqrt{\left(\frac{2\pi kT}{m}\right)}\frac{D^3}{L}\left[\frac{1 + \sqrt{\left(\frac{m}{kT}\right)}\frac{D\bar{P}}{\eta}}{1 + 1{\cdot}24\sqrt{\left(\frac{m}{kT}\right)}\frac{D\bar{P}}{\eta}}\right] \text{ c.g.s. units} \qquad (1)$$

This result can be applied to each successive element of length along the tube in our problem, Knudsen's assumption (ii) being valid for each element and (i) being valid if the rate of flow is not too large, i.e. if the diameter of the tube does not exceed 200 microns (see Guthrie and Wakerling, page 194).

The conductance C of the element, of length ΔL, of L is then

$$C = 0{\cdot}182\frac{D^4}{\Delta L}\frac{\eta_{\text{air}}}{\eta}$$

$$P + 12{\cdot}1\frac{D^3}{\Delta L}\frac{m_{\text{air}}}{m}\left[\frac{1 + 0{\cdot}255D\sqrt{\left(\frac{m}{m_{\text{air}}}\right)}\frac{\eta_{\text{air}}P}{\eta}}{1 + 0{\cdot}316D\sqrt{\left(\frac{m}{m_{\text{air}}}\right)}\frac{\eta_{\text{air}}P}{\eta}}\right] \text{ litres/sec} \qquad (2)$$

for gas at 20° C where P is now measured in μ Hg.

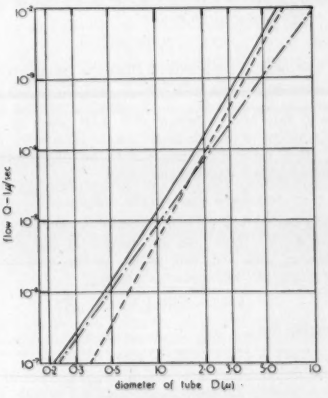
Following Guthrie and Loevinger we write this equation as

$$C = \frac{1}{\Delta L}\left(aP + b\frac{1 + cP}{1 + fP}\right) \qquad (3)$$

where a, b, c, f are the appropriate coefficients in equation 2, so that the flow, Q, through the element is $Q = C\Delta P$. In the steady state Q can be considered constant and hence integration gives

$$Q \cdot L = \int_{P_2}^{P_1}\left(aP + b\frac{1 + cP}{1 + fP}\right)dP = \left[\frac{aP^2}{2} + \frac{bc}{f}P + \frac{b(f - c)}{f^2}\ln(1 + fP)\right]_{P_2}^{P_1} \qquad (4)$$

where P_1 is the high pressure and P_2 is the low pressure at the ends of the tube. In the case considered P_1 is atmospheric pressure, i.e. $7{\cdot}6 \times 10^5$ μ Hg and P_2 is very small for small



Region of transition from viscous to molecular flow for a cylindrical tube 1 cm. long

- - - - - - assuming purely viscous flow
— · — · — assuming purely molecular flow
——— true flow

flows so that we may put $P_2 = 0$. Hence, inserting these values into the equation (with D in microns) we have

$$Q \cdot L = 10^{-6}\left\{5{\cdot}25\frac{\eta_{\text{air}}}{\eta}D^4 + 7{\cdot}4\left(\frac{m_{\text{air}}}{m}\right)^{\frac{1}{2}}D^3 + 0{\cdot}074\frac{m_{\text{air}}}{m}D^2\ln\left[1 + 24\sqrt{\left(\frac{m}{m_{\text{air}}}\right)\frac{\eta_{\text{air}}}{\eta}}D\right]\right\} \quad (5)$$

Let us consider the limiting conditions of this equation. As D becomes relatively large, the D^4 term predominates over the other two terms so that for $D > 10\ \mu$ (or flows larger than $10^{-2}\ \mu$ l/sec) the flow becomes almost entirely viscous. On the other hand, as D becomes small, the D^4 term is negligible. To see the effect of the other two terms as $D \to 0$, the limit of (4) shows that they add up to bP, i.e. the same flow as would be obtained by assuming conditions of molecular flow for the whole length of the tube. Calculation shows that this can be assumed with negligible error when $D < 1\ \mu$ (or for flows less $10^{-5}\ \mu$ l/sec). This transition in flow characteristics is shown in the graph where the flow, calculated from equation (5) for air is plotted against the diameter D (μ) for a tube 1 cm long.

It should be noted that our equations (4) and (5) should replace Guthrie and Loevinger's equations (1) and (2) on page 191 with consequent changes in the deductions from this equation. Thus the lower limit of Q/Q_{air} quoted in Table 5.1 giving the flow characteristics of circular capillary leaks for different gases should be modified to $(M_{\text{air}}/m)^{\frac{1}{2}}$ and is independent of viscosity.

Much of the argument in this communication is already located in Guthrie and Wakerling's book, but it was felt that it should be included here in the interests of continuity.

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N. Ochert
W. Steckelmacher

The conclusions reached by Ochert and Steckelmacher appear to be correct. The term $b(d - c)/f^2$ in equation (1) on page 191 of Guthrie and Wakerling is in error; it should be $b(f - c)/f^2$. The letter "d" is not defined and the last term of equation (2) is in error.

The conclusions reached are, as noted, of considerable interest to vacuum technologists.

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New Books

Dielectric Breakdown of Solids. By S. Whitehead, M.A., Ph.D., F.Inst.P. (London: Oxford University Press.) Pp. xv + 271. Price 25*s*. net.

The current output of scientific literature is so great that the first question which must be asked about any new contribution is: is it really necessary? Apart from Fröhlich's review in *Reports on Progress in Physics*, 1939, which covered only a small part of the field covered by the present book, there has been no summary of the very considerable progress in the study of the breakdown of solid dielectrics made in recent years. The scatter of publications on the subject, the highly speculative nature of many of the theoretical contributions, and the failure to apply sufficiently rigorous criteria in many of the experimental papers make an authoritative review not only necessary but also time-saving even for those intimately concerned with the subject.

The book is also opportune. Theoretical and experimental work on intrinsic breakdown has progressed to the point where the broad outlines can be said to be well established and the engineering implications plain. The state of knowledge of other forms of breakdown is not so definite, apart from thermal breakdown, and it is likely that the book will prove to be more in the nature of an interim account of this part of the subject. Even so, the progress made amply justifies the wider dissemination achieved by publication as a book.

A book covering a fairly narrow field must necessarily be addressed to all those concerned in the field of whatever background and training. In this case they include theoretical physicists, experimental physicists and engineers; it is clearly difficult to achieve a presentation of the subject both adequate and acceptable to all three. The method adopted—qualitative exposition in terms of models with a minimum of mathematics, followed by separate detailed mathematical exposition—probably goes as far as is possible towards meeting the varied requirements of different classes of reader.

The first major section of the book (Chapter II) is an account of both theories and experimental work on intrinsic breakdown. The historical survey is not complete and would indeed be less valuable if it were, since much was written on the subject before either the experimental concept had emerged with sufficient clarity, or theoretical knowledge of solid structure had advanced far enough to make the development of a theory of breakdown possible. But enough is included to trace the growth of ideas which led to the modern theories of electronic breakdown.

The main contributors were von Hippel and his associates, to whom also the experimental part of the subject owes much, and Fröhlich. The two theories differ materially and although the essence of the disagreement is stated with clarity in the book comparisons with experiment are made only for Fröhlich's theories. This may well be a reflexion of the original publications since the theoretical work of von Hippel's school was rarely carried far enough to make detailed comparison possible. Fröhlich, on the other hand, did achieve quantitative results, often without arbitrary constants, sometimes accounting for existing experimental results and sometimes predicting in advance the experimental work which was proceeding concurrently. This parallel development has determined the treatment used in the book which treats theory and the experimental comparisons aspect by aspect. The formidable record of success justifies the author's acceptance of Fröhlich's theories, and will probably convince most readers. The absolute values of the electric strength of the alkali halides, the temperature dependence of the strength of such crystalline materials and also of amorphous materials, the variation of the electric strength of mixed crystals, the increase of strength of thin specimens are examples where the theoretical results agree, generally quantitatively, with experiment.

The chapter includes a section, justifiably critical, on the experimental work and some of the associated speculation on conductivity at high stress, polarization and space charges; this part of the subject is notable for the absence of reliable data.

Thermal breakdown, which is the subject of the third chapter, has long been well understood and there is accordingly little new matter. There are, however, a number of elegant generalizations and particular solutions for thin and thick plane specimens and for specimens bounded by coaxial cylinders. The consideration of the possibility of local thermal instability in the vicinity of defects, in common with much other matter in this chapter, is due to the author himself.

A review of early work on breakdown of solids caused by discharges in associated gaseous or liquid media is followed by a fairly full account of E.R.A. work on the subject. This includes the mechanism and sequence of discharges, their detection and measurement, and the relation between such measurements and the life of dielectrics under various forms of applied voltage. Some work has been done on the mechanism of failure due to the external discharges and tracking, but much remains to be explained in these fields.

The chapter on electrochemical deterioration describes some little-known work on deterioration processes arising from electrolytic and chemical processes in impregnated dielectrics at high stress or when damp.

The last chapter on breakdown in practice deals briefly with the bearing of the rest of the book on industrial problems. Working stresses are reviewed and some possibilities of increasing them indicated, though the possibility and magnitude of service overvoltages due to faults and surges must be taken into account. The various kinds of insulation testing found necessary in industrial practice are examined.

The majority of those who consult a book of this kind are likely to be interested in only a particular section, and it is therefore desirable that the individual subjects should be easily comprehensible without the consecutive reading and understanding of whole chapters. For this, the lists of symbols chapter by chapter, together with page when first used, given in the front of the book, are extremely helpful. Although unnecessary for its avowed purpose—the exposition of principles—this casual use of the book would have been aided by the more frequent inclusion of numerical values of the quantities used, e.g. inclusion of values of θ_0, which describes the variation of conductivity with temperature, for the dielectrics listed in the table on p. 123, or the location of a source of such information would be helpful in approximate calculations. Another minor criticism is that the photographs of breakdowns are not always readily comprehensible, e.g. in Figs. 58 and 63 sketches of the specimen showing direction of field, etc., would have been helpful.

The book is a very adequate and necessary review of a subject much of the literature of which is diffuse. And it is a valuable summary of the substantial contributions to it of the Electrical Research Association, both by work in its own laboratories and extramurally in the parallel work of Fröhlich and his associates. It is likely to prove indispensable to engineers who use dielectrics and high stresses, experimenters in the field, and theoretical physicists concerned with the solid state.

A. E. W. AUSTEN

Introductory Nuclear Physics. By D. HALLIDAY. (New York: John Wiley and Sons Inc. London: Chapman and Hall Ltd.) Pp. ix + 558. Price 52*s.* net.

It may be stated at the outset that this book, in the reviewer's opinion, succeeds in its aim, namely that of providing a first text-book for a graduate course in nuclear physics, which may also be used for an advanced undergraduate course by omitting certain material. In general, the historical approach is discarded in favour of conciseness and clarity and this step seems to be justified, although no doubt some readers will find certain cherished material omitted in consequence. The point of view of operational philosophy is adopted, according to which the "irrefutable facts of nature" are recognized as pointer readings and the question of ultimate truth does not arise.

The order in which the material is presented calls for some comment. For example, the author refers much too frequently to later pages of the book. This should not be necessary and suggests that some re-arrangement of the subject matter is desirable in future editions. It is also arguable that Chapter 5, which is purely experimental, dealing with the detection of charged particles and photons, should be relegated to an appendix. The author has made a praiseworthy effort to keep the subject matter up to date and has added a "stop-press" appendix dealing with recent work, which includes the discovery of the neutral meson and the measurement of the half-life of the neutron. A surprising omission is that of all reference to radiation damage although the interaction of charged nuclear radiations and photons with matter has a chapter to itself and the interaction of neutrons with matter forms part of another chapter. A word should, perhaps, be said about two idiosyncrasies in the author's nomenclature which appear to have little to recommend them. He uses the word "negatron" on most occasions instead of "electron" (although he is not consistent in this use) and he describes a beta-ray spectrometer as a nuclear spectrometer.

The chapter headings are: Basic nuclear concepts; Alpha and gamma emission; Electrons in disintegration processes; Interaction of charged nuclear radiations and photons with matter; Detection of charged particles and photons; Neutrons—production, detection, and interaction with matter; Nuclear masses, mass numbers and mass abundances; Nuclear forces; Charged particle accelerators; Nuclear reaction; Nuclear fission; Cosmic rays; Nuclear spin and magnetism. There are two Appendices, one giving various physical constants and the other giving an account of recent developments. There is an adequate index and very few misprints.

J. THEWLIS

Applied Electronics Annual 1951. (Edited) R. B. BLAISE, A.M. Brit. I.R.E. (London: British-Continental Trade Press Ltd.) Pp. 263. Price 40*s.* net.

The sub-title of this book is "International Year Book and Directory for Radio and Electronics," which requires a justification by the contents which is not apparent in this, the first edition. For example, in the list of component suppliers there are 215 British firms listed against 9 American, and only 67 American manufacturers of radio and electronic apparatus are given for 83 British.

The main contents consist of 24 articles by various authors on aspects of electronic developments: printed circuits, B.B.C. equipment, sound recording, television, radar, microwave physics, radioactivity, to name a few, followed by survey articles on the application of electronic devices to industry and some branches of research. With the exception of that on "Electronic Aids to Industry," based on a 1944 survey by the

British Institute of Radio Engineers, there are no appended bibliographies or references, which would be a valuable addition to an annual of this type.

It is in the second part of the book—the Directory—that the reader may feel disappointed. The lists of manufacturers and suppliers are by no means complete, and the information is unbalanced. Pye Radio Ltd. has the laconic note "See advertisement," whereas A. C. Cossor Ltd. have the full entry of directors, addresses and products, in common with most other firms. Only 300-odd trade marks are listed, which must be a very low figure for an international list. Readers of this Journal will find no mention of the *British Journal of Applied Physics*, although the American one is shown. Only 5 radio and electronic publications appear out of the 24 given by Willing's Press Guide. In such a compilation it is obviously important that the contents should appear to be impartially compiled, and it is suggested that the printing of advertisers' names in heavy type is not in keeping with this requirement.

Although there are shortcomings, it is recognized that an immense amount of work has to be put into a work of this kind and it is a creditable attempt on the part of the author and publishers to meet an undoubted demand for a comprehensive reference book for the electronic industry. They have certainly laid the foundation of what may prove to be a standard work if it is expanded in future editions and some of the errors and omissions rectified. G. PARR

Advances in Radiochemistry. By E. BRODA. (London: Cambridge University Press.) Pp. x + 152. Price 15*s*. net.

The book deals primarily with advances since the appearances of the last comprehensive works on radiochemistry, the account being complete up to the end of 1949. Perhaps it should be made clear that the author uses the term "radiochemistry" in the sense used by Paneth, namely "radiochemistry is the chemistry of bodies which are detected through their nuclear radiations." The monograph deals with the production as well as the chemical properties of the radioelements, a knowledge of the basic facts of radioactivity being presumed. Radiation chemistry, that is the investigation of the chemical action of ionizing radiations, has been excluded. These chemical actions can be produced by external irradiation and this field is sometimes misleadingly referred to as radiochemistry, but is clearly a separate branch of science. While this is admirably clear, it does mean that the volume is of interest to only a limited number of people, though this opinion must be modified inasmuch as radiochemical problems in the author's sense are continually arising in fields which make use of radioactive isotopes, for example, tracer elements in medicine. Moreover, most of those dealing with these applications will find the book by Dr. Broda of great use in making them familiar with a field to which they probably have previously devoted little attention. This applies, for example, to the section on radiocolloids.

Generally, it may be said that the bibliography is very good and full, and clearly and admirably arranged. Various chapters deal with the distribution of tracer quantities between solids and liquids, tracer quantities in gases, electro-chemistry of the radioelements, production of radioelements by nuclear synthesis, chemical aspects of nuclear fission and new radioelements of special interest. Here the emphasis has been largely put on the $4n + 1$ radioactive series and on production and properties of the transuranic elements. There are, however, sections on elements 43 and 61, tritium and radiocarbon. The following chapter deals with specific radiochemistry effects, that is, the chemical excitation due to nuclear reactions. Here the emphasis is naturally on the Szilard–Chalmers effect, its yields, energetic aspects and some applications. Finally, there is a short account of new developments in the techniques of radiochemical measurements, involving short accounts of ionization chambers, electroscopes, Geiger counters, radiographic plates in radiochemistry, mass spectroscopy, and ultra-microchemistry of radioelements. The book is an admirable and useful summary of a fascinating field of work. W. V. MAYNEORD

Advances in Electronics, Vol. 3. Edited by L. MARTON. (New York: Academic Press Inc.) Pp. xii + 357. Price $7.50 net.

Volume III of this series has now appeared and follows the pattern of its predecessors. Like its companion of last year, several authors are other than American and so help to maintain an international outlook. Several of the articles are somewhat complementary to each other and are so grouped in what follows.

In "Electronic theory of the plane magnetron," L. Brillouin develops theoretical results which are stated to be applicable to modern magnetrons which, though of cylindrical construction, have a small anode-to-cathode distance. The theory is extended by the same author and F. Bloch in "Electronic theory of the cylindrical magnetron," where differences between plane and cylindrical magnetrons are discussed; apparently experimental work is still needed to verify some of the theoretical results. These two articles are more in the nature of papers by individuals and do not quite fit into the scheme of review articles which it is believed is the intention of this series. A full review of "Velocity modulated tubes" is presented by R. R. Warnecke, M. Chodorow, P. R. Guénard and E. L. Ginzton—a combined effort from the U.S.A. and France. The authors feel that it is now possible to assess the general usefulness of klystrons and have drawn upon all American, British and French information available to them. "Tube miniaturization," by J. E. White, is a short article dealing with subminiature valves. It is written in a very general way and more might have been put into it perhaps; some of the published information on life behaviour would have been of interest, for example. The field of "Subminiaturization techniques" is covered by G. Shapiro and contains useful broad information on all aspects backed by references for fuller details. The author admits that he deals only with American practice. H. F. Mayer provides a lucid account of the "Principles of pulse code modulation" and shows how the system provides a good economic balance between noise-cleaning efficiency and complexity of the transmission apparatus. It is, perhaps, a little surprising that no major contribution to communication theory seems to have been made between Hartley's paper of 1928 and Gabor's in 1946 since when a number of dissertations on the subject have appeared. These are well summarized and their principal points compared by M. Leifer and W. F. Schreiber in "Communication theory." Field-emission microscopy is by no means new and several experimenters have used the technique for studying emission and migration phenomena on electron-emitting surfaces. The art has evidently advanced considerably and the study of fairly large molecules is now possible. The review on "Field emission microscopy," by

F. Ashworth, is a very useful contribution. "A summary of modern methods of network synthesis," by E. A. Guillemin, adequately lives up to its title and deals with the essential principles and procedures together with some illustrative examples. A. J. MADDOCK

Mechanics Applied to Vibrations and Balancing. By D. LANGHARNE THORNTON, M.A. 2nd edition. (London: Chapman and Hall Ltd.) Price 50*s*. net.

This new edition is in the main a reproduction of the first edition (noticed in the January 1940 issue of the *Journal of Scientific Instruments*), with an additional chapter, "Dynamic Loading of Structures," and a rewritten "General Survey."

The author employs generalized coefficients and Lagrange's equations as a basis for the study of the vibration of beams and plates, rotating shafts and disks, mechanisms, and structures. This method will be unfamiliar to many engineering graduates of British universities but the principles are clearly enunciated early in the work where they are applied to the dynamics of the reciprocating engine and to the problem of governor stability. In later chapters the method is developed and the generality of its application to vibrations is demonstrated by examples from all branches of engineering. A chapter on the propagation of stress in elastic materials includes a discussion of stress waves in reinforced concrete, elastic hysteresis and internal friction of metals, water hammer and earthquakes.

The new chapter deals with the transmission of stress under the conditions of rapidly applied loading. Such conditions are met on guns, ships and fortifications subject to the effect of an explosion or impact, and also occur in the design of dams and other structures for seismic regions, of drives for rolling mills and paper-making machinery, and in certain metal-forming operations. This addition will obviously be of value to engineers and others concerned with the design of such plant. Elastic strain waves due to axial, tranverse and torsional impulses on uniform forms are first investigated and the theory is then extended to cover plastic behaviour. There is at the present time a lack of information on the behaviour of metals under this type of loading and the author summarizes such research as has been done and indicates profitable lines which could be followed.

The General Survey briefly recapitulates the general method of vibration study and instances practical examples from the experience of the author and others. The theory of stress waves in a turbine disk and of the pendulum absorber for torsional vibrations is covered. There is also an informative table of the measured frequencies of hull vibrations for ships of different types and displacements.

The book is written primarily for engineers, and will be of particular interest to civil engineers since most vibration literature tends to ignore their field. It should also be useful to the applied physicist. J. S. RANKIN

Experimental Spectroscopy (2nd Edition). By R. A. SAWYER. (New York: Prentice-Hall Inc.) Pp. x + 358. Price $6.65 net.

The English first edition of this book, published by Chapman and Hall, appeared in 1945, and is well known to spectroscopists and physicists in this country. In the past six years many advances in technique have been made and numerous papers on all aspects of the subject have appeared. These have been faithfully dealt with by the author in extensive additions and much re-writing. As instances at random one may mention the references to new methods of grating and replica manufacture, the use of photomultiplier tubes, improvements in vacuum ultra-violet technique, and synthetic crystals. The bibliography has been greatly enlarged and the paper and printing much improved. In short, it is a comprehensive and thoroughly up-to-date treatise, invaluable to any worker in this field, now under such intensive cultivation, especially on the applied side.

All things considered, it is not surprising that the price is also appreciably augmented ($6.65, previously 25*s*.). But it cannot be regarded as excessive. W. E. CURTIS

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SPECIAL ARTICLE

Physical aspects of the biological action of ionizing radiations

By JOHN READ, B.Sc., Ph.D., British Empire Cancer Campaign (New Zealand Branch), Box 373, Christchurch, New Zealand

The influence of such physical factors as the linear ion density created by different radiations, the presence of dissolved oxygen or certain other molecules, and the dose rate, on the production of biological effects by ionizing radiations, is discussed. Certain radiochemical experiments are outlined which show similarities with radiobiological experiments. Hypotheses are suggested which attempt to correlate and explain some of the biological observations.

X-rays and the rays from radioactive substances were discovered about 50 years ago. Accidents (such as the well-known Becquerel burn) soon showed that they had a biological effect, and the medical profession quickly explored their utility in the treatment of numerous diseases. Disappointing results caused a progressive contraction of the list until malignant growth and a few other diseases were left whose treatment by radiations was of value. Although radiotherapists have not neglected to draw on the relevant branches of science for the improvement of their technique, nevertheless, radiotherapy has to a large extent remained an art—a body of empirical knowledge developed and passed on by each generation of practitioners. The mechanisms by which ionizing radiations exert their biological effects are still little understood. On the one hand the physicist can give a fairly detailed picture of the absorption of the radiations in human tissue, and of the production there of ionization and of excited molecules. On the other hand, a great body of information on the gross effects produced in human tissues has been accumulated. The desire is to close the gap so that each link in the chain of events from the ionization to the observed effects is understood. In the early days experimenters rather naturally started as near the end point as possible, and studied the effects produced by irradiation of patients and animals. Realization of their immense complexity in subjects so highly organized led to the choice of ever simpler test material—tissue cultures, eggs, sperm, protozoa, bacteria, viruses, enzymes, inorganic solutions, down to pure water. The hope has been that if events immediately following ionization can be explained at the simplest level they will be recognized at the next higher level of complexity, disentangled, and the residuum explained, so working back up the ladder until man is again reached. The discovery of radiation induced mutations has strengthened this policy, since they are of great practical importance, yet seem to fit into the scale of complexity at a considerably lower level than the gross effects on man. Workers in numerous branches of science have been drawn into a study of this problem, and it is illustrative of our ignorance that each group has suggested hypotheses drawn from their own speciality. Some, medically trained, have drawn on pharmacology, and stressed the similarities between the effects of radiations and of drugs; the biochemist has suggested explanations based on interference with enzyme systems; while the physicist has tended to base his explanations on the random nature of the distribution of ions along the tracks of the ionizing particles. To-day there are some points of contact between the views of the different groups, which prompt the hope that there may be a synthesis within a time short compared with that spent on the earlier studies. As this paper deals with physical aspects it will largely describe the approach of the physicist. Such description must necessarily start from the book of D. E. Lea (1946),[1] which sets out the position at that time so admirably that no good purpose can be served by repetition here. We shall be content to outline subsequent work which has a bearing on his views.

THE INFLUENCE OF LINEAR ION DENSITY

An important contribution of the physicist has been the comparative study of the biological effects of radiations which produce different linear ion densities along the tracks of the ionizing particles, i.e. gamma rays, X-rays, protons recoiling from fast neutrons, alpha rays, and uranium fission fragments. Perhaps, however, the earliest experiment of this type was that of Zirkle (1935, 1940),[2] a biologist, who irradiated cells with diameters less than 5 μ with alpha rays which had a range in tissue of 32 μ. By covering the source with screens different portions of the Bragg ionization curve were intercepted in the cells, so that the effectiveness of a certain number of ions when aggregated into a greater or less linear ion density could be obtained. X-rays were also used to produce a much smaller linear density along the electron paths. He found that whereas the effectiveness of a fixed energy absorption increased with increase in linear ion density in the case of spores of a mould, *Aspergillus niger*, and of vegetative cells of a yeast, *Saccharomyces cerevisiae*, it decreased when *Bacterium coli* was the test material. It has seemed that this difference in behaviour could serve as a useful classification for all the biological effects studied.

Class A. Responses which occur with diminished efficiency when the linear ion density is increased. The inactivation of viruses and phage particles by irradiation under conditions which ensure that the effect is only due to ionization produced within the particles, and not in the surrounding medium, is an example of this class. The

active fraction diminishes exponentially with increase in dose, and neither dose rate nor temperature has an influence. The hypothesis in explanation, known as the "target hypothesis," started rather vaguely, but was made precise by Crowther (1924),[3] who suggested that a single ion pair left within some sensitive volume in the organism, would produce the effect, from which there was no recovery. His Silvanus Thompson Memorial Lecture read before the British Institute of Radiology in 1938[4] is an excellent exposition of his views. Suppose our organisms each have a sensitive volume v, and we subject them to a dose of N ion pairs/c.c. aggregated in tracks with mean specific ionization s, and suppose the mean length of track intercepted by a sensitive volume is p. Then the probability that a track will pass through a sensitive volume (geometrically speaking) is vN/sp. All passages may not be effective; suppose the probability of producing the effect by a single passage is ϕ. Then $P = \phi vN/sp$, and the fraction of organisms unaffected by the dose N is $\exp(-\phi vN/sp)$. v/p may be written A, the mean target area, while N/s may be written L, the total length of all the tracks per c.c. Lea,[1] having adopted the assumption that a single ion pair left within the sensitive volume produces the effect, has developed an elegant relationship between ϕ and sr, where r is the radius of a sphere equivalent to the sensitive volume, based on the fact that the ions are produced along the tracks in a Poisson distribution. The greater the value of sr the less efficient is the radiation, because frequently more than one ion are left in the volume, and those in excess of one are wasted. He has outlined several conditions which a response should fulfil in order that this hypothesis may be applied to it with confidence. Some authors have explained sigmoid survival curves on rather similar lines, but with the assumption that a larger number of "hits" are required (as did Crowther) (1926),[5] in his explanation of the survival curve of *Colpidium colpoda*. Generally to-day, however, it is considered that such an extension of the argument is not justifiable without supporting evidence of a different character, as any likely shape of survival curve can be fitted by a suitable choice of the necessary number of "hits." When the hypothesis applies, from the survival curve for any one radiation the total sensitive volume in an organism can be calculated. If curves are available for more than one radiation, which differ in linear ion density rather widely, e.g. gamma rays and alpha rays, then it can be deduced whether this volume forms one sensitive zone, or whether it is the aggregate of a number of smaller volumes in any one of which an ion pair is effective. In this way Lea showed that a considerable number of smaller viruses had a single sensitive volume which approximated to the whole volume of the particle as measured by other methods.

A number of biological effects (of Class B below) are known which are much increased by the presence of dissolved oxygen. Hewitt and Read (1950)[6] tested whether the X-ray sensitivity of a *B. coli* phage (to which Lea's hypothesis should apply) depended on the surrounding oxygen tension, by giving X-ray doses to the phage in broth which contained on the one hand about 15 c.c. oxygen/l., and on the other virtually none. The two inactivation curves differed very little, and from them was deduced by Lea's method that the particles had a diameter of 54 milli μ. Electron micrographs of the same phage particles have shown that the organism consists of a headpiece with mean diameters 65 and 85 mμ with a tail 100 mμ long. The mean diameter from an ultracentrifuge measurement was 45 mμ. These results fit Lea's picture very satisfactorily, and it seems reasonably safe to conclude that the Class A type of effect is not affected by the presence of oxygen. However, Lea applied the same hypothesis to the production of sex linked recessive lethal mutations in Drosophila, and concluded that the X chromosome contained some 800 genes of diameter about 6 mμ. Baker and Sgourakis (1950)[7] have X-rayed male Drosophila in either a stream of nitrogen, or of oxygen, and studied the genetic effects produced. The recessive sex linked lethal mutations were increased by the oxygen by a factor of 2 to 3. Dominant lethal mutations were also increased. Furthermore, Luce, Quastler, and Skaggs (1949)[8], and Regehr, Arnason, and Johns (1950)[9] have compared the efficiency of production of sex linked recessive lethal mutations in Drosophila by X-rays from a betatron (maximum quantum energies 20 and 32 MeV) with that of 200 kV X-rays. They were only about 0·6 times as efficient, whereas the specific ionization along the tracks should be less. Lea applied his hypothesis to this biological effect because the efficiency of alpha rays, fast neutrons, soft X-rays, and hard X-rays or gamma rays, increased in that order, i.e. in this range the less the linear ion density the greater the efficiency. It now appears that the efficiency passes through a maximum, and this effect, both in this respect and in the influence of oxygen, comes into line with the Class B effects below.

Class B. Responses which occur with increased efficiency when the linear ion density is increased. Gray and others (1940)[10] listed some 20 effects, grouped into (i) production of chromosome abnormalities, (ii) reduction of cell division subsequent to irradiation, (iii) reduction of growth of plant roots and shoots, and (iv) immediate or delayed death, to produce which fewer ion pairs per c.c. were necessary when created by particles with a greater linear ion density. The relative efficiency of two radiations has been denoted by $X/N, \gamma/\alpha, \ldots$ where X, and $N \ldots$, are the numbers of ion pairs per cc necessary when created by X-rays, fast neutrons, etc., to produce the effect in question. This ratio can be surprisingly large. For example γ/N is of the order of 20 for the regression of grafted mouse tumours—Gray and Read (1948),[11] and of 30 for the killing of mice when the irradiation is spread over 48 h—Mitchell (1947).[12] On Fig. 1 the relative efficiencies (with gamma rays taken as 1) for a number of effects are plotted against

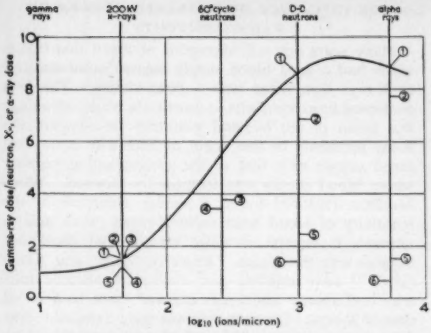

Fig. 1. The abscissa scale gives the logarithm of the mean specific ionization in ions per micron. Ordinates are erected at values characteristic of the radiations used, which are given at the tops of the ordinates.

The ordinate scale gives the ratio of the number of ion pairs per c.c. necessary to produce the chosen biological effect when gamma rays are used, to the necessary number when created by the radiation in question.

The biological effects are denoted by numbers:—

1. Lethal effect on the broad bean root.
2. Isochromatid breaks, *Tradescantia bracteata.*
3. Human skin reaction.
4. Lethal effect on tumour tissue irradiated in vitro.
5. Simple chromatid breaks, *Tradescantia bracteata.*
6. Reduction in the proportion of cells in division in tips of the broad bean root.

For clarity the points have been displaced left and right of the appropriate ordinates, to which they are connected by short lines. A curve is drawn through the points 1 only; the course of the other curves can be judged by eye

the logarithm of the mean linear ion density. Two curves pass through a maximum and the shapes of two others suggest that a maximum would be attained with a rather larger linear ion density. We have seen above that a similarly shaped curve now seems to fit the production of sex linked recessive lethal mutations in Drosophila. The maximum occurs at a different density for each particular effect. Zirkle, as long ago as 1940 (loc. cit.),[2] suggested what amounts to the following in our nomenclature. We had, on the target hypothesis, that the probability P of producing an effect $= \phi AN/s$.

Therefore $$\frac{dP}{ds} = \frac{AN}{s}\left(\frac{d\phi}{ds} - \frac{\phi}{s}\right)$$

The probability of an effect only increases so long as $d\phi/ds > \phi/s$, i.e. the increase in probability that a single passage through the sensitive volume will produce the effect must more than compensate for the diminution in the track length per c.c. caused by the increase in s. In the case of Class A results the bracketed expression is always negative. This view pre-supposes that the Class B effects are also caused by a kind of target mechanism, but that a considerable number of ions are necessary. We have other evidence that in at least two of the Class B effects a target mechanism is involved:—

(i) *The production of chromosome aberrations.* (Readers unfamiliar with the progress of cell division through prophase, metaphase, anaphase, and telophase in the case of somatic cells, and of the analogous behaviour of the reproductive cells, the appearance of the chromosomes in early prophase and their splitting into two chromatids, etc., should consult *The Chromosomes* by White, Methuen's Monographs on Biological Subjects. A description for the general reader of the production of chromosome and chromatid breaks by radiation, and of the joining of the broken ends to produce aberrant types, has been given by Catcheside (1947).[13])

If at times after a dose of radiation cells are fixed, stained to show up the chromosome threads, and inspected under the microscope, it is observed that breaks have been produced in the threads which are not seen in the unirradiated controls. If a chromosome had divided into two chromatids at the time of irradiation then a break may be seen in one and not in the other. Furthermore, a composite thread may be seen which has resulted from the breaking of two separate threads and the joining of their broken ends. When graded doses of X-rays are given in the same period of irradiation simple breaks increase in number in direct proportion to the dose. However, the aberrations which require two breaks each, increase in proportion to the square of the dose. It is inferred that the breaks are produced by the passage of a single ionizing particle through (or perhaps near) to the thread. Kotval and Gray (1947),[14] from a study of the breaks produced by alpha particles in the chromatids of *Tradescantia bracteata* concluded that if the "target" were a thread of length equal to that of the chromatids then its width must be $0{\cdot}22\,\mu$, compared with the usually accepted diameter for the chromatid of $0{\cdot}1\,\mu$. This suggests that a "near miss" may produce the break; in fact all breaks may be produced in a similar way, since particles which pass through the thread ionize the nuclear sap on the incident and emergent sides of it. Unlike X-rays, the number of two break aberrations produced by alpha particles increases with the first power of the dose (as do simple breaks). It is concluded that ϕ for X-rays is small, and it is unlikely that an electron, having broken one thread, will break a second. However L is large so that the chance of two breaks each produced by a different electron, is appreciable. In the case of alpha rays ϕ approaches unity, whereas L is small, and the two break aberrations largely result from breaks in two threads (or from two breaks in the same thread, which is more or less coiled) traversed by the same particle. Thoday (1947)[15] has studied the aberrations produced by both X-rays and alpha rays in the broad bean root and they show the dependence on dose outlined above. Alpha rays were considerably more efficient than X-rays. Lea (1946)[1] has deduced that a minimum of between 15 and 20 ions must be left in a *Tradescantia chromatid*

by a particle in order that the probability of producing a break shall be appreciable. If the effectiveness of a near miss be accepted then the 15 to 20 ionizations in the thread $0{\cdot}1\,\mu$ in diameter really characterizes a minimum linear ion density of some 200 ions per micron. We consider that something similar must apply in the case of the broad bean root. Two of the curves of Fig. 1 relate to *Tradescantia chromatid* injuries. The bean root curve is for the gross lethal effect on the roots, and though we would not claim that the production of chromosome aberrations is the sole cause of death (it certainly does lead to the death of a proportion of the cells) we have found that the gross effect and the production of aberrations run parallel.

(ii) *The reduction in the number of dividing cells.* It has been observed in many types of tissue that after irradiation the proportion of cells seen in division first falls, and then, if the dose were not too large, increases again to the normal value, or even larger. Furthermore, after a dose of gamma radiation the proportion in tips of the broad bean root fixed 3 hours after the dose decreases exponentially with increase in dose. The fall and recovery have not been studied at a range of times after neutron and alpha-ray irradiations, but at 3 hours the proportion of cells seen in division shows the same exponential dependence on dose—Gray and Read (1950).[16] The neutron exponent was $3{\cdot}5$ times greater than that for alpha rays while, as a result of the smaller specific ionization, the track length of protons for the same number of ion pairs per c.c. was also about $3{\cdot}5$ times greater. Until the way in which the number of dividing cells changes with time after doses of neutrons and alpha rays has been determined we cannot argue with complete confidence, but these results suggest that we are dealing with a target phenomenon, that ϕ is about unity for both radiations, and that the smaller efficiency of alpha rays is due to the smaller value of L.

Because the production of sex linked recessive lethal mutations in Drosophila shows the same type of maximum we conclude that an appreciable proportion are not so called "point gene mutations" resulting from a single ion cluster within a gene. It has already been concluded on genetic grounds (Muller, 1950)[17] that a proportion must be associated with the joining of broken chromosomes. It seems likely that a number of ion pairs are necessary to break a Drosophila chromosome (as for *Tradescantia*), so accounting for the maximum. That this maximum occurs at a linear ion density so much smaller than in the case of *Tradescantia* may be due to a smaller number of ion pairs being necessary, and perhaps also because there is a proportion of point gene mutations which shifts the maximum towards smaller values.

The lethal effect of whole body irradiation of animals and the response of human skin and tumours (as shown by gamma ray, X-ray, and neutron therapy) are all Class B effects.

THE INFLUENCE OF DISSOLVED OXYGEN ON RADIOSENSITIVITY

Many years ago radiotherapists observed that tissues which had a good blood supply seemed more sensitive to X-rays than those with a poor supply. This was confirmed by experiments on animals in which advantage was taken of the bilateral symmetry to compare the X-ray sensitivity of one organ or limb with its normal blood supply with that of the corresponding member whose blood supply was temporarily blocked. When Mottram (1935)[18] found a similar difference in the sensitivity of broad bean roots X-rayed in air and in nitrogen it seemed virtually certain that molecular oxygen was the agent. Recently Storer and Coon (1950)[19] have injected *para-aminopropiophenone* into mice and shortly afterwards exposed them to a lethal dose of X-rays. The mortality was much reduced. This drug converts some 50% of the haemoglobin into methaemoglobin, in which the oxygen is more firmly bound. The treatment would therefore cause a drop in the concentration of molecular oxygen in the tissues. Furthermore Dowdy, Bennett, and Chastain (1950)[20] took advantage of the fact that rats would recover after 8 mins in a 5% oxygen supply to give them X-ray doses in this period. The mean lethal dose was increased by a factor of between 2 and $2{\cdot}4$. An injection of NaCN will block the action of the respiratory enzymes, causing the same subsequent metabolic changes as the oxygen starvation, yet without reducing the molecular oxygen in the tissues. They report that this treatment had no protective effect. (This would be a point of great significance, but it is weakened by the fact that Bacq, Herve, Lecomte, and Fischer (1950)[21] report a considerable protective effect of NaCN injected into mice just before X-irradiation.)

Read (1951)[22] has studied the influence of oxygen in the water in which groups of broad bean roots were immersed during an X-irradiation. He showed (i) that each oxygen concentration acted as a simple multiplying factor on the dose. The manner in which the factor depended on the concentration is shown in Fig. 2. A dose given with 20 c.c. oxygen/l has the same effect as a dose almost 3 times larger given under oxygen-free conditions, so that this is a very pronounced effect; (ii) that oxygen supplied 37 sec after a 2 min irradiation in oxygen-free water had no effect; (iii) that when a sudden change was made in the oxygen concentration before a dose given in 13 sec, the radio-sensitivity was adjusted to the appropriate new value with a lag (if any) certainly not greater than 1 min; (iv) that the histological appearances of sections of roots X-rayed with and without oxygen were qualitatively the same, but the reduction of cell division was greater and there were many more abnormal dividing cells when oxygen had been present. The measure of effect on which (i) to (iii) were based was the ratio of the mean growth of the irradiated group of roots in the 10 days after the dose, to the growth of an untreated control group.

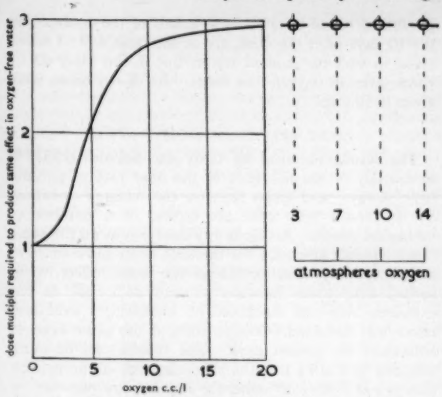

Fig. 2. The ordinate scale gives the ratio of the X-ray dose necessary to produce a chosen reduction in the growth of a group of broad bean roots when irradiated in oxygen-free water to the dose necessary when the water contains the concentration of oxygen given by the abscissa scale.

Abscissa scale 0–20:—c.c. oxygen/l in the water in which the roots were immersed. Normally aerated water at 19° C contains 6·4 c.c. (N.T.P.) oxygen/l.

Abscissa scale 3–14:—atmospheres pressure of oxygen in which the moist roots were suspended.

Thoday and Read (1947)[23] gave two sets of bean roots 143 roentgens* of X-rays, one set in water containing less than 0·2 c.c. oxygen/l, and the other in water containing 20 c.c. oxygen/l. Tips were fixed at intervals up to 72 h after the dose, and the percentages of anaphase cells which contained chromosome bridges and fragments (see Catcheside, 1947)[13] were determined. Many more were observed in the case of irradiation under oxygen-rich conditions; in fact the areas under the curves of Fig. 2 are in the ratio 3·5 : 1. Perhaps a better comparison can be based on the argument that if an average of m breaks are produced per cell then the proportion of cells which escape breaks is $\exp(-m)$. Therefore the curves of Fig. 2 can be converted into curves of m. The areas are then in the ratio 4·4 : 1. From our Fig. 2 we conclude that the doses of 143 r should be equivalent to approximately 59 r and 167 r given in normally aerated water in their effect on growth. From Thoday's (1947) study we compute that 59 r produce 66 single plus 6 two-break aberrations per 100 cells, total 78 breaks; while 167 r produce 186 single plus 48 two-break aberrations, total 282 breaks. These numbers are in the ratio 3·6 : 1, compared with the 4·4 : 1 above. It seems that the factors by which break production is increased by oxygen cannot differ very much from the factors derived from the gross effect on the growth of the roots.

An analogous experiment was described by Thoday and Read (1949),[24] but alpha radiation was used instead of X-rays. The published doses need revision, but they were undoubtedly nearly equal. The areas under the curves of Fig. 2 of that paper are in the ratio 1·21 : 1. Conversion to "m" curves as we did before can make little difference in this case, as the largest value of m involved is 0·64. The ratio obtained in a parallel study based on the growth of the roots was 1·22 (Read, 1951).[25] Here, again, there is a close correspondence between the effect of oxygen on the production of chromosome aberrations and its effect on the reduction of growth of the roots.

Giles and Riley (1949),[26] (1950),[27] and Giles and Beatty (1950),[28] have studied the effect of oxygen in increasing the X-ray production of chromosome aberrations (interchanges and interstitial deletions) in *Tradescantia*. Buds were irradiated for 1 min, either *in vacuo* or in 2 atm. pressure of oxygen, and then, within a matter of seconds the vacuum was replaced by 2 atm. of oxygen, or vice versa. These changes immediately after the irradiation had no influence. When the change was made at the mid point of the irradiation an intermediate number of aberrations was produced. It appears that the oxygen must co-exist with the radiation in order to exert its influence (compare the similar results obtained with the bean root, above). The number of aberrations produced by a certain dose depends on the percentage of oxygen in the atmosphere surrounding the buds in the manner shown in Fig. 3, curve 1 (drawn from the data of Giles and Riley, 1950).[27] If oxygen here acts as a simple multiplying factor F on the dose then, because these aberrations are two-break types curve 1 is a curve of F^2. From it curve 2 is derived by taking square roots. There is a striking similarity between these curves and Fig. 2 derived from a study of the reduction in growth of the bean root.

The values of F for a very varied group of X-ray effects all lie between 2 and 3, when the oxygen change in the surroundings is from pure oxygen, or normal aeration, to near zero:—Reduction in growth of the bean root (above) 2·8; mean lethal dose for rats, Dowdy and others[20] 2·2; skin reaction of rats, Evans, Goodrich and Slaughter (1942)[29] 2·2; reduction in the growth of plants grown from X-rayed germinated barley seed to 60% of the control value, Hayden and Smith (1949)[30] 2·7; lethal effect on yeast cells, Anderson and Turkowitz (1941)[31] 2·2; production of sex linked recessive lethal mutations in Drosophila, Baker and Sgourakis[7] 2·5. Perhaps it may be argued from this that the effect of oxygen is intimately connected with the ionization rather than with the biological factors involved.

* The roentgen is defined as "that quantity of X- or gamma-radiation such that the associated corpuscular emission per 0·001293 g of air produces, in air, ions carrying 1 electrostatic unit of quantity of electricity of either sign." One roentgen (written 1 r) produces from 1·6 to 1·9 ion pairs per cubic micron of wet tissue in the wavelength range 8 to 0·015 A.

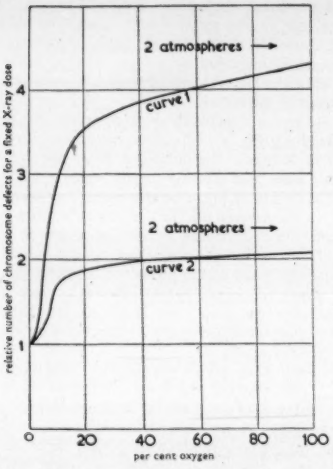

Fig. 3. Curve 1:—Ordinate scale: Relative numbers of interstitial deletions and interchanges produced in the chromosomes of *Tradescantia bracteata* by a certain X-ray dose when the buds were irradiated in a mixture of helium and oxygen at a pressure of 1 atm.

Abscissa scale: Percentage of oxygen in the mixture.

Drawn from the data of Giles and Riley (1950)[27].

Curve 2:—Square roots of the relative numbers given by curve 1.

From the alpha-ray experiments of Thoday and Read (1949)[24] and of Read (1951),[25] as a consequence of the relatively long half lives of radium *B* and *C*, some 60% of the dose was given to the roots during their immersion in water of chosen oxygen concentration (20 c.c. per litre or near zero), and the remaining 40% while they were in normally aerated water. If we assume that the same factors *F* apply as were obtained with X-rays, Fig. 2, each to its own fraction of the dose, then an overall factor of 1·72 is to be expected, compared with the observed 1·22. The difference corresponds to 3·6 times the standard deviation. We conclude that oxygen has a much less effect on an alpha ray dose than on an X-ray dose, in fact we estimate that if the whole alpha ray dose had each been given under its chosen oxygen conditions a factor of 1·38 would have been obtained compared with 2·82 for X-rays. When we consider that about 25% of the alpha ray ions are actually in the delta rays, which are more akin to X-rays ionization than to the primary alpha ray track, it seems likely that the biological effect of the primary alpha ray track is little affected by oxygen, if at all.

Evidently the efficiency ratio X/α depends on the oxygen concentration. Doses which reduce the mean growth of broad bean roots to a half of the controls, in the 10 days after the dose, are in the ratio 4·9 : 1 when given in well oxygenated water, but in the ratio 10 : 1 when given in oxygen-free water. All X-ray doses were given in 10 min.

THE EFFECT OF DOSE RATE

The results obtained by Gray and Scholes (1951)[32] in a study of the influence of the dose rate of gamma rays, X-rays, and alpha rays on the damage sustained by the broad bean root are typical of a number of biological effects. An alpha ray dose produced the same result whether given in a few minutes, or spread over 24 h. A gamma ray dose produced the same effect when spread over times between 12 and 24 h, but as the exposure time was shortened an increasingly enhanced effect was obtained. X-rays showed the same type of behaviour as gamma rays. The results can be summarized by stating that the effect depends on an expression $A \,.\, D + B(t) \,.\, D^n$ where A and $B(t)$ are parameters, A depending only on the type of radiation while $B(t)$ depends also on the duration of irradiation, t, and diminishes to zero as the time increases. D is the dose and n an exponent considerably greater than unity. $B(t) \,.\, D^n$ is negligible compared with $A \,.\, D$ in the range of alpha ray doses used, but in the X-ray case it is only negligible either when t is great, or when D is small (and the degree of damage chosen as the criterion of effect is small). Evidently the efficiency ratios γ/α and X/α depend on the duration of exposure and degree of damage chosen as criterion. $X/\alpha = 12{\cdot}5$ when the maximum reduction in growth rate is 10% after an X-ray dose of 30 r given in 8 minutes, but falls to 4·5 when the reduction is 70% caused by 105 r, also given in 8 minutes. When the duration of irradiation is $\sim$ 12 hours X/α is 12·5 for all degrees of damage. It is tempting to identify the two components $A \,.\, D$ and $B(t) \,.\, D^n$ with the production of the single and two hit types of chromosome aberrations, but the relative numbers observed by Thoday (1947)[15] do not support this view, and we would have to assume that breaks in the bean root chromosomes remain open for hours compared with an estimated mean of 4 min in the case of *Tradescantia*. Nevertheless, it seems reasonable to conclude that the $A \,.\, D$ component results from effects caused by a single ionizing particle, and the other from the co-operative effect of more than one, which must follow each other at time intervals not too widely separated.

The mean lethal doses of neutrons and gamma rays used in whole body irradiation of small animals shows a similar behaviour. As the duration of irradiation is prolonged γ/N increases to a maximum.

CHEMICAL EFFECTS OF RADIATIONS. THE INDIRECT EFFECT

Bonet-Maury and Lefort (1950)[33] have studied the production of H_2O_2 when water is irradiated. Alpha

rays produce 0·27 molecules per ion pair (denoted by $M/N = 0{\cdot}27$) irrespective of whether dissolved oxygen is present or not, nor does the yield depend on the dose rate. They state that no H_2O_2 is produced by X-rays in pure oxygen-free water, though other workers have reported a very small production ascribed to the densely ionized "tails" of the electron tracks, which simulate alpha ray track (Allen, 1948).[34] When the water contains oxygen X-rays produce H_2O_2 in much greater quantities. As the irradiation is prolonged the amount approaches an equilibrium value, because the X-rays also promote a back reaction which destroys the H_2O_2. M/N depends on the circumstances, ranging from rather more to rather less than the alpha ray value. However, if the reagents used for the titanium colorimetric determination are present during the X-irradiation the yield is considerably increased, M/N approaching 0·9. The yield varies strongly with pH. None of these circumstances affects the alpha ray yield. We conclude that the reactions which lead to H_2O_2 take place in a narrow sheath surrounding an alpha ray track in which few of the foreign molecules are present, and there is no co-operative effect between tracks. In the X-ray case the products of the ionization disperse through the bulk of the liquid before the reactions are completed.

Weiss (1944,[35] 1947[36]) has proposed an explanation for these and other radiochemical reactions in aqueous solutions, and Lea (1947)[37] has elaborated it. H_2O^+ ions are produced along the track of an ionizing particle. The ejected electrons may make another ion or two until their energies are below 12·6 eV, the lowest ionization potential of water. They can then excite electron levels at ~ 6 eV, and rotational levels at $\sim 0{\cdot}4$ eV. Presumably thereafter their energy can only be reduced to thermal values by elastic collisions. Eventually this migration is terminated by their capture to form H_2O^-. Almost immediately after formation both types of ion dissociate to produce in the one case $H^+ + OH$, and in the other $OH^- + H$. The following reactions (with others) may then occur:—

(1) $H + OH = H_2O$; (2) $H + H = H_2$;
(3) $OH + OH = H_2O + O$; (4) $O + O = O_2$;
(5) $H + O_2 = HO_2$; (6) $HO_2 + HO_2 = H_2O_2 + O_2$;
(7) $H_2O_2 + H = H_2O + OH$.

The energy expended per ion pair is over twice the ionization potential so that probably about as many excited molecules as ions are formed. These can also dissociate to $H + OH$ but, as they are formed in close proximity and perhaps held together by the surrounding cage of molecules, they are generally assumed to recombine. It is the migration of the ejected electrons from their parent atoms so that the H and OH can be formed in different localities, which enables the competitive reactions to occur. The average spacing of OH radicals along an alpha ray track is about 0·25 millimicrons (m μ), while along an electron track resulting from X-rays it is about 10 mμ (Gray, 1947).[38] In normally aerated water the average distance between oxygen molecules is 36 mμ. Read (1949,[39] 1951)[40] has suggested that the H atoms may be formed closer to the axis of the track of an alpha particle than of an X-ray electron, because the initial migration of the electrons creates an electric field, which is very great when the specific ionization is great, and the electrons may be drawn back somewhat. These various factors would seem to co-operate in making reactions (3) to (6) go so efficiently with alpha rays that dissolved oxygen has a negligible influence. At best, on the above scheme, 0·5 molecules of H_2O_2 per ion pair can be expected. Bonet-Maury and Lefort found 0·27 (alpha rays) while Allen[34] gives a production of 2 molecules of H_2 per 100 eV energy absorbed, which just about accounts for the rest of the H atoms. The much greater X-ray spacings makes the sequence of reactions (3) to (6) improbable, while any H_2O_2 produced in the "tails" will be in danger of destruction by reaction (7) from the H atoms produced by all the rest of the ionization. The relative isolation of the H atoms, however, favours an effect from dissolved oxygen by reactions (5) and (6).

It seems probable that one or more of these very reactive products H, OH, O, HO_2, H_2O_2, can be identified with "activated water" which has been postulated to explain the results of earlier radiochemical experiments (Fricke and Hart, 1935).[41] Dale's experiments (1940,[42] 1947)[43] are examples of great radio-biological importance. When aqueous solutions of pure crystalline carboxypeptidase were X-rayed the active fraction diminished exponentially with increase in dose. The exponent was inversely proportional to the initial concentration of the enzyme. It appeared that some inactivating agent was produced in the water in proportion to the dose, which could be collected by the enzyme molecules with equal facility whether they were still active or not. When one of certain other substances was also present the quantity inactivated was reduced. The substance exerted a protective effect by competing with the enzyme for the active principle. The inactive enzyme may be regarded as protecting the still active fraction, so causing an exponential instead of a linear dependence on dose. Previous experiments on the X-ray inactivation of enzymes had been done either with very concentrated or impure preparations (i.e. containing protective substances as impurities) and, as the inactivation doses under these circumstances were far greater than caused severe damage to cells it was concluded that this damage could not result from interference with enzyme systems. Dale's work shattered confidence in this argument as, with dilute solutions yet of the same order of concentration as those within cells, the necessary doses were well within the therapeutic range. The question now remains whether the protoplasm contains a large enough concentration of protein to exert a protective effect, without itself being changed by this role in a manner lethal to the cell.

DISCUSSION

Explanations of the effect of oxygen on the X-ray sensitivity have taken two main lines. (1) The oxygen alters the metabolic state of the cell to one of greater sensitivity. Crabtree and Cramer (1933)[44], for example suggested that "the functional activity and chemical condition of the respiratory mechanism with respect to its state of oxidation, reduction, or combination play a significant role." (2) The oxygen reacts with the immediate products of the ionization to produce a toxic product, the resulting damage being little dependent on the metabolic state. Crabtree and Cramer's view was based on the fact that whereas anaerobiosis reduced radiosensitivity, HNC and cold increased it. This increase would here be ascribed to the rise in intracellular oxygen tension on blocking its consumption. Perhaps also the inhibition of catalase by HCN plays a part. Dry seeds are very radioresistant compared with soaked seeds. This can be attributed to the speeding up of metabolism by the soaking or, on the other hand, to the creation in the water of the H and OH. Living organisms are, generally speaking, 80 to 90% water. The remaining material is largely aggregated into structures, so that we visualize that over regions large compared with the distribution of the ions along a track we may have pockets of very nearly pure water, in which the reactions (1) to (7) above occur. The role of oxygen, on this view, is to compete with some harmless reaction, e.g. $H + H = H_2$, and divert the H into the production of HO_2 or H_2O_2. If the track passes near to some important cell structure, such as a chromosome, irreparable damage may be done. If it does not, and the products are dispersed by diffusion, then perhaps the protective action of solute molecules and the destruction of the H_2O_2 by catalase can avert an effect. This would reconcile radiochemical hypotheses with our desire to retain something from the target hypothesis. We visualize that all of an alpha ray track can be biologically effective, but that the X-ray effect when no oxygen is present is due only to the densely ionized "tails" of the electron tracks, which simulate alpha ray track in their indifference to oxygen. Read's value for X/α of 10 for the bean root (without oxygen) suggests that about 10% of the X-ray ions can act in this way and the rest are wasted. Gray and Scholes found $X/\alpha = 12{\cdot}5$ with very prolonged X-irradiations in normally aerated water. Here 8% of the X-ray ions behaved like alpha ray ions; perhaps the 10% and 8% are measures of the same group. This view implies, however, that oxygen augments the $B(t) \cdot D^n$ component only, which is not easy to reconcile with Read's claim that oxygen acts as a simple multiplying factor on the first power of the dose, nor with the observation of Thoday and Read that chromosome aberrations are increased by oxygen by about the same factor. Further experiments are necessary. We quote Crowther (1938):[4] "An hypothesis," as my old master, Sir J. J. Thomson, used to remind me, "is a policy, not a creed."

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ORIGINAL CONTRIBUTIONS

The stretching and relaxing of polyethylene

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By using an X-ray beam of very small cross-sectional area, a series of X-ray diffraction patterns has been obtained over the region of the neck that is produced when randomly oriented polythene is stretched. These indicate that the orientation over the region of the neck produced by stretching at room and lower temperatures is abnormal in that it does not consist of a gradual alinement of the polymer chains with the direction of stretch, but that at intermediate stretches the preferred orientation is such that the (011) axis is parallel to the stretch direction. At higher stretches, however, there is a gradual tilting of the crystallites until they are finally alined with their chain axis parallel to the direction of stretch. At temperatures in the region of 100° C, this intermediate step does not take place and the crystallites gradually aline themselves with their chain direction parallel to the axis of the monofilament. The relaxing of a polythene neck at 110° C has also been shown to be non-random. The direction of preferred orientation changes as relaxation proceeds from that in which the *a* axis of the crystallites is perpendicular to the axis of the monofilament, to that in which the *a* axis is parallel to this direction. A tentative explanation has been put forward to explain this behaviour.

Polyethylene is a long chain hydrocarbon which in addition to its well known electrical and moulding applications is also extruded in the form of monofilaments for the manufacture of fabrics.[1] In the monofilament process the extruded filament is first passed through a quenching bath and is then given a cold stretch to increase the tensile strength. This cold stretch does not result in the gradual thinning down of the filament, but at points along the filament, necks are formed, giving portions of the thread which are elongated by 400–500%, separated by abrupt shoulders from the rest of the filament which remains essentially unstretched. As stretching proceeds the necks move rapidly towards each other until the complete monofilament is of uniform diameter. The work described in this paper was undertaken to provide information on the deformation processes occurring during this phase of the manufacture of these monofilaments.

The above behaviour seems common to crystalline polymers. By carefully stretching a filament of polyethylene in a jig, however, intermediate elongations can be obtained, and Wood,[2] by an X-ray examination of specimens of this type, demonstrated the non-random orientation of the crystallites during stretching. By means of a micro-X-ray examination as described by Fankuchen and Mark[3] it has been possible to show that similar non-random behaviour of the crystallites occurs over the region of the neck during the normal stretching of polyethylene.

It has been shown by several workers that 50–75% of the CH_2 groups present in the polymer are arranged in an orderly manner.[4,5,6] The X-ray diffraction pattern will, therefore, consist of relatively sharp maxima superimposed upon a more diffuse band from the amorphous regions. The structure of the crystalline regions has been elucidated by Bunn[7] who showed that the unit cell was orthorhombic and body-centred, the chain direction being along the *c*-axis. The (100), (010) and (001) reflexions are, therefore, absent, the strongest reflexions being the (110) and (200). The diffuse maximum from the amorphous regions falls just inside the intense (110) ring and can be seen from the patterns reproduced in Fig. 1.

NECKING OF POLYTHENE AT ROOM TEMPERATURE

The monofilaments were extruded from grade 20 Alkathene of average molecular weight 20 000. The diameter of the unstretched extruded filament was of the order of 50 mils. This led to a stretched monofilament on the other side of the neck of 20 to 25 mils diameter, i.e. equivalent to an elongation of 400 to 500%. A cylindrical collimator was used giving an X-ray beam of circular cross-section, 8 mils in diameter, i.e. approximately one-third of the diameter of the stretched monofilament. The specimen was mounted directly in front of the collimator, and forward reflexion patterns obtained with a specimen-to-plate distance of 4 cm on a North American Philips Type 41D X-ray Diffraction Unit

using CuKα radiation. With this arrangement diffraction patterns were obtained after thirty minutes' exposure. Patterns were obtained at eight different positions in the region of the neck and these are reproduced in Fig. 1.

Position 1: *Unstretched filament.*—The typical pattern of randomly oriented polyethylene is obtained. The strong (110) and the less intense outer (200) crystalline rings are clearly seen, together with the inner, more diffuse, halo from the amorphous regions.

Position 2: *Upper shoulder of neck at edge of specimen.*—The crystallites now show signs of orientation, the complete circular rings of position 1 splitting up into two separate arcs. Also the pattern is tilted, indicating that the orientation follows the contour of the side of the neck. A closer examination reveals, however, that whereas the outer (200) line has behaved normally, the inner intense (110) reflexion now possesses four distinct maxima.

Position 3: *Centre of neck at edge of specimen.*—Further orientation has been produced and the axis of symmetry is again parallel to the contour of the neck. Both the (200) and the (110) reflexions now possess a single maxima indicating that the preferred orientation direction is now parallel to the direction of stretch.

Position 4: *Bottom shoulder of neck at edge of specimen.*—There has been a further increase in the degree of orientation indicated by the shortening of the arcs.

Position 5: *Stretched monofilament.*—A typical fibre pattern is obtained showing a high degree of orientation parallel to the monofilament axis. The unoriented diffuse halo from the amorphous regions, however, is still visible.

Position 6: *Upper shoulder of neck at centre of specimen.*—A similar pattern to that of position 2 is obtained except that in this case the arcs are symmetrically placed with respect to the axis of the monofilament. The angular separation of the maxima of the (110) line, however, has increased, indicating that the forces producing orientation vary as we pass across the specimen.

Position 7: *Centre of neck at centre of specimen.*—Apart from the direction of the axis of symmetry, which is now parallel to the fibre axis, the pattern is equivalent to that obtained in position 3.

Position 8: *Bottom shoulder of neck at centre of specimen.*—Equivalent pattern to position 4 obtained, although the degree of orientation appears to be somewhat less.

NECKING OF POLYTHENE AT − 55° C

A sample of the extruded polythene was placed in a cooling mixture of solid carbon dioxide and methanol and left for several minutes to obtain thermal equilibrium. It was then removed and rapidly stretched. The force to produce necking at this temperature is considerably greater than that required at room temperature and the extent of the neck is somewhat smaller. The diffraction patterns were obtained by the method already described and taken with the specimen at room temperature and not at the temperature of stretch.

The patterns follow a similar sequence to those obtained by stretching at room temperature. At intermediate stretches the (110) arcs again show a double maximum which disappears as stretching proceeds.

NECKING OF POLYTHENE AT 96° C

The extruded polythene was stretched in a water bath at 96° C. At this temperature no distinct neck occurs but the specimen gradually thins down until in the region of 500% elongation it breaks. The diffraction patterns were again obtained with the specimen at room temperature, and are reproduced in Fig. 2.

At this temperature the intermediate stage appearing in the stretching at lower temperatures, namely the splitting of the (110) ring into four distinct maxima, does not take place. As the stretch ratio increases there is a gradual alining of the crystallites with their chain axis parallel to the direction of stretch, demonstrated in the patterns by the decreasing length of the arcs as we pass to the thinner portions of the specimen.

Discussion of results

The diffraction patterns show that similar non-random changes in orientation occur over the small region of the neck as reported by Wood[1] for carefully stretched specimens. He showed that by assuming a preferred orientation at intermediate stretches in which the (011) axis is parallel to the direction of stretch, the observed diffraction patterns could be explained. Fig. 3 shows the reciprocal lattice[8,9] of polythene for this position. The reciprocal lattice point representing the (110) planes is raised above the meridian plane and consequently the splitting of the 110 maximum is explained. The (200)

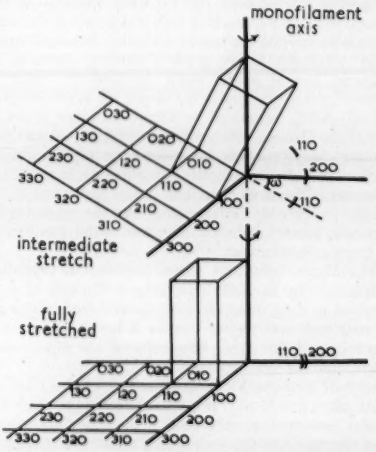


Fig. 3. Reciprocal lattice of polythene when "necked"

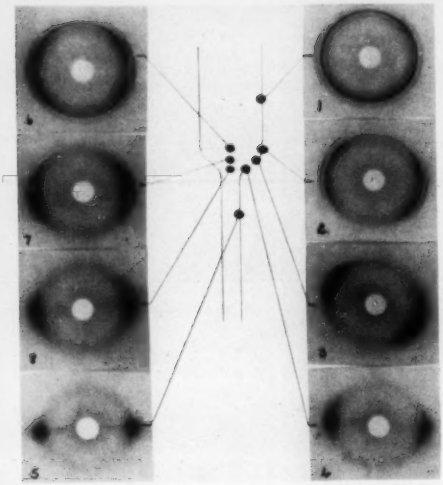


Fig. 1. X-ray diffraction patterns obtained from necking of polythene at room temperature

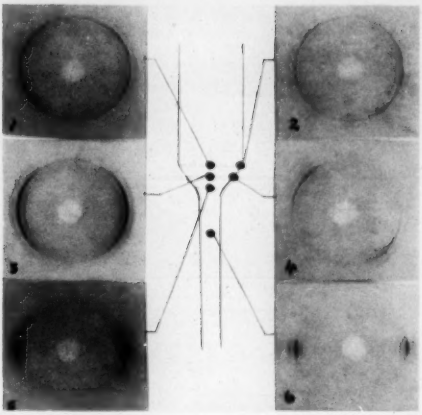


Fig. 5. X-ray diffraction patterns obtained when neck relaxed 10 min at 80° C

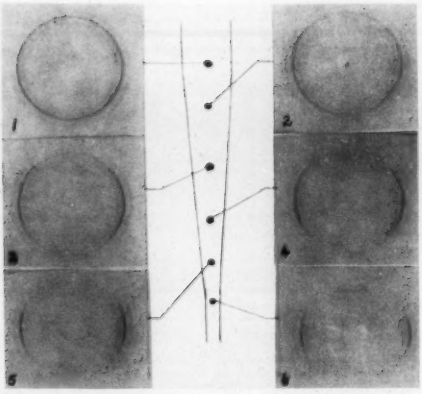


Fig. 2. X-ray diffraction patterns at room temperature obtained from necking of polythene at 96° C

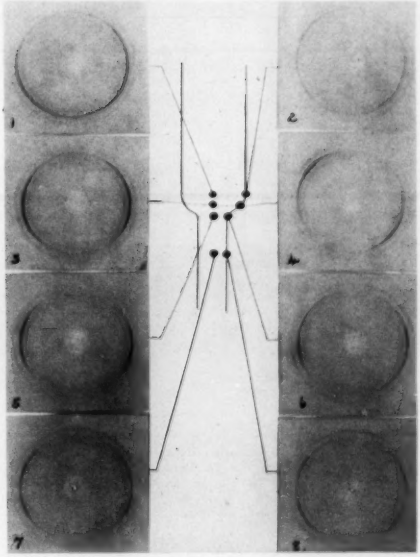
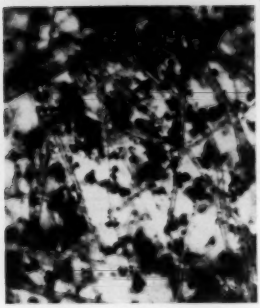


Fig. 6. X-ray diffraction patterns obtained when neck relaxed for a few seconds at 110° C



Cloud *A*

The photoelectric measurement of coal dust stains on filter paper

By C. N. Davies
and
Mary Aylward

See pages 352–359

Cloud *D*

Fig. 5. Photomicrograph (× 160) of dust deposited on paper No. 3

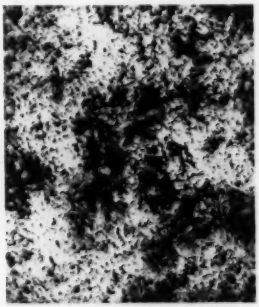
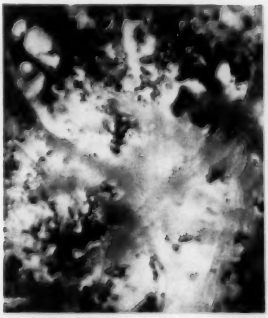


Fig. 7. Photomicrograph (× 40) showing patchy deposit on paper No. 8 (cloud *D*)

Cloud *A*



Cloud *D*

Fig. 8. Photomicrographs (× 160) of dust deposition on paper No. 8

point, however, will still intersect the sphere of reflexion in the meridian plane, as is observed experimentally. The angular shift of the maxima of the (110) line can be obtained experimentally from microphotometer traces, and this can be compared with the value calculated from the known dimensions of the reciprocal lattice as follows:

Using Bunn's figures for the unit cell of polyethylene[7]

$$a^* = \lambda/a = 1{\cdot}54/7{\cdot}40 = 0{\cdot}208$$
$$b^* = \lambda/b = 1{\cdot}54/4{\cdot}93 = 0{\cdot}312$$
$$c^* = \lambda/c = 1{\cdot}54/2{\cdot}534 = 0{\cdot}608$$

Referring to Fig. 4, O is the origin of the reciprocal lattice, BCO is the direction of X-ray beam, and OAD is the unit sphere of reflexion with centre C. P is the point on the reciprocal lattice representing the (110) planes taking the (011) axis as the direction of preferred orientation. PN is the perpendicular from P on to the meridian plane, and NX the perpendicular from N, to the diameter BCO. The angular shift (ω) of the maximum measured on the photographic film will then be equal to the angle PXN.

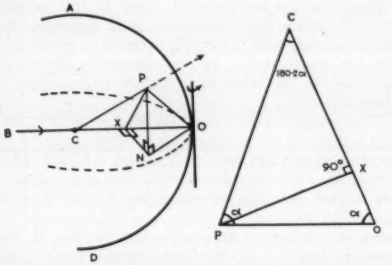


Fig. 4. Illustrating explanation of patterns obtained

Considering the triangle POC, we have

$$CP = CO = 1 \text{ (radii of sphere)}$$
$$PO = \sqrt{(a^{*2} + b^{*2})} = 0{\cdot}375$$

Also from the triangle POC,

$$PO/\sin(180 - 2\alpha) = 1/\sin\alpha$$

therefore $\cos\alpha = PO/2 = 0{\cdot}1875$
therefore $\alpha = 79° 12'$
therefore $PX = PO \sin\alpha = 0{\cdot}375 \times 0{\cdot}9823 = 0{\cdot}368.$

Now PN is equal to the height of the point (110) above the meridian plane, which is equal to the height of the point (010) above this plane.

Therefore $PN = b^* \sin\phi$
where $\tan\phi = b^*/c^* = 0{\cdot}513$
giving $\phi = 27° 9'$
therefore $PN = b^* \sin\phi = 0{\cdot}142$
therefore $\sin\omega = PN/PX = 0{\cdot}387$
giving $\omega \simeq 23°$

From microphotometer curves of the negatives the following experimental values of ω were obtained:

(1) Edge of neck at 20° C $\omega = 25°$
(2) Centre of neck at 20° C $\omega = 29{\cdot}5°$
(3) Edge of neck at − 55° C $\omega = 28°$
(4) Centre of neck at − 55° C $\omega = 29{\cdot}5°$

In the upper region of the neck the crystallites thus tend to aline themselves so that their (011) axes are parallel to the direction of stretch.

Fig. 3 also gives the reciprocal lattice for the higher values of stretch, in which the chain axis is parallel to the direction of stretch. As the lattice rotates the (110) and (200) points will both intersect the sphere of reflexion in the meridian plane as is observed experimentally.

By a suitable combination of the above two positions all the patterns observed over the region of the neck can be satisfactorily explained.

RELAXING OF THE NECK AT HIGH TEMPERATURES

Neck relaxed 10 *minutes at* 80° *C*

The neck obtained by stretching at room temperature was immersed for ten minutes in a water bath at 80° C. A further series of X-ray diffraction patterns were obtained and these are reproduced in Fig. 5. No definite decrease in orientation has been obtained for the higher stretches, but the splitting of the (110) line is less pronounced. The crystalline lines are much more distinct, however, indicating that the crystallites have increased in size.

Relaxing at 110° *C*

In this case the neck produced by stretching at room temperature was relaxed for a few seconds at 110° C. In contrast to the behaviour at 80° C, there was a definite change in the shape of the neck, the stretched portions swelling considerably. The series of diffraction patterns obtained are reproduced in Fig. 6.

Positions 1 *and* 2: The pattern obtained is essentially that of unoriented polythene, the preferred orientation parallel to the (011) axis having completely disappeared.

Position 3: Considerable relaxation has occurred, the complete (110) and (200) rings being visible. The preferred orientation, however, is still with the chain axis parallel to the direction of stretch.

Position 4: Signs of non-random relaxation are visible. The inner (110) rings are not symmetrical and there are signs of splitting of the outer (200) ring.

Position 5: The (110) arcs are symmetrical but the (200) ring shows signs of double maxima.

Position 6: Both crystalline bands now show definite splitting, and the (200) arcs appear to be moving round to the opposite pair of quadrants to which they occupied on the earlier patterns.

Position 7: The splitting of the inner ring is more sharply defined and there are signs of three maxima on each of the (200) arcs, which have rotated through 90°.

Position 8: The splitting of the (110) line is now quite distinct, and the outer (200) arcs are almost symmetrical, occupying the opposite pair of quadrants to those in the case of the stretched sample.

EXPLANATION OF DIFFRACTION PATTERNS

The changes that have occurred are again most easily explained by reference to the reciprocal lattice of polyethylene. They are due to a gradual tilting of the crystallites from the position in which their a axis is perpendicular to the direction of stretch (Fig. 3) to that in which the a axis is parallel to the direction of stretch. Fig. 7 gives the reciprocal lattice with the crystallites at

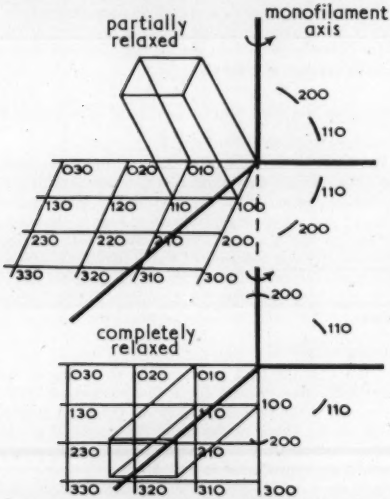


Fig. 7. Reciprocal lattice with crystallites at an intermediate stage and for a complete rotation through 90°

an intermediate stage, and for a complete rotation through 90°. Considering them in the above order, for intermediate relaxation the (110) point is below the meridian plane, but at a shorter distance than it would be had the crystallite turned through a complete right angle. The (110) arcs will thus appear to be split on the diffraction pattern. For a similar reason the (200) arcs will also split into a double maximum, and as relaxation proceeds, it can be seen that the arcs will move round from their original position towards an equivalent one in the opposite pair of quadrants. For the final position only the (110) arcs will split, as the (200) point of the reciprocal lattice now lies on the axis of rotation. Remembering that the specimen was only immersed for a few seconds, the edge of the specimen will have relaxed more than the central core. The patterns taken in the central positions will thus be a combination of various degrees of relaxation. In this way all the patterns of Fig. 7 can be explained.

As in the previous case we can calculate a value for ω, the angular shift of the (110) maxima, and compare this with values obtained experimentally from the microphotometer curves.

The calculation is analogous to that worked out previously and the numerical value of the line PX will be unaltered.

i.e. $$PX = 0{\cdot}368$$

In this case, however,

$$PN = PO \sin\psi \text{ where } \tan\psi = a^*/b^*.$$

Therefore $$PN = a^*$$

therefore $$\sin\omega = PN/PX = 0{\cdot}565$$

giving $$\omega \simeq 34°$$

Of the series of X-ray diffraction patterns of Fig. 6 the most relaxed will be position 8.

From the microphotometer curve of this negative, $\omega = 30°$, which is in reasonably good agreement with the calculated value, especially when it is remembered that the specimen may not be completely relaxed and consequently the value of ω should be less than the calculated value.

The assumption that relaxation involves gradual tilt is also borne out by the following. Measuring up the photometer curve of position 7, which is of the central core of the stretched monofilament, and is consequently less relaxed than the corresponding position 8 at the edge of the specimen, we obtain $\omega = 24°$, indicating that the crystallites are in an intermediate position as illustrated in Fig. 7. On the same pattern there is also evidence of a splitting of the outer (200) arcs which is required at this intermediate stage.

DISCUSSION OF RESULTS

It has been demonstrated that the diffraction patterns indicate that for intermediate values of stretch, the preferred orientation is in the direction of the (011) axis, and that upon relaxing from the highly stretched condition, the orientation changes from that in which the chain axis is parallel to the direction of stretch to one in which the chain axis is perpendicular to this direction. At present, however, it is uncertain as to why such orientation phenomena should occur. The alinement of the crystals with a minor axis parallel to the direction of stretch is similar to the cold drawing of metals. It is explained in the case of metals by the presence of preferred glide planes in the crystal, the slipping bringing about a change in orientation of the crystal lattice with reference to the stress direction, which can be described as a rotation. In the case of stretching experiments the slipping plane gradually approaches a position more and more parallel to the axis of stress. For compression it

approaches one perpendicular to this direction. This mechanism applied to the case of polythene, however, has two disadvantages. Firstly, in metals the intergrain boundaries are considered more resistant to strain than certain planes in the crystals at which slip occurs, whereas in the case of polythene the grain boundaries consist of amorphous material which is presumably softer than the actual crystallites. Secondly, slipping along the (011) axis would require a breaking of the hydrocarbon chains which also appears unlikely. The most probable cleavage planes are the (110) and the (100) which would not lead to the observed orientation at these intermediate stretches, but to an alinement of the chain axis parallel to the axis of stretch observed during the later stages. It thus seems uncertain that the preferred orientation at intermediate stretches can be explained in terms of favoured glide planes along which the van der Waals' forces are least, unless the shape of the crystallites is such that the resultant force upon them at these tensions is not along the monofilament axis, but inclined at an angle of approximately 60° to it, that is the observed direction of the chain axis at intermediate stretches.

The observations on relaxing are, however, consistent with the assumption of preferred glide planes. During this process there will be a compressive force along the monofilament axis. As already stated the most probable glide planes for polythene are the (100) and (110), and slipping along these planes would cause a tilt of the crystallites in such a direction as to bring them perpendicular to the stress direction. That is, the chain direction in the ordered regions would tilt until it finally became perpendicular to the monofilament axis, which is what has been observed experimentally.

An alternative approach can be made from the thermodynamic point of view. A tension would cause a decrease in melting point of the crystallites, this decrease being least for the axis that expands the least with temperature. The most stable position of a crystallite would, therefore, be that in which the axis with the lowest coefficient of expansion is parallel to the stress direction. Bunn and Alcock[10] have obtained the following figures for the coefficient of expansion of the unit cell of polythene. The *a* axis of the orthorhombic cell increases from 7·42 A at 18° C to 7·65 A at 100° C, while the *b* axis remains approximately constant at 4·93–4·95 A. There are no similar figures for the *c* axis, but this being the repeat distance within the molecule it would also be expected to remain constant. The crystallites would, therefore, be expected to take up a position in which the chain axis is parallel to the direction of stress, i.e. a position in which the *a* axis is perpendicular to the stress direction. This would explain the behaviour for stretching at high temperatures but is again unsatisfactory for intermediate elongations at room temperature. Considering relaxation, as a compression causes an increase in melting point, the crystallites would in this case tend to reform a position such that the axis which exhibits the greatest coefficient of expansion and consequently the greatest increase in melting point, would be parallel to the direction of stretch. This from Bunn's figures is the *a* axis which is in agreement with the experimental observations.

The presence of preferred glide planes and the melting and reformation of crystallites in preferred directions depending upon the relative coefficients of expansions of the unit cell axis, gives a satisfactory explanation of the relaxing and of the stretching at elevated temperatures of polythene. The results of the necking at room temperature lend support to the qualitative explanation of the necking of crystalline polymers put forward by Bunn and Alcock.[10] They suggested that the shearing of crystallites most probably occurs when they are oriented in such a position that their chain axis makes an angle with the stretching direction. The potential barriers are then broken progressively instead of simultaneously as in the case of slipping. Break-up by tearing is consequently a much easier process. The mechanism in the neck at room and low temperatures can thus be explained by the following three processes. Firstly, at low elongations there is a gradual tilt of the crystallites until they are alined in such a direction that their (011) axis is approximately parallel to the monofilament axis. This appears to be the optimum position for shearing by tearing which occurs during the second stage. This slipping presumably continues until it is prevented by the tie molecules. The torn-off portions of the crystallites will also tend to be oriented in the stretch direction, so that further slipping is prevented as the van der Waals' forces along the whole length of a crystal have now to be overcome before shearing can take place. The third stage thus consists in the further alinement of the torn crystallites with their chain axis parallel to the stretch direction. This stage is probably also accompanied by further crystallization in the stretched portions of the monofilament.

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The photoelectric measurement of coal dust stains on filter paper

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Deposits of coal dust collected on filter paper from airborne clouds have been studied in connexion with their light screening power. This depends on the nature of the paper as well as the size and concentration of the particles. Nine kinds of paper were investigated, two of them in detail. Some idea has been obtained of the qualities necessary in a filter paper in order that the dust deposits will have maximum optical density. Theoretical expressions are derived for light screening according to how the particles are dispersed in the paper. Maximum screening would result if each particle shadowed individually, but this is only possible in very sparse deposits; otherwise, overlapping of the particles has to be allowed for. Diffraction of light by the particles and the effects of diffusion of light in the paper are discussed.

As a rapid method of assessing the concentration of airborne coal dust in mines considerable use has been made of the hand pump and densitometer described by Watson[1] and Watson and Hounam.[2] Dusty air is aspirated through a piece of filter paper having a clear area of $\pi/4$ cm^2 and the resulting stain is measured photo-electrically. An account of the apparatus and its calibration with experimental clouds of coal dust has been published by Davies and Aylward,[3] but no attempt was made then to analyse the fundamental principles of the measurement or to investigate the properties of the paper most suitable for collecting the deposit. We now go more deeply into the theory of the matter and obtain some idea of the ways in which instrumental factors and the nature of the filter paper affect the results.

Coal dust particles between 1 μ and 10 μ in diameter have been used in the way described in reference [3]. Such particles do not penetrate through the paper to a sufficient extent to necessitate an allowance being made for loss of light screening power on this account. Penetration is unlikely to be a factor in deciding between different kinds of filter paper which are otherwise suitable. All the papers used in the following experiments allowed a considerable proportion of fine particles below 0·5 μ to pass through them, but the number of such fine particles in coal dust is small enough for their optical screening effect to be negligible. Transmission of light through coal particles of the sizes used is also unimportant.

INSTRUMENTAL FACTORS

The arrangement of the 6 V, 6 W lamp and selenium photocell for measuring the dust stain is shown in Fig. 1. Light passes down a cylindrical hole 1·3 cm diameter bored in a block of Bakelite. The paper slides across in a brass clip holder and is measured with the dust deposit towards the lamp. The current from the photocell is proportional to the total light flux falling on it. The paper clip can be inserted into the nozzle of the hand pump and by aspiration of dusty air the clear area of $\pi/4$ cm^2 of paper is covered uniformly with particles.

If the air contains N particles/cm^3 and n pump strokes, each of volume V, are taken, the number of particles/cm^2 of paper is

$$x = 4nNV/\pi \qquad (1)$$

Light from the lamp is intercepted by each particle in proportion to its projected area of cross-section. In addition, light passing near a particle is diffracted. Light scattered forwards from the particle is all collected in the paper and further scattered before being passed on to the photocell. A fraction is lost by reflexion and absorption in the paper and by absorption at the walls of the Bakelite tube leading to the photocell. Assuming, for the moment, that a constant proportion of light is

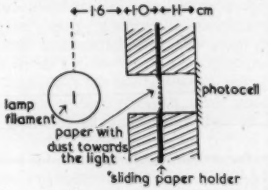


Fig. 1. Arrangement of paper and photocell in densitometer

lost, the screening effect of the particles is measured by the area of cross-section presented by them in the plane of the paper. This conclusion is dependent on the collection of all the forward diffracted light in the paper and would not be true if the dust were deposited on a transparent material unless the photocell subtended a wide enough angle to receive the forward scattered light directly.

The maximum screening that could be exerted by x particles distributed over unit area of filter paper is therefore

$$\pi x\overline{d^2}/4 \qquad (2)$$

where the mean of the squares of the diameters can be identified with the value obtained from sizing a dust sample under the microscope, on the assumption that the average orientation of particles is the same in both cases. In sizing particles which do not present a circular image, the diameter is taken equal to that of a circle judged to have the same area as the image of the particle. It is found by experiment that only very sparse deposits of coal dust exert so great a screening effect as this. In denser deposits the particles overlap to some extent and much less screening is observed.

Before proceeding to develop the theory of screening and to consider the measurements it is advantageous to describe some simple experiments which were carried out to throw light on instrumental factors. A number of holes was drilled in a piece of brown Bakelite $\frac{1}{8}$ in thick and the same size as the paper holder; they all fell inside a circle 1 cm in diameter and lay in the path of the light beam when the Bakelite was inserted into the slot normally occupied by the paper holder. The photocell current was measured with light passing through the holes and a piece of filter paper, which acted as a diffuser, in contact with the face of the perforated Bakelite. A much greater current was noted when the paper was on the side towards the photocell than when it was nearest the lamp. The difference was more marked for holes of a given total area when they were small and numerous. For 17 holes $\frac{1}{16}$ in diameter the ratio was 3 : 1. Although no lens was used to direct light from the lamp, its filament was only 3 mm long, so that most of the light reached the paper quite near to normal incidence. When light entered the holes in the Bakelite direct from the lamp, reflexion from the inside walls occurred mainly at glancing incidence so that loss by absorption at the walls was minimal. On the other hand, when diffusion was introduced in front of the holes, a large part of the light could only get out after multiple to and fro reflexions across the holes and the loss due to absorption was larger.

Measurements were also made of the photocell current after light had passed through a number of thicknesses of clean filter paper. Allowing for absorption in the paper, a linear relation would be expected between the logarithm of the current and the number of sheets. Allowance for reflexions backwards and forwards between the sheets introduces only a very slight curvature. What was actually observed is shown in Fig. 2. Absorption in a single sheet is relatively large, while the absorption/sheet decreases with the number of sheets that the light has passed through until it ultimately becomes constant. Thin paper shows this behaviour better than thick.

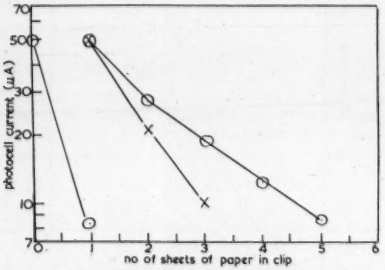


Fig. 2. Relationship between photocell current and the number of sheets of paper in the clip

○ = paper No. 7. Current was reset to 50 μA after one sheet to enable up to 5 sheets to be measured.

× = paper No. 3.

This phenomenon is also connected with the diffusion of light. When there is no paper in the clip, light passes down the tube to the photocell more or less parallel to the axis and such reflexions as take place off the wall are at glancing incidence. After passing through a sufficient thickness of paper, however, the emergent light is perfectly diffused and the distribution in direction of the intensity from the surface of the paper becomes independent of the number of sheets. Owing to the diffuse emission a large proportion of the light leaving the paper now falls upon the tube wall at small angles of incidence and a greater fraction is absorbed than before.

The proportion of light scattered by the paper which reaches the photocell is constant only if the number of sheets of paper is sufficient to thoroughly diffuse the emergent light. After this, each additional sheet reduces the photocell current in the same ratio. The first sheet of paper introduces diffusion into the light reaching the photocell and therefore has a higher apparent absorption than subsequent sheets. If it is not thick enough, it still allows a portion of directed light to proceed so that the absorption of the second sheet must seem greater than that of the third. For paper No. 7 (Fig. 2) the current was reduced in constant ratio after the third sheet.

An error might arise in this manner when measuring dust stains on thin paper because the particles themselves cause diffusion of light, by diffraction, which would make the degree of diffusion of the light leaving the filter paper depend on the quantity of dust. The apparent absorption due to the dust would therefore be diminished at high densities.

The error is avoided if papers of sufficient thickness are used, and it would certainly be better if the greatest possible amount of the light scattered from the filter paper was caught directly on the photocell, which should therefore be placed close to the paper or at least have a wide angle of acceptance.

Another improvement would be to the zero adjustment of the instrument which depended on varying the distance between the lamp and the paper and so altered the angle of the illuminating cone.

THE THEORY OF LIGHT SCREENING BY OPAQUE PARTICLES

Consider a very small particle placed at random upon a unit area, A, in the plane of which it has a projected area C_1. If a point P in the area A is not to be covered by the particle, it is necessary that the centre of C_1 should fall outside an identical projected area centred on P and rotated 180° in the plane of A relative to C_1. The probability of this occurring is equal to the proportion of A which is not covered by C_1, namely $(1 - C_1)$. If, now, a second particle of size C_2 is placed at random on A the chance of P not being covered by either is

Table 1. *Properties*

Paper No.	$I/50$ for 5×10^6 dust particles/cm² Cloud A	Cloud D	Thickness (cm)	Weight (g/cm²)	Fibre diameter (μ)	Pressure drop at 1 l/min/cm² cm of mercury
1	0·234	—	0·0295	0·0109	14·0 ± 3·32	2·0
2	0·258	—	0·0358	0·0128	24·5 ± 6·24	2·0
3	0·258	0·62	0·0240	0·0082	8·7 ± 2·74	2·0
4	0·286	—	0·0257	0·0088	12·3 ± 3·48	2·0
5	0·336	—	0·0284	0·0150	21·5 ± 6·04	2·7
6	0·364	—	0·0247	0·0086	26·9 ± 10·4	2·0
7	0·396	0·76	0·0188	0·0090	22·0 ± 6·80	3·5
8	0·400	0·79	0·0157	0·0085	20·7 ± 4·51	5·7
9	0·470	—	0·0360	0·0121	26·6 ± 11·4	1·9
Calculated (No overlap, eq. 5)	0	0·716				
Calculated (eq. 4)	0·284	0·753				

$(1 - C_1)(1 - C_2)$, which is also equal to the mean value of the uncovered area when pairs of particles are set down successively at random. In the same way, if x_1 particles of size C_1, x_2 of size C_2 . . . x_r of size C_r, are placed on unit area at random, the mean area left uncovered is

$$(1 - C_1)^{x_1}(1 - C_2)^{x_2} \ldots (1 - C_r)^{x_r}$$

and the total number of particles per unit area is

$$x = \Sigma x_r$$

Since the x's are large and the particles are small, the mean proportion of the area which is not covered is given by

$$\exp(-C_1x_1 - C_2x_2 - \ldots - C_rx_r) = \exp(-x\bar{C})$$

and if the areas are all circles,

$$= \exp(-\pi x\overline{d^2}/4) \qquad (3)$$

This agrees with the corresponding quantity obtained from expression (2) only when x is not too large, otherwise the screening is less.

The variance of the mean area with equal circles has been discussed in more general cases by Garwood,[4] whose paper suggested the argument above; however, for the numbers of circles in the present problem, the variance is negligible. Variation in the shape of particles has been shown by Armitage[5] to exert quite a small effect on the frequency of overlap and it is unlikely that an appreciable error could be introduced, due to shape, when the mean of the squares of the particle diameters is taken, as previously mentioned, by sizing the dust microscopically.

Suppose I_0 is the light flux falling on the photocell when the paper in the clip is clean and the current is adjusted to 50 μA; then I is the flux for dust covered paper and gives current i. Hence,

$$I/I_0 = i/50 = \exp(-\pi x\overline{d^2}/4)$$

or, substituting for x from equation (1),

$$i/50 = \exp(-nNV\overline{d^2}) \qquad (4)$$

This formula predicts a linear relationship between the concentration of airborne dust, by number, and the logarithm of the current. An equation of similar form has been derived by Silberstein[6] in connexion with photographic emulsions.

TYPES OF FILTER PAPER

Nine different kinds of paper were compared in these experiments and certain of their qualities are set out in Table 1, where they are placed in order of effectiveness of light screening when 5×10^6 particles of coal dust are distributed per cm² of paper. Two of the papers, Nos. 3 and 8, were selected for detailed examination using four coal dust clouds, *A* to *D*, of increasing fineness and a wide range of concentration. These two papers were of high quality and free from defects which might introduce inaccuracies into the readings. They were, respectively, the best and the worst of the nine which did not appear to suffer from any inherent faults. It is at once apparent that none of the qualities listed in Table 1 correlates with light screening capacity when loaded with coal dust, in spite of the fact that this attribute varies considerably from one paper to another. This problem will be taken up in the next section. The relationship of pressure to air flow followed a common

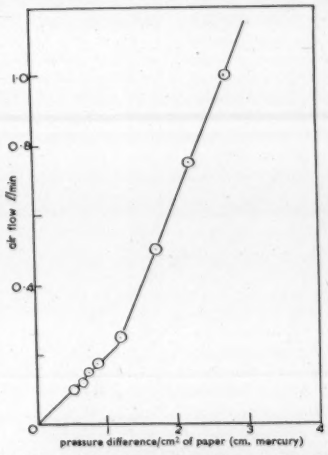


Fig. 3. The pressure drop across paper as function of the rate of flow of air (paper No. 7)

of filter paper

Colour	*Texture*	*Appearance by transmitted light*
white	soft, smooth	slight ripple water mark: fairly homogeneous, a few blemishes
slightly grey	firmer than 1, slightly rough	slight screen mark: slightly patchy: fair number of blemishes
cream	hard, firm, smooth	slightly inhomogeneous: very free from blemishes
very slightly grey	firm	fine screen mark: very homogeneous but a few blemishes
white	hard, rough	slightly inhomogeneous: no blemishes
white	very soft	very patchy: a number of fine holes visible to naked eye
white	hard, rough	quite homogeneous: no blemishes: many fine holes
white	hard, slightly rough feel	slightly inhomogeneous: no blemishes: a few fine holes visible
dirty white	soft and coarse	patchy: many blemishes: many holes

pattern for all the papers. As shown in Fig. 3, with flows up to about 0·2 l/min . cm^2 of paper the pressure drop was proportional to the flow. Near this value, however, a discontinuity occurred in the graph, and while the relationship still continued to be linear within the range of the measurements, the pressure then increased more slowly. Presumably this was due to a change in the nature of the flow around the fibres.

Lack of homogeneity in the paper, when viewed by transmitted light, was considerable in some, but as the current was set to 50 μA before exposure to dust this caused no error. Opaque blemishes were frequent in some pieces and were, of course, avoided. Such defects would be fatal in paper used for routine dust measurements or in automatic recording apparatus. Papers Nos. 6, 7 and 9 revealed a number of very fine holes when held up to the light. These were probably not actual apertures but small areas where the paper was extra thin; a much smaller number was seen in No. 8. The leakage of particles through No. 8 was appreciable in the case of the finest cloud, *D*, but was not large enough to cause a significant error. Leakage through 6, 7 and 9 was not checked.

INTERPRETATION OF EXPERIMENTAL DATA ON LIGHT SCREENING

For particles spread in a plane without overlapping expression (2) shows that

$$i/50 = 1 - \pi x\overline{d^2}/4 \qquad (5)$$

while equation (4) holds when they fall at random. Theoretical values of $i/50$ for each condition are shown at the foot of column 2, Table 1, for 5×10^6 particles/cm^2 of clouds *A* and *D*. The particles sizes in the clouds are given in Table 2.

Table 2. *Root mean square diameter of dust particles*

Paper	*Cloud*	*A*	*B*	*C*	*D*
8	$\sqrt{d^2}$	5·66	4·74	3·95	2·69 μ
1–7 and 9	$\sqrt{d^2}$	5·23	5·16	4·40	3·44 μ

Looking up column 2 of Table 1, we see that the photocell current agrees better with the overlap equation than with (5), but it does in fact vary from values which are a little too low, in the good papers, to some which are much too large in the bad.

In the latter case the degree of overlapping of the dust particles is considerably greater than would be obtained by placing the particles at random in a plane and can be explained if the effective area over which the particles are distributed is less than the area of the paper; this can be brought about if the paper is composed for the most part of tightly packed fibres with occasional clear spaces through which the air prefers to find its way. It would be in the restricted area of these clear spaces that dust deposition took place.

On the other hand, a smaller current than is predicted by equation (4) means less overlap, which can be accounted for by deposition of particles in depth, within the paper; since dust particles and paper fibres scatter the incident light there is the chance of a certain amount of blurring of the region of light and shadow beyond dust on the surface of the paper, with the result that particles lying deeper would overlap with surface ones, in a photometric sense, less completely than they did geometrically. This effect is very much smaller than the hole or restricted area phenomenon observed with other papers and calls for only a slight correction to equation (4).

Having put forward these ideas we shall now show how they can be made quantitative for paper 3 and paper 8.

RESULTS FOR PAPER 3

This was a good quality esparto paper of low weight. Owing to the small size of the fibres, and their uniform dispersion, it was quite thick and porous and had a low air resistance. These facts, together with its optical performance, homogeneity and freedom from blemishes, led us to consider it the best for dust measurements amongst the papers listed.

The results of the measurements with this paper are shown in Table 3. A separate cloud was dispersed in a dust chamber for each experiment by the standard method which has previously been described by the authors.[3] The number of particles/cm^3 was determined from sedimentation cell samples[7] taken at the same time as the hand pump ones. For successive clouds, measured the same time after dispersal, the dust concentration only varied slightly, so the number of particles/cm^2 of paper was adjusted by taking a different

Table 3. *Experimental results for paper No. 3*

Cloud	No. of particles/cm³ N	No. of pump strokes n	Photocell current i μA	No. of particles/cm² x
A	9 160	3	20·5	2·97 0×10^6
	10 745	5	11·5	5·81
	9 630	7	6·5	6·99
	9 445	9	4·1	9·18
	10 300	10	3·0	11·13
B	8 490	3	21·5	2·75
	9 665	5	14·5	5·22
	9 220	7	8·8	6·97
	8 915	9	7·5	8·67
	9 630	11	4·3	11·43
C	4 660	5	32·5	2·52
	5 810	10	20·0	6·27
	5 445	15	12·5	8·83
	5 495	20	10·3	11·87
	4 900	25	10·0	13·23
	4 860	30	6·8	15·75
	4 650	35	5·8	17·47
	4 610	40	3·2	19·92
D	2 035	20	32·0	4·40
	2 362	40	20·5	10·21
	1 872	60	16·0	12·13
	2 183	90	7·1	21·22
	1 955	120	7·5	25·34
	1 809	150	5·4	29·31
	1 720	180	5·0	33·44 0×10^6

number of strokes of the hand pump for each experiment. The volume, *V*/stroke was 85 cm³.

The mean particle size in the samples was controlled by varying the interval between dispersal and sampling from 1 min, for cloud *A*, to 60 min for cloud *D*.

On Fig. 4 the photocell current is shown related to the number of particles deposited on the paper. Equation (5) gives a straight line and equation (4) a curve, both passing through 50 μA at zero particles. The experimental points are shown as crosses. For the finer clouds we see that the experiments agree well with the theory based on overlap in a plane, but that coarser particles overlap less than this, especially at low numbers per cm², and the light screening is rather greater than is predicted by equation (4). However, only for very

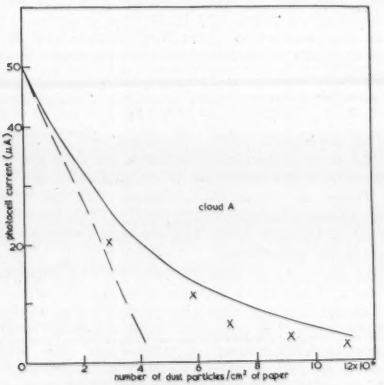
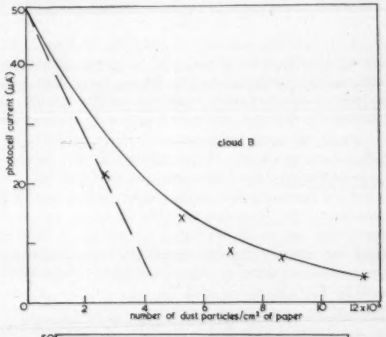
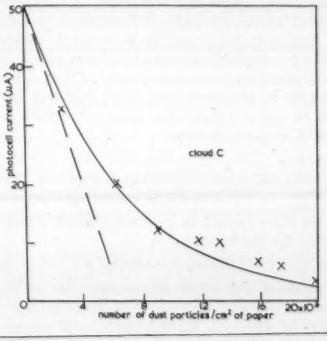


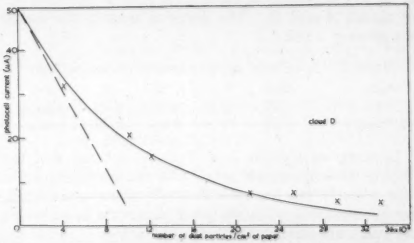
Fig. 4. Performance of paper No. 3

× = experimental points.
——— equation 4 particles spread at random in a plane.
– – – – equation 5 particles lying without overlapping.

Fig. 4.—*continued*







low numbers indeed do the particles screen independently according to equation (5).

Equation (4) can be made to fit the results for clouds *A* and *B* by multiplying the argument of the exponential function by 1·2.

Photomicrographs (× 160) of the upper surface of the paper are shown in Fig. 5 (p. 348). They were taken of specimens exposed to cloud *A* and cloud *D* having optical densities of the same order and gave photocell currents of 3·0 and 5·0 μA respectively. The number of particles/cm^2 was $11{\cdot}1 \times 10^6$ for cloud *A* and $33{\cdot}4 \times 10^6$ for cloud *D*.

According to equation (4) the screening is a unique function of the total projected area of the dust, independent of particle size, so the proportion of overlap is the same in these two papers, apart from the small deviation from the equation for cloud *A*.

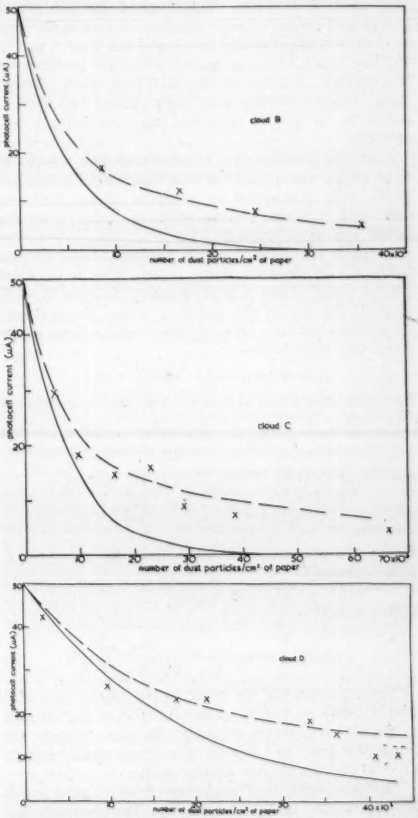
It is rather noticeable in the photographs how the fine particles are distributed on the surface of the fibres while the coarse ones lie mostly on the sides or else form bridges across the angles where fibres cross.

At the usual speed of operation, the hand pump gives a mean velocity in the pores of the paper of about 65 cm/sec, which makes the Reynolds number of flow past the fibres nearly 0·4. Collection of particles under these conditions is due to their inertia causing them to impinge upon the fibres. The collection efficiency of an isolated fibre 8·8 μ diameter, at the above speed, has been calculated to be 10% for 2 μ diameter particles, 25% for 3 μ, and 100% for 6 μ; the efficiency is expressed as the fraction of the space swept by the projected area of the fibre which is denuded of particles. The conditions of the calculation do not hold exactly in filter paper, but the figures indicate that fine particles which strike a fibre must approach within a narrow zone of 10–25% of the fibre width centred upon its axis. These particles will therefore deposit near the middle of the fibre. Large particles, however, can be intercepted when they approach with their centres well out of line with the fibre because they are comparable in diameter with the fibre and heavy enough to experience only slight deflexion by the air flow as it parts to sweep by on either side. Hence, more large particles are caught on the sides of the fibres than on the front. The inertia of the larger particles may also be a factor in causing them to penetrate more deeply into the paper so that they are distributed in depth and the photometric overlap is less, as described in the previous section. Departures from equation (4) due to this would be less marked for large values of x.

RESULTS FOR PAPER NO. 8

This was a high grade, hard, chemical filter paper made of cotton fibres which were very tightly packed.

Fig. 6.—*continued*



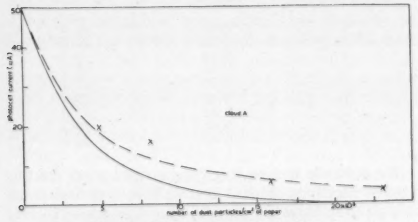


Fig. 6. Performance of paper No. 8

× = experimental points. – – – – – – calculated by equation 6.
——— calculated by equation 4.

Cloud *A*. $\alpha = 0{\cdot}6, \beta = 0{\cdot}88$.
Cloud *B*. $\alpha = 0{\cdot}6, \beta = 0{\cdot}905$.
Cloud *C*. $\alpha = 0{\cdot}6, \beta = 0{\cdot}94$.
Cloud *D*. $\alpha = 0{\cdot}6, \beta = 0{\cdot}95$.

The experimental data are given in our previous paper[3] and are plotted on Fig. 6 for the four clouds. Graphs are also reproduced of equation (4), and this time we see that the screening is considerably less than the equation predicts, so that the particles must be overlapping to a greater extent.

Several attempts were made to fit the experimental data on different hypotheses, but the argument which follows was the only successful one.

The appearance of the deposits on this paper, to the naked eye, was distinctly granular in comparison with those on paper 3, which were of a smooth grey colour and quite homogeneous. A low power photomicrograph (× 40), reproduced in Fig. 7, shows that the dust is concentrated to a large extent in patches with relatively clear areas in between. The fibres of this paper were coarse and packed closely together, so that some areas were almost impermeable to air and the greater part of the flow was directed to regions where the packing was less perfect. Almost all the flow took place through these "holes," and the dust, being carried with the air, lodged in the porous areas and made the deposit look granular.

Since the particles were concentrated over a series of areas which were smaller in total than the surface of the paper, they overlapped to a greater extent than was calculated on the basis of the latter and the light screening power was correspondingly less.

Suppose a fraction β of the particles is laid upon a fraction α of the area of the paper. This is the total porous area. Then a corresponding fraction $(1-\beta)$ falls on the rest of the paper $(1-\alpha)$. The numbers of particles per unit area on these two fractions of the paper are, respectively,

$$\beta x/\alpha \quad \text{and} \quad (1-\beta)x/(1-\alpha)$$

Hence the proportion of area α not covered by particles is

$$\exp\left(-\frac{\beta x}{\alpha}\cdot\frac{\pi\overline{d^2}}{4}\right)$$

and the proportion of area $(1-\alpha)$ not covered is

$$\exp\left(-\frac{(1-\beta)x}{1-\alpha}\cdot\frac{\pi\overline{d^2}}{4}\right)$$

Hence, the proportion of unit area of the paper which is not covered is

$$t/50 = \alpha\exp\left(-\frac{\beta x}{\alpha}\cdot\frac{\pi\overline{d^2}}{4}\right) + (1-\alpha)\exp\left(-\frac{(1-\beta)x}{1-\alpha}\cdot\frac{\pi\overline{d^2}}{4}\right) \qquad (6)$$

This equation fits the experimental results very well and is shown on Fig. 6. β is a function of particle size, a greater proportion of large particles missing the permeable areas and depositing on impermeable regions $(1-\alpha)$ because of their greater inertia.

Photomicrographs of the deposits from clouds *A* and *D* are reproduced in Fig. 8 (× 160). The photocell currents for these specimens were 15·8 and 15·2 μA, respectively, while they bore $8{\cdot}2\times10^6$ and $36{\cdot}4\times10^6$ particles/cm². Owing to the coarseness of the fibres and the tightness of the packing the different localization of coarse and fine particles is not so striking as it was for the esparto paper, but areas devoid of particles, due to tight masses of fibres hindering the air flow, can be seen clearly.

GENERAL DISCUSSION

It is of great importance to secure optically homogeneous paper for the photometric comparison of stains. This is the first criterion of suitability. If fine particles were to be significant the question of their penetration through the sheet would have to be explored. After these points there remain the optical homogeneity of the stain and the procuring of maximum light screening power.

These experiments show that it is best if the paper is of a fairly open texture so that the air permeates uniformly through its bulk and is not liable to seek a limited number of channels. If the particles deposit in depth, to some degree, overlapping is less than if they all lie in the surface and the optical density can be increased above that predicted by the theory of overlapping in a plane.

If we neglect variations in the density of the fibres, the porosity of the paper can be defined as

$$1-\frac{\text{mass of paper/cm}^2}{\text{thickness of paper}}$$

and, within limits, it is clear from Table 4 that the porosity of the paper is a more important factor than fibre diameter. A soft texture, which goes with open structure, is advantageous. Papers 6 and 9 were exceptions to these statements. Both were poor; the texture was uneven and contained holes. Probably loss by penetration of dust contributed to their low efficiency.

Table 4. *Porosity and fibre diameter of papers*

Paper No. in order of screening efficiency	*Porosity*	*Fibre diameter* μ
1	0·63	14·0
2	0·64	24·5
3	0·66	8·7
4	0·66	12·3
5	0·47	21·5
6	0·65	26·9
7	0·52	22·0
8	0·46	20·7
9	0·67	26·6

We conclude that in specifying a filter paper for this kind of measurement it is best to have a porous paper in which the fibres are dispersed homogeneously so that the air can flow freely past them all and is not constrained to avoid some areas where numbers of fibres are jammed into clumps. Other things being equal, fibres of small diameter are probably advantageous; this would certainly be true if very fine particles had to be dealt with.

ACKNOWLEDGMENTS

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The measurement of the viscosity of highly viscous plastic mixes at high rates of shear

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Viscosity measurements on very viscous pastes at high rates of shear present special difficulties. It has been found that the pendulum viscometer[1] can be used where the more conventional types of viscometer fail. This paper describes the construction of an instrument suitable for the testing of pastes with viscosities of several hundred poises at rates of shear of from 5 000 to 25 000 reciprocal seconds. The average rise of temperature resulting from the work done on such a paste is calculated to be of the order of 13° C at the highest rate of shear. The errors introduced by the change in linear velocity of the pendulum as it passes through the paste and the variation in velocity with the distance from the point of suspension are shown to be negligible in comparison with that arising from the temperature change. Typical results are presented and it is shown that the instrument may be used to detect thixotropy or shear hardening.

Certain leather cloth and related finishes are obtained by spreading on suitable fabrics highly viscous pastes containing polyvinyl chloride (p.v.c.) powder, plasticizers, pigments and fillers. To control the flow properties of such pastes adequately it is desirable to measure their apparent viscosities over a wide range of rates of shear, for they usually exhibit some form of anomalous viscosity behaviour. Measurements at a rate of shear of about 10 000 reciprocal seconds may prove especially useful, since this is the order of magnitude of the velocity gradient between the moving cloth and the stationary spreading blade as the paste is being applied to the cloth.

Measurements at rates of shear below 100 reciprocal seconds offer no difficulty and may be made with a coaxial cylinder viscometer or the cone and plate viscometer.[2] As the rate of shear is increased, however, it becomes more and more difficult to measure the viscosity of the pastes, principally because of the heat which is produced. Some of the mixes examined during the present work have had apparent viscosities of about 1 000 poises at a rate of shear of 1 000 reciprocal seconds, falling to about 200 poises at 10 000 reciprocal seconds. When such pastes are sheared continuously at high rates, slippage occurs at the boundaries. The reason for this is not known with certainty. It may be associated with the fact that when polyvinyl chloride mixes are heated, the polymer particles imbibe plasticizer and swell, and a coherent gel is produced. The heat generated in a rotational viscometer operating at a high rate of shear may be sufficient to cause gelation and hence slippage. This behaviour precludes the use, for making measurements at high rates of shear, of any viscometer in which the paste is sheared for an appreciable time.

The conditions of flow in the pendulum viscometer[1] should reproduce fairly closely those which occur in the region immediately beneath the spreading blade, for with this instrument the duration of shear is extremely short and there is seldom any sign of slippage. Since the paste is tested in the form of a very thin film bounded by metallic surfaces, the rise of temperature is reduced to a minimum. Even so, it may be as large as 13° C at the highest rate of shear because of the low thermal conductivity of the paste. Consequently, the measured values of the apparent viscosity may differ appreciably from the true value at the initial temperature. A similar rise of temperature must occur, however, when the pastes are applied to fabrics, and it is probable that the variation of viscosity with temperature is at least as important as the variation with rate of shear. Provided the rise of temperature is known, the pendulum viscometer may be used to measure viscosities at any desired temperature. Thus, if the temperature coefficient at the rate of shear in question is known, the required viscosity can be calculated directly from the measured value. Alternatively, the initial temperature of the paste and viscometer may be controlled to secure the appropriate final temperature.

DESCRIPTION OF THE INSTRUMENT

An instrument for measurements on very viscous plastic mixes (Fig. 1) has been constructed conveniently from a Goodbrand ballistic yarn tester[3] by changing the pendulum and adding appropriate shearing surfaces. The pendulum comprised a steel block *A* (4 in square, $1\frac{1}{2}$ in thick) attached by two T-sectioned steel bars ($1\frac{1}{2} \times 1\frac{1}{2}$ in, $\frac{3}{16}$ in thick, $14\frac{1}{2}$ in long) and a steel rod ($\frac{1}{4}$ in diameter, $14\frac{1}{2}$ in long) to a horizontal spindle

mounted in ball bearings. The total mass was about 6 kg and the moment of inertia about the axis of rotation approximately 7×10^6 g.cm^2. The faces of the steel block were machined exactly perpendicular to the horizontal spindle after the pendulum had been assembled. Moving shearing surfaces were formed by raised areas (4 cm × 2 cm × 2 mm thick) on two plates B (2½ in square) screwed to the faces of the block A. Similar plates C, carried on the inner faces of two steel plates D (4 in square, ½ in thick), formed the fixed shearing surfaces. Both plates D were screwed to a steel block (4 in long, 2½ in wide, 1¼ in thick) fixed to the base of the ballistic tester, with both fixed shearing surfaces exactly parallel to the plane of rotation of the pendulum.

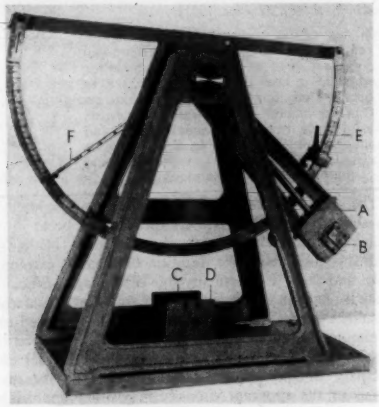

Fig. 1. Goodbrand ballistic yarn tester modified for use as a viscometer

The rate of shear may be varied either (*a*) by replacing the plates B by similar plates of different thickness or by placing pieces of thin steel behind them, or (*b*) by altering the height from which the pendulum is released. Actually, the width of the shearing gaps has been varied from 0·033 in to 0·003 in, giving rates of shear from 1 000 to 25 000 reciprocal seconds. The normal pendulum release mechanism E and indicator pointer F have been retained.

CALCULATION OF RESULTS AND ESTIMATION OF ERRORS

All quantities used are expressed in c.g.s. units, the following notation being used:

η = apparent viscosity
$\overline{V}$ = rate of shear
I = moment of inertia of pendulum about its axis of rotation
M = mass of pendulum
R = distance measured from the axis of rotation
R_1 = distance of upper edge of shearing surface from axis of rotation
R_2 = distance of lower edge of shearing surface from axis of rotation
m = mass of a simple pendulum of length $\frac{1}{2}(R_1 + R_2)$ whose moment of inertia about its axis of rotation is I
$r = (R_2 - R_1)$ = height of shearing surface
l = horizontal length of shearing surface
d = thickness of sheared layers of paste
x = distance of leading edge of moving shearing surface from the edge of the fixed shearing surface at which shearing commences
$\dot{x}_0$ = horizontal velocity of the pendulum when shearing commences
$\dot{X}_0$ = lowest horizontal velocity of the pendulum when shearing commences which will shear the paste
h_1 = height to which centre of gravity rises above its lowest position with no paste
h_2 = height to which centre of gravity rises with paste
y = vertical distance measured from upper edge of shearing surface
z = distance measured perpendicularly from either shearing surface
θ = temperature at any point at any time
θ_0 = initial temperature of paste and temperature of boundaries
$\overline{\theta}_x$ = mean temperature across gap at any time
$\overline{\theta}_{xt}$ = average temperature during shearing
t = time interval after shearing commences
τ = time for which any volume element of paste is sheared
A_0 = heat produced per unit time per unit volume
K = thermal conductivity
k = diffusivity

The expressions[1] used to calculate the apparent viscosity and rate of shear are:

$$\eta = [Id/l(R_2^2 - R_1^2)](2Mg/I)^{\frac{1}{2}}(h_1^{\frac{1}{2}} - h_2^{\frac{1}{2}}) \text{ poises} \quad (1)$$

$$\overline{V} = [(R_1 + R_2)/4d](2Mg/I)^{\frac{1}{2}}(h_1^{\frac{1}{2}} + h_2^{\frac{1}{2}}) \text{ sec}^{-1} \quad (2)$$

These are strictly applicable only for Newtonian fluids, the rate of shear being assumed to be the mean of its initial and final values at the upper and lower edges of the shearing surfaces. For non-Newtonian fluids, such as plastic mixes, the expression for the viscosity is incorrect, as the rate of shear varies with the distance travelled by the moving shearing surfaces through the gap formed by the fixed shearing surface (lateral variation) and with the distance from the axis of rotation (radial variation). The correct expression can only be found when the relation between viscosity and rate of shear is known; by assuming a reasonable form of this function, however, it can be shown that the errors resulting from the application of equations (1) and (2) to plastic mixes are negligible for an instrument with the above dimensions. For this purpose, it is convenient to consider the

effects of the radial and lateral variations of rate of shear separately.

Within the range of rates of shear covered by the pendulum viscometer, the logarithms of the apparent viscosities of plastic mixes are related almost linearly to the logarithm of the rate of shear (Fig. 2). Thus

$$\log \eta = \log \eta' - \alpha \log \bar{V}$$

or

$$\eta = \eta'/\bar{V}^{\alpha}$$

where η' is a constant and $-\alpha$ is the slope of the logarithmic flow curve.

LATERAL VARIATION OF RATE OF SHEAR

When the paste is being sheared, the movement of the pendulum is substantially horizontal; equating the work done by the pendulum to its loss of kinetic energy,

$$\tfrac{1}{2}m\dot{x}_0^2 - \tfrac{1}{2}m\dot{x}^2 = \int_0^x \left[\eta'\dot{x}^{(1-\alpha)}/d^{(1-\alpha)}\right]2xr \, . \, dx$$

when the value of x lies between 0 and l.

Differentiating with respect to time

$$\ddot{x} + a\dot{x}^{(1-\alpha)}x = 0 \text{ where } a = 2\eta' r/md^{(1-\alpha)}$$

The solution of this differential equation, after inserting the appropriate boundary conditions, is:

$$\dot{x} = \left[\dot{x}_o^{\alpha+1} - \tfrac{1}{2}a(\alpha+1)x^2\right]^{1/(\alpha+1)} \quad (3)$$

Similarly, as the volume of paste being sheared decreases with time and x lies between l and $2l$,

$$\dot{x} = \left[\dot{x}_o^{(\alpha+1)} - a(\alpha+1)(2lx - l^2 - x^2/2)\right]^{1/(\alpha+1)} \quad (4)$$

From these expressions the work done in a small displacement of the pendulum may be deduced; the total work W_1 done by the pendulum is then found by integration:

$$W_1 = \tfrac{1}{2}m\left\{\dot{x}_0^2 - [\dot{x}_0^{(\alpha+1)} - a(\alpha+1)l^2]^{2/(\alpha+1)}\right\}$$

Using the mean of the initial and final rates of shear in place of the true rate of shear, the total work W_2 done by the pendulum is:

$$W_2 = aml^2\left\{\tfrac{1}{2}\dot{x}_0 + \tfrac{1}{2}[\dot{x}_0^{(\alpha+1)} - a(\alpha+1)l^2]^{1/(\alpha+1)}\right\}^{(1-\alpha)}$$

Since the calculated viscosity is inversely proportional to the work done, it follows that the error which arises when the simple expression (1) is used is: $[(W_1/W_2) - 1]$ or

$$\left[\left[\dot{x}_0^2 - \left\{\dot{x}_0^{(\alpha+1)} - a(\alpha+1)l^2\right\}^{2/(\alpha+1)}\right]/ 2al^2\left[\tfrac{1}{2}\dot{x}_0 + \tfrac{1}{2}\left\{\dot{x}_0^{(\alpha+1)} - a(\alpha+1)l^2\right\}^{1/(\alpha+1)}\right]^{1-\alpha}\right] - 1 \quad (5)$$

Table 1. *Percentage errors arising from the variation of rate of shear as the volume of paste being sheared varies, for pastes with linear flow curves on a log-log scale passing through the point* $\eta = 316$, $\bar{V} = 3162$

$\dot{x}_0 = 220$	200,	184,	165 cm/sec
% error ($\alpha = \frac{1}{4}$) 0·4	1·5	6·2	
$\dot{X}_0 = 181$ cm/sec			
% error ($\alpha = \frac{3}{4}$) 0·14	0·18	0·4	1·1
$\dot{X}_0 = 153$ cm/sec			

When $\eta = 316$ poises, $\bar{V} = 3\,162$ reciprocal seconds, $\alpha = \frac{1}{2}$ and $\frac{1}{4}$, the errors arising from the use of expression (1) are as given in Table 1. The error increases rapidly as the value of $\dot{x}_0$ approaches the critical value, $\dot{X}_0$, which is just sufficiently large for the pendulum to shear the paste. Such conditions need never be used, however; in practice, the initial height is chosen so that the pendulum has a substantial velocity at the end of the shearing, and the error due to this variation of rate of shear can be neglected.

RADIAL VARIATION OF RATE OF SHEAR

The rate of shear varies linearly from $R = R_1$ to $R = R_2$ and hence,

$$\dot{x} = \dot{x}_{y=0}(1 + y/R_1)$$

The work done when a horizontal strip of height dy, of paste is sheared

is $$P(1 + y/R_1)^{(1-\alpha)}dy$$

where $$P = 2l^2\eta'\dot{x}_{y=0}{}^{(1-\alpha)}/d^{(1-\alpha)}$$

(the factor 2 arising since two layers of paste are sheared).

Integration between the limits $y = 0$, $y = r$ gives the total work done W_1:

$$W_1 = PR_1[(R_2/R_1)^{(2-\alpha)} - 1]/(2 - \alpha)$$

By taking the mean of the two extreme rates of shear, the work W_2 done by the pendulum is:

$$W_2 = P(R_2 - R_1)(R_1 + R_2)^{(1-\alpha)}/(2R_1)^{(1-\alpha)}$$

Hence the error in the viscosity arising from the use of expression (1) is: $[(W_1/W_2) - 1]$ or

$$\left\{2^{(1-\alpha)}(R_2^{(2-\alpha)} - R_1^{(2-\alpha)}/ (2 - \alpha)(R_2 - R_1)(R_1 + R_2)^{(1-\alpha)} - 1\right\} \quad (6)$$

When $\eta = 316$, $\bar{V} = 3\,162$, the errors for various values of α (Table 2) are so small that they can be neglected in all cases. When the paste in contact with the plate is renewed between successive measurements, the viscosities at a constant rate of shear (calculated by using expression (1)) agree within $\pm$ 7%.

Table 2. *Percentage errors arising from the radial variation of rate of shear for pastes having linear flow curves, on a log-log scale passing through the point* $\eta = 316$, $\bar{V} = 3162$

α	% *error*
0	0·0
$\frac{1}{4}$	0·07
$\frac{1}{2}$	0·09
$\frac{3}{4}$	0·16
1	0·0

($\alpha = 0$ corresponds to Newtonian behaviour and the work done is proportional to the rate of shear; $\alpha = 1$ implies that the work done is independent of the rate of shear.)

RISE OF TEMPERATURE

Certain assumptions must be made in order to calculate the rise of temperature. They are: (*a*) that the work done in shearing the paste is converted instantaneously into

heat; and (*b*) that the rate of shear is uniform throughout and is equal to the mean of the initial and final rates of shear.

As the temperature of any volume element of paste increases, its viscosity falls, and to permit an exact treatment, the resultant variation of rate of shear across the gap at any time would have to be taken into account. For the present purpose, however, it is considered justifiable to neglect this effect.

The total heat produced in a vertical strip of width dx is:

$$\eta\bar{V}rl\,dx/J$$

Thus the total heat produced in the whole volume of paste is:

$$\eta\bar{V}rl\int_0^l dx/J$$

or

$$\eta\bar{V}rl^2/J$$

The time τ for which any element of paste is sheared is $\tau = l/\dot{x}$, or as $\bar{V} = \dot{x}/d$, $\tau = l/d\bar{V}$. Also the volume of paste is (rld); hence the heat A_0 produced per unit time per unit volume is $\eta\bar{V}^2/J$.

To calculate the temperature distribution across the gap, the system is regarded as a thin plate of poorly conducting material in which heat is produced at a uniform rate, this plate being contained between conducting boundaries which remain at their initial temperature throughout the duration of shearing.

The distribution of temperature in such a system is given by Carslaw and Jaeger[4] as:

$$\theta - \theta_0 = (A_0/2K)\{z(d-z) - (8d^2/\pi^3)\sum_{n=0}^{\infty}[(\exp\{-k(2n+1)^2\pi^2 t/d^2\})(\sin\{(2n+1)\pi z/d\})/(2n+1)^3]\}$$

Averaging over all values of z gives the mean temperature in the paste at any time

$$\bar{\theta}_x - \theta_0 = (A_0 d^2/2K)\{(1/6) - (16/\pi^4)\sum_{n=0}^{\infty}[(\exp\{-k(2n+1)^2\pi^2 t/d^2\})/(2n+1)^4]\}$$

The time average of the temperature during the shearing can be obtained by integrating $(\bar{\theta}_x - \theta_0)/\tau$ with respect to t between the limits 0 and τ. Thus, after substituting for A_0,

$$\bar{\theta}_{xt} - \theta_0 = (d^2\eta\bar{V}^2/2KJ)\{(1/6) - (16/\pi^4)\sum_{n=0}^{\infty}[\{d^2/\pi^2(2n+1)^6 k\tau\}\{1 - \exp(-k(2n+1)^2\pi^2\tau/d^2)\}]\}$$

or

$$\bar{\theta}_{xt} - \theta_0 = (d^2\eta\bar{V}^2/2KJ)\left[(1/6) - (16c/\pi^4)\left\{1 - \exp\left(-\frac{1}{c}\right)\right\}\right] \quad (7)$$

where $c = d^2\bar{V}/\pi^2 kl$, and terms in which n is greater than zero are neglected as they are negligibly small.

The thermal conductivity of a typical p.v.c. paste was 3×10^{-4} cal cm^{-1} sec^{-1} per 1° C or $1{\cdot}3 \times 10^{-3}$ joule cm^{-1} sec^{-1} per 1° C as measured by a modified Lee's disk method; its diffusivity was $7{\cdot}5 \times 10^{-4}$ cm^2 sec^{-1}.

Using expression (7), the temperature rises during shearing of the paste were calculated (Table 3).

Table 3. *Temperature rise during shearing*

η poises measured	η poises corrected for temperature rise	rate of shear sec^{-1}	d cm	$\bar{\theta} - \theta_0$ °C
112	200	25 400	$7{\cdot}6 \times 10^{-3}$	12·6
139	200	16 900	$7{\cdot}6 \times 10^{-3}$	8·7
800	851	1 240	$8{\cdot}4 \times 10^{-2}$	1·4

MODE OF OPERATION

The pendulum was released from a selected height and the height to which its centre of gravity rose was observed. As the work done in moving the pointer over the ratchet was appreciable, this procedure was repeated several times, the pointer being set initially within 1 cm of its anticipated final position. In this way, the effect of friction between pointer and ratchet was eliminated. The release mechanism was set at suitable points along the whole of the right-hand side of the scale and the height to which the centre of gravity rose was noted in each case.

Next, sufficient paste was spread on the four shearing surfaces to ensure that the gaps would be filled when the pendulum was at its lowest point. The pendulum was released and the height to which its centre of gravity rose was noted. The observation was repeated several times with the pointer set initially to within 1 cm of its final position, a fresh application of paste being made after each observation. The initial height from which the pendulum was released was then varied in suitable steps. Corresponding series of observations were made in which the gaps between the plates were varied.

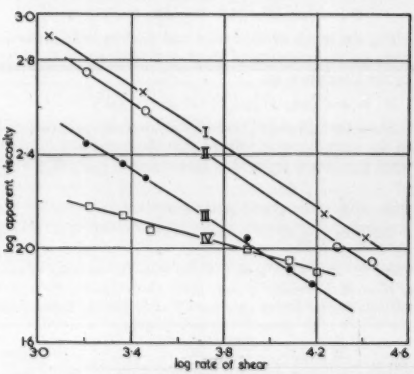

Fig. 2. Variation of apparent viscosity with rate of shear

Curves I and II: p.v.c. in dioctylphthalate (pigmented). Curve III: p.v.c. in tricresyl phosphate. Curve IV: pigment (light ultra blue) in tricresyl phosphate.

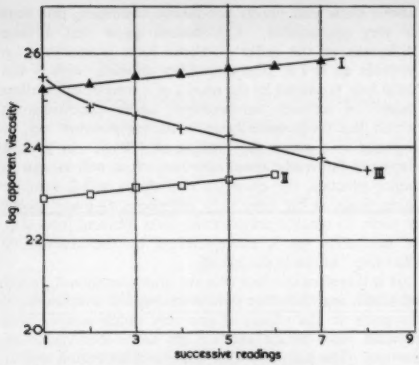

Fig. 3. Examples of thixotropy or shear hardening

Curves I and II: p.v.c. in tricresyl phosphate. Curve III: p.v.c. in tricresyl phosphate (pigmented).

SOME TYPICAL RESULTS

The results obtained with a few typical industrial pastes are depicted in Fig. 2, where $\log \eta$ is plotted against log (rate of shear). The variation of the viscosity of paste No. 1 with temperature, at a low rate of shear, was measured with a cone and plate viscometer. Using the values of $(\theta_{xt} - \theta_0)$ calculated for this paste, the viscosities measured with the pendulum viscometer were corrected on the assumption that the temperature coefficient of viscosity is the same at high rates of shear as at low (Table 3). It is probable, however, that the temperature coefficient will be less at high rates of shear, and the actual errors may be smaller than the calculations suggest.

The pendulum viscometer has also been used to detect either thixotropy or shear hardening in plastic mixes at high rates of shear. To do this, the sample of paste is sheared several times at a constant rate of shear. Between successive tests, the paste is redistributed over the shearing surfaces and left for about a minute. Under these conditions, the viscosity of a thixotropic paste decreases progressively, whereas that of a paste which exhibits shear hardening rises. The changes in viscosity are not caused by temperature changes, for even with such poor thermal conductors, the temperature falls to that of the boundaries well within the time interval between successive observations. The results for three pastes are shown in Fig. 3; two exhibit shear hardening and one thixotropy.

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Steam heat transfer to small drops of water

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In cases of drop heat transfer in which a saturated atmosphere condenses into the liquid of the drop, a point is reached when the drop is at the atmospheric temperature. If formal treatment is given to the heat transfer up to that time, the end temperature difference being zero, the conception of logarithmic mean temperature difference cannot be used and an arbitrary choice which may give misleading results must be made. That heat transfer does continue with this apparent zero difference is obvious and therefore a potential difference must exist which is shown to be expressible as an equivalent temperature difference.

The rate of flow of heat into a curved surface is by no means the same as that into a plane surface of the same area, since the lines of heat flow, i.e. the orthogonals to the isotherms, are parallel in the latter case but not in the former. In addition, the vapour pressure in the vicinity of a convex surface is greater than that adjacent to a plane surface with the temperature (potential) difference apparently the same. One of the first difficulties encountered in establishing a method of measuring heat transfer between a drop of water projected into a steam atmosphere, especially at the stage beyond which evaporation begins, is the correct interpretation of temperature difference. Because, at saturation temperature, there is apparently no temperature difference, the use of the simple concept of logarithmic mean temperature difference (l.m.t.d.) is impossible. Some experimenters fall back on an arbitrarily chosen arithmetic mean, between the conditions at which the drop enters the system and those at saturation point. Since the temperature rise might be, and usually is, rapid in the early stages, however, there is a disproportionate part of the drop's path in the steam in which the temperature difference is very small and the choice of the arithmetic mean can therefore only lead to unreliable results. There is, of course, no doubt that energy exchanges actually take place at what is apparently a zero temperature difference (e.g. condensation and evaporation) and there must therefore be a potential

energy difference which can be expressed in terms of a temperature difference. This difference, however minute, will enable a finite value to be assigned to the l.m.t.d.

TEMPERATURE DISTRIBUTION UNDER STATIC CONDITIONS

Investigations of the temperature distribution inside a spherical drop of water in a steam atmosphere can be found in many text-books on theoretical heat flow. For example, reference can be made to Carslaw and Jaeger.[1] The results are based on a Fourier solution of radially conductive heat flow and are usually expressed in the form of a curve showing temperature distribution with radius for different values of a dimensionless parameter which is a combination of physical properties, drop size and time. For static conditions, the assumptions usually made for temperature distribution in a sphere, initially at a uniform datum temperature, after a certain time of exposure to an infinite atmosphere at a second uniform temperature, are that the boundary sphere is at atmospheric temperature throughout (N.B.—zero temperature difference for heat transfer); the temperature at all other radii is datum at zero time, and varies with time; the temperature at infinite time is atmospheric at all radii.

When these end conditions are applied to the general equation of spherical conduction,

$$\frac{2}{r}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial r^2}=\frac{\rho s}{k'}\frac{\partial\theta}{\partial t} \tag{1}$$

we obtain finally the temperature at any radius r (in a sphere of radius r_0) and time t as,

$$\theta_{r,t}=\theta_s+\frac{2r_0\theta_s}{\pi r}\sum_{n=1}^{n=\infty}\frac{(-1)^n}{n}\exp\left(-\frac{k'}{\rho s}\frac{n^2\pi^2}{r_0^2}\right)\sin\frac{n\pi r}{r_0} \tag{2}$$

This of course, cannot be used in any consideration of surface heat transfer, for at $r=r_0$, independently of t:—

$$\theta_{r_0,t}=\theta_s \tag{3}$$

However by considering the heat capacity of concentric elementary spherical shells, making only the assumption of constant specific heat, the mean temperature of the whole sphere can be given, by suitable integration, in terms of t and r_0, and, of course, independent of r:—

$$\frac{\overline{\theta}_t}{\theta_s}=1-\frac{6}{\pi^2}\sum_{n=1}^{n=\infty}\frac{1}{n^2}\exp\left(-\frac{k'}{\rho s}\frac{n^2\pi^2 t}{r_0^2}\right) \tag{4}$$

It will be found, that by equating (4) to (2), the radius at which the representative (heat capacity) temperature exists, varies with time for any given sphere.

It soon becomes obvious that conventional methods of heat transfer are quite intractable when this kind of conductive distribution is assumed. This is, of course, no argument for making an arbitrary choice of method. Equation (4), however, indicates the time required for a given heat capacity to be reached. In fact, under the above static and purely conductive condition, this time is very appreciable. Calculations show that a time difference of the order of about $\frac{1}{4}$ sec is necessary to provide an 0·1 in diameter drop of water with $\frac{1}{2}$ the total heat predicted by the results of a simpler calculation based on uniform temperature, under conditions in which the drop, initially at room temperature say, is exposed to a steam atmosphere at 250° F. In $\frac{1}{4}$ sec a drop can fall freely about *nine feet*. It is well known in boiler practice, for example, that drops which may be three times as big need only fall about as many *inches* in order to reach a temperature little different from that of saturation (as is made evident by the absence of "hunting" in the boiler cycle).

It is therefore obvious that the above treatment, based on static, and therefore purely conductive, conditions, is no guide to the effects of the very much greater heat transfer rates which are set up under conditions of motion. The adoption of a uniform distribution with a mean temperature, however, gives results very close to those which can be measured. As long as this is done, i.e. a mean temperature assumed, the standard treatment which results in the application of a l.m.t.d., can be applied. (The error in so doing may be almost completely balanced out by the assumption of a slightly distorted time scale—usually having no very significant difference.) Some very generalized experiments in the case of conditions similar to those of boiler feed have indicated that it is possible to treat drops up to the order of $\frac{1}{16}$ in diameter, at any rate, as uniform as far as temperature distribution is concerned.

ESTIMATE OF TEMPERATURE DIFFERENCE

Both theory and experiment, e.g. Houghton,[2] show that the diameter of a drop varies as a parabolic function of the time when evaporation or condensation occur, so that the surface area changes directly with time. This leads to

$$\frac{dM}{dt}=-2\pi Ka(\rho_0'-\rho') \tag{5}$$

whence

$$a^2=a_0^2-\frac{8K}{\rho}(\rho_0'-\rho')t \tag{6}$$

where $\frac{dM}{dt}$ = mass change rate,

K = diffusivity of the vapour in the atmosphere surrounding the drop,

a = drop diameter at time t,

a_0 = drop diameter at time t_0,

ρ = liquid density,

and $\rho_0'-\rho'$ = vapour density difference between surface of drop and surroundings.

Now for radial heat conduction in the spherical shell immediately surrounding the drop,

$$\frac{dH}{dt}=k4\pi r^2\frac{d\theta}{dr} \tag{7}$$

whence $$\frac{dH}{dt} = -2\pi k(\theta_s - \theta_0')a \qquad (8)$$

where $\frac{dH}{dt}$ = heat transfer rate,

k = thermal conductivity of the atmosphere,

ρ', θ_s = ambient density and temperature,

and ρ_0', θ_0' = saturation conditions at the immediate drop surface.

It will be seen that upon multiplying equation (5) by l, the latent heat, substitution can be made in equation (8) and, finally,

$$\theta_s - \theta_0' = \frac{Kl}{k}(\rho_0' - \rho') \qquad (9)$$

which gives a relation between the local temperature and the local vapour pressure differences. It is obvious that ρ_0' and θ_0' in equation (9) are inter-related, and without further information, the expression could only be used by means of trial and error methods. One can, however, consider the interchange of energy taking place at the surface of a drop in relation to the surface energy.

When heat exchanges take place at a plane surface, no surface energy is gained or lost. When a curved surface is involved in condensation or evaporation however, there is generally a change in the total surface area, and therefore energy, which must be accounted for in any energy balance. For evaporation under so-called constant temperature conditions energy has to be supplied to provide the latent heat of the vapour formed and part of this is obtained from the surface energy which is lost by the area decrease. Thus a drop will continue to evaporate into a space which would be in vapour equilibrium with a plane surface and, in fact, there can be no true equilibrium between a drop and the surrounding saturated vapour. As the drop diminishes, a regime will be reached at which the surface energy loss is sufficient to provide *all* the latent heat. (The converse is also true, and a drop therefore requires a nucleus for its initial formation.)

Kelvin[3] first examined the equilibrium of a drop of a given size with the surrounding vapour. He showed that as a consequence of the surface tension, the equilibrium depended on the state of supersaturation of the vapour in the vicinity of the convex surface. Along these lines, he obtained the relationship

$$ln\frac{p_0'}{p'} = \frac{4S}{R\theta_s\rho a} \qquad (10)$$

where p_0' and p' = vapour pressures at surface and surroundings, respectively,

S = surface tension of fluid to atmosphere,

and R and θ_s = gas constant and absolute temperature of surrounding atmosphere.

Equation (10) gives the vapour pressure ratio for equilibrium with a curved surface of diameter a and a flat surface. For a given degree of supersaturation, drops smaller than a will continue to evaporate, while those larger will continue to grow. The effect is clearly very small and for an idea of the magnitude the following values for water at 65° F are quoted from Saha and Srivastava.[4]

drop diameter $mm \times 10^{-6}$	$\frac{p_0}{p'}$
100	1·01
10	1·12
1	2·97

For the case of an ideal gas, the ratio of the vapour pressures is equal to the ratio of the vapour densities, and in the case of steam, this is nearly true over a fairly wide range of working pressures. If, therefore, this approximation is made, equation (10) can be arranged in the form

$$\rho_0' - \rho' = \rho'\left[\exp\left(\frac{4S}{R\theta_s\rho a}\right) - 1\right] \qquad (11)$$

Combining with equation (9) and noting that $K = \frac{k}{\rho c}$, where c is the specific heat of the atmosphere,

$$\theta_s - \theta_0' = \frac{l}{c}\left[\exp\left(\frac{4S}{R\theta_s\rho a}\right) - 1\right] \qquad (12)$$

Now the index of the exponential is very small, and it is therefore quite reasonable to expand it ignoring all powers of the index above the first. Finally, therefore, an expression is obtained which gives a value to the effective temperature difference operative at the surface of a drop of water in a steam atmosphere, when both are apparently at the same (saturation) temperature. This is

$$\theta_s - \theta_0' = \delta\theta = \frac{l}{c}\frac{4S}{R\theta_s\rho a} \qquad (13)$$

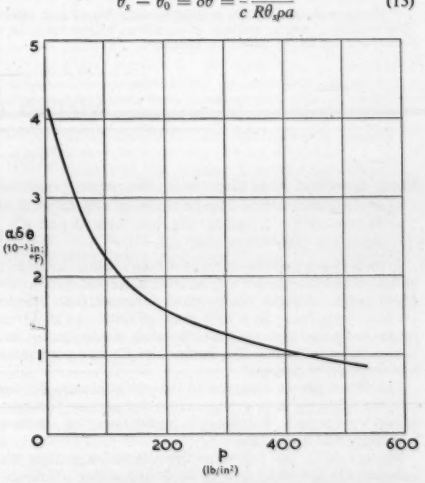

Fig. 1. Temperature difference at saturation conditions

For convenience of evaluation, the expression may be re-grouped as

$$a \,.\, \delta\theta = \frac{4lS}{cR\theta_s\rho} \qquad (13a)$$

where the left-hand side, which includes the drop diameter, is now a function of the physical properties only.

Fig. 1 shows the value of $a \,.\, \delta\theta$ in terms of the absolute pressure corresponding to various saturation conditions. From such a curve,* the operative temperature difference at saturation point may be estimated for any drop size. An example of the small magnitude of this difference can be shown for the case of a drop of water $\frac{1}{32}$ in. in diameter at the saturation temperature corresponding to a steam atmosphere at 300 lb/in^2 abs., when the value of $\delta\theta$ is 0·040° F. It should be stressed, however, that the use of this figure in the calculation of a l.m.t.d., provides a finite value, however small, which is different from the arithmetic mean value to which recourse must be made if this point is assumed to have zero temperature difference. If the same drop entered the steam at a temperature of 100° F, the arithmetic mean temperature difference would be about 159° F; the l.m.t.d. would be zero if the end difference were assumed to be zero, and actually about 35° F for the above calculated potential difference, or less than a quarter of the arbitrary value. Thus it can be seen at once that measurements of heat transfer coefficients for small drops which attain saturation temperature and which are not based on the conception of l.m.t.d. may lead to results which cannot be considered reliable.

* The sources used in this evaluation were Marks and Davis' "Steam Tables," and an estimate of the surface tension based on a linear variation to the critical temperature.

NOMENCLATURE

M = mass flow of fluid
K = Diffusivity of the vapour in the atmosphere surrounding the drop
t = time
r = radius
a = drop diameter at time t
a_0 = drop diameter at time t_0
r_0 = drop radius at time t_0
ρ = density of fluid
ρ' = steam density
ρ'_0 = vapour density close to a small drop
θ = temperature
θ_s = steam saturation temperature
θ'_0 = temperature close to a small drop
H = quantity of heat
k = thermal conductivity of the atmosphere
k' = thermal conductivity of the fluid
l = latent heat of evaporation
S = surface tension of fluid to atmosphere
s = specific heat of fluid
c = specific heat of atmosphere
p' = vapour pressure of steam atmosphere
p'_0 = vapour pressure close to a small drop
R = gas constant

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(3) KELVIN. *Proc. Roy. Soc. Edin.*, 7 (1870).
(4) SAHA, M. N., and SRIVASTAVA, B. N. *A Treatise on Heat* p. 261 (Calcutta: Indian Press, 1935).

NOTES AND NEWS

New Books

Some Aspects of Fluid Flow, being the papers presented at a Conference of The Institute of Physics held in March 1950. (London: Edward Arnold and Co.) Pp. vii + 292. Price 50*s.* net.

In recent years great advances have been made in the science of fluid flow in connexion with aircraft, ships and certain new power units. Possibly due to these advances, fresh attacks are now being made on a wide range of fields and problems related to the processes that form the bases of many industries. It is to these fields that The Institute of Physics' conference was particularly directed.

The fifteen papers presented to the conference are divided into the following four groups: industrial problems; fundamental problems; techniques; applications of present knowledge and techniques.

Reports from the following five discussion groups are included: (1) combined flow of fluids and solids, (2) fundamentals of mixing and flow patterns, (3) flow problems in industries employing high temperature furnaces (steel, glass, etc.), (4) flow problems in the process industries (gas, oil, chemical, etc.), (5) flow problems in industries based on steam generation (power industries).

The concluding statement and summary of the conference is also included in this volume. Sir Charles Goodeve, the Director of the British Iron and Steel Research Association, contributes the foreword, in which he writes: "This volume should prove of great value to the many scientists and engineers all over the world who are concerned with fluid flow problems."

Advanced Dynamics, Parts I–II. By E. HOWARD SMART, M.A. (London: Macmillan and Co. Ltd.) Pp. v + 419. Price 40*s.* net each volume.

These two volumes are based on E. H. Smart's former lecture courses to B.Sc. General and B.Sc. Special Mathematics students at Birkbeck College, London. Volume I and

part of Volume II together cover adequately the dynamics of the B.Sc. General course, and the remainder of the second volume deals with the more advanced parts of the subject included in the B.Sc. Special Mathematics examination. The author died in 1945 and, unfortunately, saw only the first proof of Chapter I of the first volume. The books have been seen through the press by F. G. W. Brown, who himself died in 1949, and one of his colleagues has completed the task.

The treatment of the subject is excellent and obviously moulded by many years' lecturing experience. The fundamental principles are carefully explained, their scientific character and not merely their mathematical formulation being emphasized. The growth of the science of mechanics in history is helpfully made clear by short notes on some of the men who have contributed to its development; these include, of course, Galileo, Newton, D'Alembert, Lagrange and Hamilton.

Volume I discusses the motion of a particle in one and two dimensions, and there is a final chapter on motion in three dimensions. All this is very competent, detailed and orthodox, with the discussion of Newton's laws outstanding. Volume II deals with the motion of a rigid body; the basic equations are derived from D'Alembert's Principle and consequently a fundamental theorem in statics has to be invoked. In keeping with this, dynamical problems are solved by equating the system of effective forces to the system of external forces. This will not appeal to all teachers, particularly those who would give a more prominent place than is here given to the principles of linear and angular momentum. But teachers and students alike will appreciate the reference (p. 71, Vol. II) to "the equations of emotion." Incidentally the figure on p. 71 would be improved by the inclusion of *all* the forces acting on the system. The suggestion that beginners should draw two diagrams labelled "cause" and "effect," as on p. 91, Vol. II, is admirable. Other chapters in the second volume give a sound introduction to Lagrange's equations, Hamilton's equations, and the theory of small oscillations. Throughout the text numerous examples are solved and should prove helpful to the reader; an ample number of exercises is included. The short chapter on dimensions and dynamical similarity would, however, be improved if a few exercises were given. The addition law for vectors and a few applications are discussed early in Volume I, but Cartesian methods are used throughout.

The publishers express the hope "that the complete work will be not only a memorial to a distinguished mathematical scholar and teacher, but also a valuable course of Dynamics for students of both pure and applied mathematics." It is certainly a clear and careful exposition of classical dynamics, and will be found useful by physicists and others whose work overlaps this field. J. TOPPING

Textbook of Electrochemistry. By G. KORTÜM and J. O'M. BOCKRIS. (London: Elsevier Publishing Co. Ltd.) Pp. v + 351. Price 50*s*. net.

Kortüm's *Lehrbuch der Electrochemie* dealt with the fundamental thermodynamic principles of electrochemistry, especially with applications to physico-chemical problems. An English edition, based on a revised second German edition and with a new chapter on experimental technique and an extended bibliography has now been prepared by Bockris and published by the Elsevier Publishing Co.

The text, it is stated, is intended primarily for university students and calculations which extend beyond the fundamental conceptions of differential and integral calculus have been omitted. Technical processes are only lightly touched upon with restricted account of their industrial importance. The text is in two parts and the reviewer has, so far, only seen volume one. It is excellently produced, on expensive thick paper, with large print and wide margins. There is little regard for economy in set up. At a price of £2 10*s*. for 351 pages it is expensive (with the second volume of approximately 500 pages the total cost will be of the order of £6)—three times that of the original German edition. The complete text will be required as the subject matter of the two volumes is interconnected and the index is in volume two. It is evident, therefore, that the text must be regarded primarily as a library reference book mainly for the physical chemist.

It is now fourteen years since the Joint Committee of the Chemical, Faraday, and Physical Societies issued a report on the standardization of symbols. It should not be necessary at this stage to record over several pages a large number of physico-chemical symbols and their notation. Even with this elaboration there is some confusion of symbols, e.g. E is used for the electromotive force of a cell, on p. 247 and for potential, on p. 9, and N is employed for molar fraction in the table, yet on p. 171 it indicates ions per c.c.

Chapters I and II review the physical and thermodynamic principles of electrochemistry. Some of this treatment could be omitted or written more concisely, e.g. part of the introduction to chemical thermodynamics and the matter on partial derivatives which is to be found in many standard texts. The book is not easy reading and reference to footnotes, to some extent, upsets the sequence. Chapter III is on the theory of interionic attraction. These three chapters constitute about one-third of the book. The other six chapters of Vol. I are more on practical applications and cover solvation of ions, weak and strong electrolytes, conductance measurements, electromotive force, potentiometric measurement, and equilibria in solutions of weak electrolytes.

Chapter VIII, to take one section in some detail, gives applications of potentiometric measurements and the following topics are considered. Determination of equilibrium constants and solubilities, pH measurements, definition of pH values, pH scale from measurements with the hydrogen electrode, the quinhydrone glass and oxide electrodes; potentiometric titrations: (i) neutralization reactions, (ii) precipitation reactions, and (iii) redox reactions.

The treatment in this volume is largely theoretical, but possibly in the sections on practical technique in the second book more details and wider applications will be recorded.

For the convenience of laboratory workers and on economy grounds there would be many advantages in having the practical methods in a small separate text. J. REILLY

The High Pressure Mercury Vapour Discharge. By W. ELENBAAS. (Amsterdam: North Holland Publishing Co.) Pp. xi + 350. Price F14.50.

The widespread use of the high pressure mercury vapour discharge as a light source has led to an extensive study by many workers of the complex physical processes involved. Investigation of the luminous properties of the discharge has resulted in the evolution of techniques by which the discharge can be studied under conditions of thermal equilibrium at vapour pressures above 100 atm and at energy loadings of 10 000 W/cm of arc.

Among the workers in this field Dr. Elenbaas is an authority, and this book presents the results of many years

of investigation. The special conditions of the mercury vapour discharge in the high pressure state are fully considered and discussed. The book is concerned with the physical processes involved, and, although references are made to the use of the radiation in practical light sources, emphasis is laid on theory rather than practice. The book is consequently not likely to appeal to the illuminating engineer interested in the application of the high-pressure mercury vapour discharge to lighting; but it can be recommended to physicists and others working on gas discharge phenomena.

The subject is treated by assuming temperature equilibrium in the arc; and the data are often related to a typical discharge which is used as an example and fully defined in an appendix. After an introductory discussion of the elementary theory, the most interesting and comprehensive chapters are devoted to thermal conditions, radiation properties and convective flow. This latter aspect is particularly fully treated. In the past convective disturbances of the gas stream have not been treated so fully; possibly because, as the author points out, convection does not play a very important part in the behaviour of practical high-pressure mercury vapour discharge lamps. However, arc instability due to vapour flow can limit lamp design in certain circumstances, and becomes of major importance in high pressure discharges through gases and vapours of lower density than mercury vapour. Indeed, much of the theory given in this book can be applied to other types of high pressure discharge, and the chapter on convection in particular will prove a valuable guide to their behaviour.

It is unfortunate that, in an otherwise comprehensive work, there is little information concerning electrode stabilized arcs. A great deal of work on this most interesting type of discharge has been done in England and Germany, and it seems probable that the future development of high-pressure discharge lamps will depend upon the use of electrode stabilization or combined wall and electrode stabilization.

The book is well produced and illustrated and a comprehensive bibliography and subject index are included.

E. H. NELSON

Integral Transforms in Mathematical Physics. By C. J. TRANTER. (London: Methuen and Co. Ltd.) Pp. ix + 118. Price 6*s*. net.

The present volume, in common with others of the series, is directed towards "the Honours student and the research worker in other branches of physics, those engaged on work in related sciences and those who are no longer in contact with active scientific work." The plan and structure of the book is admirable for the purpose. It deals with those transforms which are the most useful in practice, namely, the Laplace, Fourier, Hankel and Mellin transforms. The essential formulae are given briefly but adequately. Worked and unworked examples show how solutions of natural and interesting problems can be obtained. Short sections on numerical evaluation and relaxation methods indicate how problems which do not lend themselves to expression in known functions can be tackled, apart from notes on methods of approximation in various places. There is a useful chapter on finite transforms. It is, however, a defect that too little space is given to explaining the elementary difficulties which those for whom the book is intended usually find troublesome. For example, the use of contour integration is not clearly described and the same integral is in one place called a line integral and, further on, a contour integral. Thus a class of reader, omitted from the opening quotation, namely those who wish to revive their memories and bring themselves up to date, will probably be better served, because they will not notice these defects. On the other hand, it is fair to say that the beginner can overcome his difficulties by consulting the books to which references are made. No doubt van der Pol and Bremmer's "Operational Calculus" appeared too late to be mentioned, but the Hilbert transform deserved inclusion owing to its value in linear systems. The references to Doetsch seem unduly generous, particularly in view of his superciliously unjust observations on Heaviside. In conclusion, although a few improvements would be welcome, the book is an exceedingly practical and useful monograph, published at a very moderate price.

S. WHITEHEAD

The Photographic Study of Rapid Events. By. W. D. Chesterman, B.Sc., F.Inst.P. (London: Oxford University Press.) Pp. x + 167. Price 21*s*. net.

Mr. Chesterman's book will form a very valuable reference book for the research worker, who may be faced with a problem, in the investigation of which high-speed photography can assist him. The available information upon the wide variety of specialized techniques and equipment, which have been devised and used for such investigations, is scattered throughout the vastness of the world's scientific literature. This book serves a very useful purpose in classifying the various methods and equipment according to the character, speed and duration of the phenomena to be investigated, and liberal reference is made to a bibliography of the most important original papers describing technique and equipment.

Chapter 3 deals with the electrical and luminous characteristics of the various types of light source available for use, and deals with the photographic characteristics of these sources, and of photographic materials when they are used under conditions of ultra-short exposure times, and very high illumination intensity. Chapter 7 deals with the use of the Schlieren method and its optical arrangements for photographing the effect of violent disturbances in transparent media, through their effects upon the refractive index of such media. The book is well illustrated with many useful curves and tables which will be most valuable to the investigator in helping him to determine which method, or type of equipment, he should adopt for his purpose, bearing in mind the various important considerations involved.

The only criticisms of the book are relatively minor ones, and relate to the use of expressions quite understandable, but, nevertheless, noticeable by their peculiarity. For example, on page 58 the expression "high-intensity *low*-time exposure" is used; *short*-time being the more usual and appropriate adjective. Again, on page 89, "*high*-diameter drums" and "*low*-diameter drums" are referred to, whereas *large* and *small* are the more usual adjectives. Among other expressions, which tend to offend by their lack of precision, or clumsiness, may be mentioned "camera instigated stroboscopic light" (page 36), and "tubes filled with 75 cm of argon" (page 51). Among the minor errors may be mentioned that on page 82, paragraph 1.3, under the sub-title "Frame Division." As the data refers to drum camera equipment for which 35 mm *unperforated* film can be used, the maximum square picture size possible is 35 × 35 mm, and not 24 × 24 mm. The American Standards *Institution* is mentioned on page 58, paragraph 1.3; this, of course, should be *Association*.

These minor criticisms are completely offset by the obvious

value of this book to the research worker in all spheres of scientific investigation, and Mr. Chesterman and the Oxford University Press are to be congratulated upon its preparation and publication. E. D. EYLES

English-Russian Technical and Scientific Dictionary. Compiled by I. CHERNUCHIN. (New York: International University Press. London: George Allen and Unwin Ltd.) Pp. 674. Price 80*s.* net.

It is not easy to review a dictionary. It is always possible that the small part of the field which the reviewer is really competent to judge may not constitute a representative sample. Nevertheless, a technical and scientific dictionary should, throughout its whole scope, enable the user to find the correct technical equivalents for those terms which, in ordinary parlance, lack precision. Thus, in an ordinary dictionary, a single word may perhaps justifiably be given to represent strength, power, force, energy and action, but the physicist turns to a technical dictionary to find the correct equivalent for each of these concepts as dimensionally defined. In referring to the work under review, he will be quite at a loss to know in which sense or senses the word "silo" (СИЛО) is to be used. Such cases could easily be clarified by the addition of dimensional symbols, e.g. MLT^{-2} for force, etc. Again a general dictionary might allow the use of "viscosity" to mean "stickiness," but a scientific dictionary is of little use if the technical word for "viscosity" and two words for "stickiness" are all given indiscriminately under both headings. "Stress" and "strain" are also badly confused. Still confining himself to his own field, the reviewer has looked in vain for the semi-official list of technical translations of very important terms used to describe viscosity-concentration relations ("intrinsic viscosity," "reduced viscosity," "specific viscosity," etc.) given, to 1949, by Slonimskiĭ, Voyutskiĭ and Margolina (*Kolloidnyi Zh.*, **11**, p. 116). The dictionary, unfortunately, bears no date, but it is to be presumed that this important paper was available in time to be included.

For an American publication, the printing is poor; not only the Russian, but even the English words are often difficult to read. When a word has several meanings, these are marked 1, 2, 3 . . ., but it is often difficult to tell to which sense of the English word the Russian term refers.

It is also a pity that the accentuated syllables in the Russian words are not marked, and even the ë's are not indicated. The reader might as well be helped not only to read but also to pronounce the words. Even the transliteration of the author's name is misleading. The Russian characters are phonetic; in English-speaking countries the first and sixth letters of the author's name are conventionally distinguished by transliteration as "Ch" and "Kh" respectively. (It may be, of course, that if the author is a Russian who has settled in America, he has long ago chosen, for some reasons unconnected with dictionaries, to transliterate his name in this unfortunate way.)

A good technical English-Russian dictionary has long been needed; unfortunately, one cannot feel that the work under review fulfils this requirement. G. W. SCOTT BLAIR

Principles of Voltage Regulation. By B. B. RAY, F.Inst.P. (Calcutta: Thacker Spink and Co.) Price 15 Rs.

This is a book primarily for electrical engineers and it deals with automatic voltage regulation of generators, transmission lines and feeder circuits. There has been little in the way of separate works on this subject up to the present and the book under review is therefore welcome.

The author deals firstly with the principles of automatic voltage regulation and with supplying power to a consumer at constant voltage. He then treats of the vibrating-contact type of voltage regulator, the sector disk and carbon pile direct-acting regulators and the motor-operated rheostat type of regulator with voltage-sensitive relay. All these, appropriate to generator voltage-control, are discussed in some detail and the author is obviously on familiar ground here as a practising engineer; even maintenance and setting-up information is given for these types. Equally fully treated is the chapter dealing with tap-changing gear for transformers. The latest types of regulator using thyratrons, amplidynes or transductors are discussed more briefly and only serve to give the broad outlines of these types though the former, being a fast-acting type, is of importance and is interesting, say, when applied to diesel-engine-driven generators where some correction for cyclic irregularity can be achieved. As the book does not deal with supplies for instruments the inclusion of a section (p. 174 *et seq.*) on a particular type of neon-tube stabilizer is out of place; this, together with the single sentence referring to constant-voltage transformers, may lead an inexperienced reader to think that everything about voltage regulation is covered in this book.

It is gratifying to note the speed of publication; issued in 1950, the book carries bibliographical references to papers published in technical journals for the same year.

A. J. MADDOCK

Notes and Comments

Elections to The Institute of Physics.

The following elections have been made by the Board of The Institute of Physics:

Fellows: W. G. Darvell, N. Pentland, J. Yarwood, S. L. Martin, W. C. Price.

Associates: D. Allan, M. H. H. Bunton, M. K. Gharpurey, W. F. Harling, J. M. McAlister, J. M. Rooum, L. W. Williams, R. E. Burnett, J. Ewels, B. L. Ginsborg, L. J. Hastewell, W. P. Osmond, W. E. Rowlands.

Forty-one Graduates, ten Subscribers and fourteen Students were also elected.

Acustica.

We have received the first issue of the new international journal entitled *Acustica*. The Editor-in-Chief is Dr. C. W. Kosten of Delft, Holland, and the English editor is Dr. E. G. Richardson of Newcastle-upon-Tyne. The preferred languages are English, French and German, but papers in other languages may be accepted. Abstracts in the three languages mentioned are given for each paper. The new journal aims to cover the science of acoustics in all its aspects, theoretical and experimental, purely scientific and applied. *Acustica* will be issued in six numbers per annum of approximately 48 pages each; and each page carries approximately

the same number of words as this journal. In the issue before us, there are the following papers: "Ultra-sons dans l'air et leurs applications," by F. Canac and V. Gavreau; "A tentative method for the measurement of indirect sound transmission in buildings," by E. Meyer, P. H. Parkin, H. Oberst and H. J. Purkis; "Les gammes et le temperament égal," by A. D. Fokker; "Die Abstrahlung von Impulsen durch ebene Kolbenmembranen in starrer Wand," by F. A. Fischer; and "Investigation of sound diffusion in rooms by means of a model," by T. Somerville and F. L. Ward.

The journal is published by S. Hirzel, Claridenhof, Gotthardstrasse 6, Zurich, the subscription rate being 36 Swiss francs per volume or the equivalent in other currency.

New examination for Graduateship of The Institute of Physics.

In September 1949 The Institute of Physics established a new grade of membership—Graduateship (Grad.Inst.P.)—for those holding a recognized degree or other qualification in physics but not having the necessary period of professional experience required for Associateship of the Institute. Nearly 500 applicants have been elected to the new grade.

The Board of the Institute now announces the establishment of an examination by which those not holding a recognized university degree may satisfy the academic requirements for election to Graduateship and, subsequently, when the necessary approved experience has been acquired, for the acquisition of professional status as a physicist by transfer to the corporate grade of Associate of the Institute.

Candidates for the examination will normally be required to have followed a suitable course of study at a college or other institution recognized for this purpose by the Institute. The subjects of the examination are physics (three papers and a practical examination), mathematics (two papers) and a third subject (one paper) chosen by the candidate from (*a*) applied physics, (*b*) more advanced physics, (*c*) mathematical physics and (*d*) statistics. Those choosing applied physics will be required to offer only one of the sections (i) electronics, (ii) high vacuum technology, (iii) acoustics, (iv) spectroscopy, (v) X-rays or (vi) temperature measurement.

Direct election to Graduateship without examination will remain open to holders of recognized university degrees in physics. Certain other degrees or other qualifications such as the Higher National Certificate in Applied Physics will entitle the holders to exemption from up to four of the six papers of the examination.

The first examination will be held in June 1952, for which the closing date for the return of application forms is 31 December, 1951.

Copies of a booklet containing the full regulations and syllabuses of the Graduateship examination may be obtained free, on request, from The Institute of Physics, 47 Belgrave Square, London, S.W.1.

Symbols, signs and abbreviations recommended for British scientific publications.

A report by the Symbols Committee of The Royal Society representing The Royal Society, The Chemical Society, The Faraday Society and The Physical Society has been published. It supersedes the Report of the Joint Committee of The Chemical Society, The Faraday Society and The Physical Society on *Symbols for Thermodynamical and Physico-Chemical Quantities and Conventions relating to their Use*, issued in 1937. Its recommendations are grouped under (i) General Principles and (ii) Specific Usages which include symbols for physical quantities; examples of subscripts and other modifying signs; examples of symbols and abbreviations denoting mathematical operations and constants; abbreviations (symbols) for the names of units; and some abbreviations for words other than names of units. An alphabetical index of recommended symbols for physical quantities is also included. This authoritative publication is available, price 9d. per copy or 12s. per 25 copies (post free), from the Royal Society, Burlington House, London, W.1.

Journal of Scientific Instruments

Contents of the December issue

ORIGINAL CONTRIBUTIONS

A new comparator of high precision for the measurement of line standards of length. By J. S. Clark, L. O. C. Johnson and V. W. Stanley.

An interferential lens centring lathe and edging machine. By G. A. Harle.

An apparatus delivering water at constant temperature. By E. R. Dymott and J. C. Evans.

The measurement of refractive index gradients in small glass specimens. By J. W. Gates.

Determination of mercury level in a steel-tube manometer. By R. Meakin.

An a.c. stabilizer. By B. Collinge and T. N. Marsham.

Notes on the ionization gauge. By L. Riddiford.

A repetitive spark source for shadow and Schlieren photography. By G. K. Adams.

CORRESPONDENCE

A frequency compensated direct-coupled amplifier for use with a four-channel pen recorder. From D. C. Pressey: J. A. Tanner and B. G. V. Harrington.

Filling procedures and performance of bromine-quenched G. M. counter tubes. From W. R. Loosemore: D. H. Le Croisette and J. Yarwood.

LABORATORY AND WORKSHOP NOTES

A simple thermistor bolometer for ultra-violet radiometry. By R. E. Dodd.

A rubber gasket cutter. By P. W. Allen, H. Mackle and L. E. Sutton.

Indicator for non-luminous flames. By C. Atkins and F. A. Lytton.

NOTES AND COMMENTS

Erratum

In Fig. 4 of the article *A three dimensional electrical potential analyser*, by S. C. Redshaw, published in the October issue, *for* Tier 1 *read* Tier 0 and in the legend to Fig. 5 *for* Tier 0 *read* Tier 1. The remainder of the legends to both figures should read: Model size: Delta, 12 mesh units semi-span, 12 mesh units root chord. Uniform voltage of 1·000 at Tier 9.

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